PLANE TRIGONOMETRY AND TABLES

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PLANE TRIGONOMETRY

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PLANE TRIGONOMETRY

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PREFACE

THE introduction into American textbooks of general or unified mathematics has opened the way for the exploitation of a wide variety of individual ideas. While there can be no doubt that some of these innovations have had a beneficial effect, others have led to mere eccentricity. It is perhaps due to this fact that so few teachers care to adopt the unified texts as they appear today.

This book is not radical. It is a painstaking effort to utilize all that is good in recent texts, to modernize the older ideas, and to moderate the newer ones.

We wish to call attention to the following outstanding features:

1. THE BREVITY OF THE TEXT. The conciseness of the treatment will commend itself to teachers.

2. THE FULL USE OF COÖRDINATES. The periodicity properties are discussed both in rectangular and in polar coördinates. So far as we are aware the use of polar coördinates to show the periodicity properties is new. We think it greatly to the student's advantage to become acquainted with both systems early in his training.

3. THE USE OF THE THEORY OF PROJECTION. The remarkably easy and beautiful proof of the addition theorems is thereby made possible.

4. NO UNNECESSARY DUPLICATION OF MATERIAL. This is accomplished by the early introduction of the definitions of the trigonometric functions for the general angle.

5. THE INTRODUCTION OF HISTORICAL NOTES. These are presented wherever they would seem to be of especial interest. This has been done in such a way as not to interrupt the continuity of the text.

6. CAREFULLY SELECTED EXERCISES. A list of approximately eight hundred exercises is given. These have been gathered from many fields and are calculated to give breadth of training as well

Grace

as to develop the power of the student to think independently and to cultivate in him an inquiring mind. The text has been planned to meet the needs of students in both engineering and the liberal arts, as is evidenced by the wide variety of applications.

7. Accuracy of Numerical Results. Methods of detecting gross errors in the solution of triangles and complete methods of checking are provided, as well as precise statements regarding the degree of accuracy of results.

8. The Form of the Tables. While no great change in the form of the tables has been introduced, considerable study has been made with reference to type and spacing in order to make the tables easy and pleasant to use. The accompanying explanations are clear and concise. These tables may be obtained separately or bound with the text according to the needs of the instructor.

Herbert E. Buchanan
Pauline Sperry

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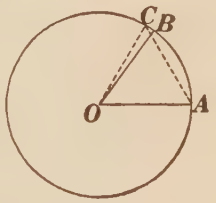
CHAPTER I

ANGLES AND THEIR MEASURES

1. Radian. A **radian** is an angle, with its vertex at the center of a circle, subtended by an arc equal to the radius of the circle. In the adjoining figure the arc AB is the same length as the radius OA. Obviously a radian is slightly less than 60°, for if the radius is used as a chord, the central angle is 60°. In the figure the chord AC equals the radius OA and the angle AOC is 60°.



Since the radius is contained in the circumference 2π times, it follows that there are 2π radians in 360 degrees. We have, therefore, 2π radians $= 360°$.

Hence $$\textbf{1 radian} = \frac{180°}{\pi} = \textbf{57°.2958},$$

and $$\textbf{1°} = \frac{\pi}{180}\ \textbf{radians} = \textbf{.017453 radians}.$$

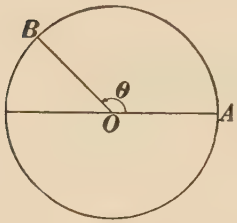
These equations enable us to change from one unit of measure to the other.

Thus $$30° = 30 \times \frac{\pi}{180}\ \text{radians} = \frac{\pi}{6}\ \text{radians} = .52360\ \text{radians},$$

or $$\frac{7\pi}{6}\ \text{radians} = \frac{7\pi}{6} \times \frac{180°}{\pi} = 210°.$$

2. A central angle and its arc. We find the number of radians in a central angle by dividing the length of its arc by the radius.

(Why?) In the adjoining figure let θ be the number of radians in angle AOB. Let s be the length of the arc AB and let r be the radius of the circle. Then from the definition of a radian,

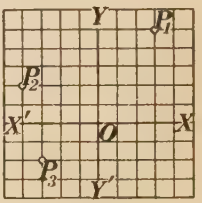


$$\theta = \frac{s}{r} \text{ (when } \theta \text{ is expressed in radians),}$$

or $$s = r\theta.$$

This formula stated in words is: *The length of any arc of a circle equals the radius times the radian measure of its central angle.* The formula is not true if θ is expressed in degrees.

3. Rectangular axes. Let $X'X$ and $Y'Y$ be two straight lines at right angles to each other. Then the position of any point in the plane of $X'X$ and $Y'Y$ may be determined by its perpendicular distances from $X'X$ and $Y'Y$.



In this figure the point P_1 is 3 units to the right of $Y'Y$ and 5 units above $X'X$. How may the position of the points P_2 and P_3 be described?

The two lines $X'X$ and $Y'Y$ are called **axes**. The line $X'X$ is the ***x*-axis** and $Y'Y$ is the ***y*-axis**. The two lines together make a set of **rectangular axes**. Their point of intersection is the **origin**.

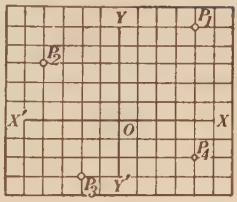
4. Coördinates. The distances of a point from the axes are called the **coördinates** of the point. The distance from the y-axis, measured along or parallel to the x-axis, is the **abscissa**. The distance of a point from the x-axis is called its **ordinate**. Thus in the above figure the coördinates of P_1 are (3, 5), the abscissa being 3 and the ordinate 5.

In writing the coördinates of a point we always write the abscissa first and the ordinate second with a comma between and enclose the pair in parenthesis. Thus (5, 3) means the coördinates of a point 5 units to the right of the y-axis and 3 units above the x-axis.

5. Signs. When a point is to the right of the y-axis, its abscissa is positive; when it is to the left of the y-axis, its abscissa is negative.

Similarly the ordinates of points above the x-axis are positive and the ordinates of points below the x-axis are negative.

In the adjoining figure the coördinates of P_1 are $(4, 5)$; of P_2 $(-4, 3)$; of P_3 $(-2, -3)$. What are the coördinates of P_4?



EXERCISES

Plot the following points:

1. $(1, 2)$, $(3, -2)$, $(-3, 2)$, $(-3, -2)$, $(\frac{1}{2}, \frac{3}{2})$.

2. $(-7, -1)$, $(0, 5)$, $(5, 0)$, $(-5, 3)$, $(-\frac{5}{2}, -\frac{2}{3})$.

3. Where are all the points whose abscissas are a? Does the number a have a sign? May a be -3?

4. Where are all the points whose ordinates are b? Does b have a sign? May b equal 2? -4?

5. Where do all points lie whose abscissas are zero?

6. The coördinates of a point P are $(4, 2)$. From P lines are drawn perpendicular to the axes and to the origin. If each line segment is produced its own length, what are the coördinates of the other ends? These points are said to be **symmetric to** P with respect to the axes and origin.

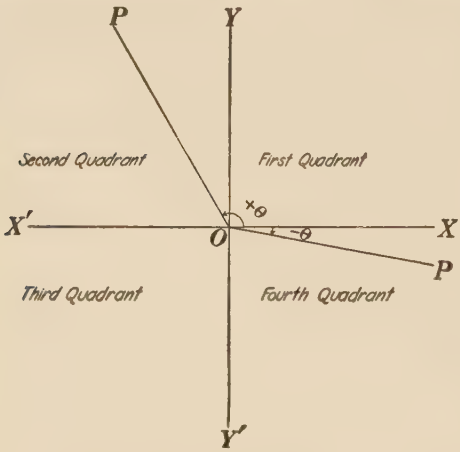
7. Where are all points for which $y = -2$? $y = k$?

6. General numbers. We denote a general point by two general numbers x and y. Thus the point $P(x, y)$ may be any point in the plane. The numbers x and y carry their own sign. They may assume any real values whatever, either positive or negative, integral or fractional, rational or irrational.

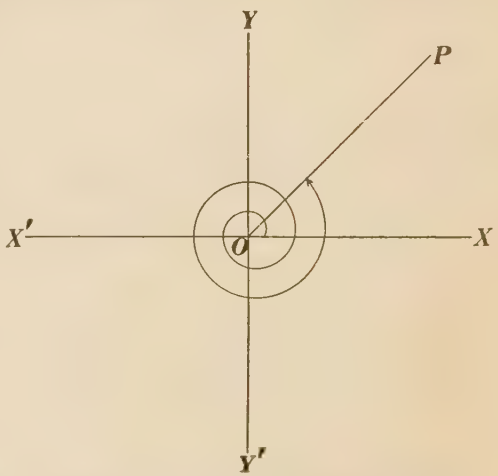
7. Angles by rotation. Draw a set of rectangular axes. They divide the plane into four parts called **quadrants**. The part above the x-axis and to the right of the y-axis is called the first quadrant. The second quadrant is above the x-axis and to the left of the y-axis. The third and fourth quadrants are below the x-axis as indicated in the figure on the next page.

We think of an angle as being generated by revolving a line OP about O from an initial position coincident with the positive end of the x-axis. We call OX the **initial side**, and OP the **terminal side** of the angle XOP. Any angle may be so placed that

its vertex is at O and its initial side on OX. It is then said to be in **standard position**. If an angle is generated by revolving OP counter-clockwise, we call it a **positive** angle. If OP is rotated clockwise, the angle generated is **negative**. The figure shows both a positive and a negative angle. An angle in standard position is said to be an angle of the first, second, third, or fourth quadrant according to the quadrant in which its terminal side lies. Thus $\theta = 290°$ and $\theta = -78°$ are both angles of the fourth quadrant.

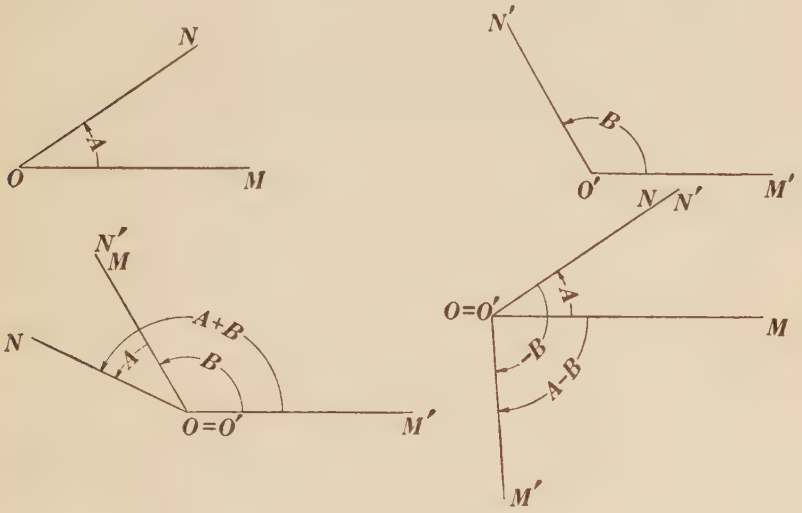


8. Angles of any magnitude. In plane geometry we used angles less than 360°. It will be necessary to extend our notion of angles so as to include angles of any size, either positive or negative. The explanation in the preceding paragraph enables us to do this for we may rotate the line OP from its initial position OX in a positive or negative direction so as to produce an angle of any size, even greater than 360°. The figure adjoining shows how we may generate an angle of $+ 765°$.



9. Graphical addition of angles. To add angle A to angle B, *place A in standard position. Place the initial side of B on the terminal side of A, their vertices coinciding, and read from the initial side of A to the*

terminal side of B. This rule serves whatever the signs of A and B. In the adjoining figures let MON be angle A and $M'O'N'$ be angle B. Then $N'O'M'$ is $-B$.



Hence $A + B = B + A = M'O'N' + MON = M'ON$ and $A - B = A + (-B) = MON + N'O'M' = MOM'$.

10. EXERCISES

Change to radians:

1. 20°, 40°, 45°, 90°, 135°, − 60°, 180°, 270°.
2. 10°, 750°, 480°, − 150°, 215°, 615°, 315°.

Change to degrees:

*3. $\pi, \frac{\pi}{3}, 2, 3, \frac{\pi}{4}, \frac{5\pi}{6}, \frac{9\pi}{5}, -\frac{2\pi}{3}, 4\pi, \frac{3\pi}{2}, \frac{\pi}{2}, -\frac{5\pi}{3}, \frac{7\pi}{4}$.

4. How many degrees in an arc of 20 feet on a circle with radius 10 feet?

[1]5. How many degrees in an arc of 4,728 miles on a great circle of a sphere with radius 3,963 miles?

[1]6. Through how many radians does a point on the earth's equator move in five hours?

* When no denomination is indicated it is understood that the unit of measurement is a radian.

7. The velocity of a point on the circumference of a wheel is 1500 feet per second. If the radius of the wheel is 10 feet, how many revolutions is it making per second?

8. In what quadrants are the terminal sides of the following angles? Draw an approximate figure in each case.

(*a*) 29°, 140°, − 85°, − 280°, 317°.
(*b*) $\frac{1}{2}, \frac{4}{3}, -\frac{8}{3}, -\frac{9}{4}, \frac{7}{3}, 6, -5$.

9. Develop a formula similar to that in article **2** when the central angle is expressed in degrees.

10. Construct the angles A and B and find graphically $A + B$, $A - B$, $-A + B$, $-A - B$ in the following:

(*a*) $A = 45°, B = 30°$
(*b*) $A = 30°, B = 60°$
(*c*) $A = 120°, B = 90°$
(*d*) $A = 180°, B = 120°$

11. If D is the number of degrees in an angle and R the number of radians prove that $\frac{D}{180} = \frac{R}{\pi}$.

12. What angles do the hour and minute hands of a clock describe in five hours and twenty-five minutes? In fifty hours and forty minutes?

13. If θ is a negative acute angle, in what quadrant is 2θ? 3θ? $\frac{4\theta}{3}$? $-\frac{\theta}{2}$? Is more than one answer possible in each case?

14. If a wheel makes 40 revolutions a second, how large an angle in radian measure does its radius describe in one minute?

15. How long does it take the minute hand of a clock to turn through an angle of 5°? 3 radians?

16. On a mountain road a railway track describes a semicircle with a radius of one mile. What is the speed of the train if it has turned through an angle of 5° in 25 seconds?

17. Two angles are **complements** if their sum is 90°. They are **supplements** if their sum is 180°. Find the complements and supplements of the angles in exercises 1 and 2.

CHAPTER II

MEASUREMENTS AND COMPUTATIONS

11. The meaning of trigonometry. The word trigonometry means the measure of a triangle. As the name indicates, the subject was developed to furnish arithmetical means for the solution of triangles. Important as this still is, we shall see that modern trigonometry includes many other considerations.

12. The need for new methods. Consider the right triangle ABC with right angle at C. Let the hypotenuse be c and the remaining sides a and b. From plane geometry we know that

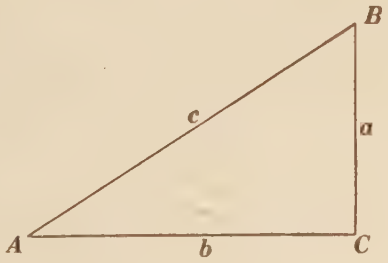
$$A + B = 90°$$

and

$$a^2 + b^2 = c^2. \quad \text{(Theorem of Pythagoras)}$$

The first relation enables us to find one acute angle when the other one is given. The second provides a means of computing any one of the three sides if two sides are given. Nothing that we have learned in plane geometry will enable us to find one acute angle when two sides are given. A new method is needed. We develop in this chapter a method for finding all the parts of a right triangle when any two parts are given, provided one of the known parts is a side.

13. The trigonometric ratios of an acute angle. Draw a right triangle ABC, letting C be the right angle, and a, b, and c, the sides. The side a is said to be **opposite** the angle A, the side b is said to be **adjacent** to A. We define the trigonometric ratios as follows:



The **sine** of A is the opposite side divided by the hypotenuse.

The **cosine** of A is the adjacent side divided by the hypotenuse.

The **tangent** of A is the opposite side divided by the adjacent side.

The **cotangent** of A is the adjacent side divided by the opposite side.

The **secant** of A is the hypotenuse divided by the adjacent side.

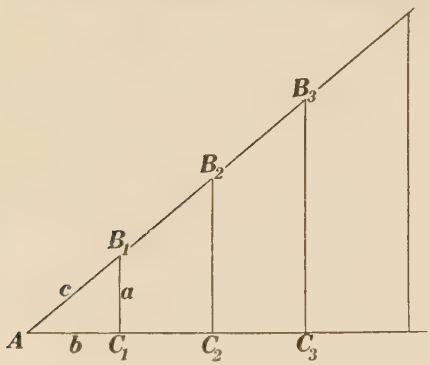
The **cosecant** of A is the hypotenuse divided by the opposite side.

We may write these definitions in symbols as indicated below:

$$\sin A = \frac{a}{c}, \cos A = \frac{b}{c}, \tan A = \frac{a}{b},$$

$$\csc A = \frac{c}{a}, \sec A = \frac{c}{b}, \cot A = \frac{b}{a}.$$

14. The trigonometric ratios as functions of A. Construct an angle A. Choose any number of points B_1, B_2, B_3 $\cdots$ on one side and drop perpendiculars to the other side. Thus we form any number of right triangles AB_1C_1, AB_2C_2, AB_3C_3 $\cdots$ and all of them are similar. But by plane geometry the ratios of corresponding sides of similar triangles are equal. Hence



$$\sin A = \frac{C_1B_1}{AB_1} = \frac{C_2B_2}{AB_2} = \frac{C_3B_3}{AB_3} \cdots,$$

$$\cos A = \frac{AC_1}{AB_1} = \frac{AC_2}{AB_2} = \frac{AC_3}{AB_3} \cdots,$$

$$\tan A = \frac{C_1B_1}{AC_1} = \frac{C_2B_2}{AC_2} = \frac{C_3B_3}{AC_3} \cdots.$$

Similar statements may be written for csc A, cot A, and sec A.

It follows that the six trigonometric ratios are unchanged by any change in the length of the sides of the triangle so long as the angle A is unchanged. If the angle A is changed, then the ratios are changed. For if not, we should have two right triangles in which the ratios of corresponding sides are equal. The triangles would then be similar and the corresponding angles equal.

Now one variable is said to be a **function** of a second variable if for every definite value assigned to the second variable the first assumes one or more definite values. By this definition sin A, cos A, tan A, cot A, csc A, sec A, are functions of the angle A for when A is assigned a value, the six ratios each assume a definite value. For this reason they are spoken of as **trigonometric functions.**

15. The reciprocal relations. From the definitions of article **13** we see at once that

$$\sin A = \frac{1}{\csc A}, \text{ or } \csc A = \frac{1}{\sin A}.$$

For $$\sin A = \frac{a}{c} \text{ and } \frac{1}{\csc A} = \frac{1}{\frac{c}{a}} = \frac{a}{c}.$$

Similarly $$\cos A = \frac{1}{\sec A}, \text{ or } \sec A = \frac{1}{\cos A},$$

and $$\tan A = \frac{1}{\cot A}, \text{ or } \cot A = \frac{1}{\tan A}.$$

These are called the **reciprocal relations.** By means of these relations the secant, cosecant, and cotangent may be found when the cosine, sine, and tangent are known, and conversely.

16. The functions of complementary angles. In the right triangle ABC let us apply the definitions of article **13** to the angles A and B. We find

$$\sin A = \frac{a}{c}, \quad \sin B = \frac{b}{c},$$

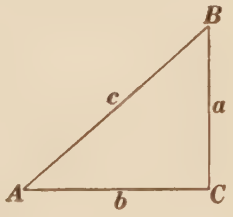
$$\cos A = \frac{b}{c}, \quad \cos B = \frac{a}{c},$$

$$\tan A = \frac{a}{b}, \quad \tan B = \frac{b}{a},$$

$$\cot A = \frac{b}{a}, \quad \cot B = \frac{a}{b},$$

$$\sec A = \frac{c}{b}, \quad \sec B = \frac{c}{a},$$

$$\csc A = \frac{c}{a}, \quad \csc B = \frac{c}{b}.$$



Then the following relations appear:

$$\sin B = \cos A,$$
$$\cos B = \sin A,$$
$$\tan B = \cot A,$$
$$\cot B = \tan A,$$
$$\sec B = \csc A,$$
$$\csc B = \sec A.$$

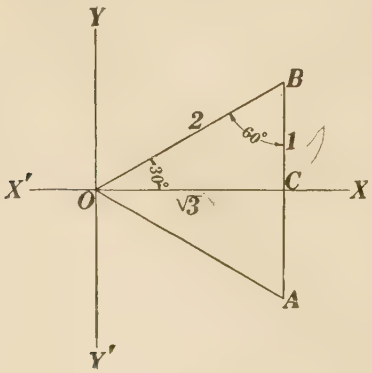
But $B = 90^\circ - A$, therefore we may write the above relations

$$\sin (90^\circ - A) = \cos A,$$
$$\cos (90^\circ - A) = \sin A,$$
$$\tan (90^\circ - A) = \cot A,$$
$$\cot (90^\circ - A) = \tan A,$$
$$\sec (90^\circ - A) = \csc A,$$
$$\csc (90^\circ - A) = \sec A.$$

These formulas enable us to find the functions of every acute angle greater than 45° if we know the functions of all angles less than 45°. For example, $\cos 65^\circ = \sin 25^\circ$, $\sin 53^\circ = \cos 37^\circ$, $\tan 82^\circ = \cot 8^\circ$.

Ex. Sum up in a single rule the formulas of this section.

17. Historical note. The above relations are interesting from an historical standpoint also, for this relation $\sin(90^\circ - A) = \cos A$ gave rise to the name cosine. In the middle ages when Latin was the universal language of scholars the cosine, being the sine of the complement, was called **complementi sinus**, which was later abridged to **cosinus**, hence our cosine. The words cotangent and cosecant had a similar origin.

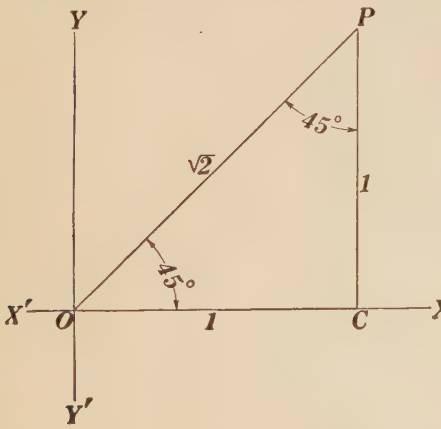


18. The functions of 30° and 60°. Before we take up the solution of right triangles it will help in certain problems to know how to find the values of the functions of a few special angles without using tables.

Construct an equilateral triangle with one vertex at the origin and with the x-axis bisecting the opposite side, as in the figure on page 10. It is obvious that $CB = \frac{OB}{2}$ and that angle $COB = 30°$. Now if we choose $OB = 2$, which we may do by article **14**, we find that $CB = 1$, $OC = \sqrt{3}$, and angle $B = 60°$. We may then read from the triangle OCB the functions of $30°$ and $60°$ as follows:

sin 30°	cos 30°	tan 30°	cot 30°	sec 30°	csc 30°
$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	$\sqrt{3}$	$\frac{2\sqrt{3}}{3}$	2
cos 60°	sin 60°	cot 60°	tan 60°	csc 60°	sec 60°

19. The functions of 45°. Through the origin draw a line making an angle of $45°$ with the x-axis. From a point C on the x-axis one unit from the origin draw a perpendicular to the x-axis to cut the $45°$ line at P. By plane geometry theorems we find that $CP = 1$ and $OP = \sqrt{2}$. Then the values of the functions of $45°$ read from the triangle OCP, are



$$\sin 45° = \cos 45° = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2},$$

$$\tan 45° = \cot 45° = 1,$$

$$\sec 45° = \csc 45° = \sqrt{2}.$$

20. EXERCISES

1. Express as functions of the complementary angles:

$\cos 20°$, $\cos 30°$, $\sin 60°$, $\tan \frac{\pi}{4}$, $\sin \frac{\pi}{6}$, $\sin 76° 10'$, $\tan 37°$, $\tan \frac{\pi}{3}$, $\cos \frac{5\pi}{12}$.

2. Express as functions of an angle less than $45°$:

$\cos 65°$, $\sin \frac{\pi}{3}$, $\tan \frac{2\pi}{5}$, $\sin 73° 15'$, $\cos 61° 17'$.

3. Given $\sqrt{3} = 1.7321$ and $\sqrt{2} = 1.4142$, find the values of the following functions to four decimal places and save your results for later verification:

$\sin 60°$, $\cos 45°$, $\tan \frac{\pi}{6}$, $\cos 60°$, $\cot \frac{\pi}{3}$, $\tan \frac{\pi}{3}$, $\sin 30°$, $\sec 60°$, $\csc 30°$, $\sec 45°$, $\sin 45°$, $\tan \frac{\pi}{4}$.

4. Is $\tan 60° = 2 \tan 30°$?
Is $\sin 60° = 2 \sin 30°$?
Is $\cos 60° = 2 \cos 30°$?

What conclusion may you draw from the answers to these questions?

Find the value of:

5. $\sin 60° \cos 30° + \cos 60° \sin 30°$.

6. $\cos 60° \cos 30° - \sin 60° \sin 30°$.

7. Prove that $\frac{2 \tan 30°}{1 - \tan^2 30°} = \tan 60°$. *

8. Prove that $\sin 60° = 2 \sin 30° \cos 30°$.

9. Prove that $\cos 60° = \cos^2 30° - \sin^2 30°$.

10. Construct the angle θ if $\sin \theta = \frac{3}{5}$; if $\cos \theta = \frac{7}{12}$.

11. Construct the angle θ if $\tan \theta = \frac{1}{2}$; if $\sec \theta = \frac{3}{2}$.

12. Evaluate $\sin 30° \cos 45° - \cos 30° \sin 45°$.

13. Which is smaller, $\sin 32°$ or $\cos 32°$? $\tan 50°$ or $\cot 50°$? Why?

14. What is θ if $\sin \theta = \cos \theta$?

15. Is the equation $\sin 2\theta = 2 \sin \theta \cos \theta$ true if $\theta = 30°$?

16. Is the equation $\sin^2 \theta + \cos^2 \theta = 1$ true if $\theta = 30°$? $45°$? $\frac{\pi}{3}$?

17. Find A, B, C if

$$\sin A = \cos 2A$$
$$\tan B = \cot 4B$$
$$\sec 3C = \csc 6C.$$

18. Construct the angles $15°$ and $75°$ in standard position. In each case choose some point on the terminal side, measure its coördinates and distance from the origin, and determine the values of trigonometric functions.

19. Give an example from everyday experience of a variable which is a function of another variable.

20. Draw a right triangle ABC. Denote AC by $\cos A$ and AB by 1. Then $CB = \sqrt{1 - \cos^2 A}$. Show that all the trigonometric functions may be expressed in terms of the cosine as follows:

$$\sin A = \sqrt{1 - \cos^2 A}, \tan A = \frac{\sqrt{1 - \cos^2 A}}{\cos A}, \cot A = \frac{\cos A}{\sqrt{1 - \cos^2 A}},$$

$$\sec A = \frac{1}{\cos A}, \csc A = \frac{1}{\sqrt{1 - \cos^2 A}}.$$

* $\tan^2 \theta$ is written for $(\tan \theta)^2$.

21. In a similar fashion by the proper naming of the sides as in the last exercise, express all of the functions in terms of the sine; in terms of the tangent.

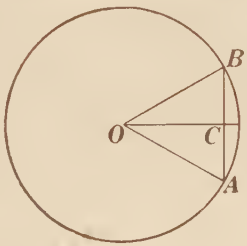
22. By keeping AB constant in length but allowing it to rotate about A show that as the angle A increases the sine, tangent, and secant increases. Use the reciprocal formulas to show the variation in the cosine, cotangent, and cosecant.

23. Find an acute angle A if $\sin A = \cos(60° + A)$. [*Hint.* This relation is satisfied if A is the complement of $60° + A$.] Find A if $\sin A = \cos 3A$; $\tan(45° + A) = \cot 4A$; $\sec(20° + A) = \csc A$.

21. Tables of the trigonometric functions. Since $\sin A = \frac{a}{c}$, we have $a = c \sin A$, and since $\cos A = \frac{b}{c}$, $b = c \cos A$. Then we may find either leg of a right triangle if we know the length of the hypotenuse and the values of $\sin A$ and $\cos A$. Now the values of $\sin A$ and $\cos A$ may be found if A is known, but the method of computation is too difficult to present in an elementary course.

We give a table of values of $\sin A$, $\cos A$, $\tan A$, and $\cot A$ for values of A at intervals of one minute from 0 to 90°. These values are called the **natural functions** to distinguish them from the logarithms of these functions which we study later. They are given in Table II of the tables which accompany this book.* The values of $\sec A$ and $\csc A$ are omitted because they are not often used. They may be found by taking the reciprocals of the values of $\cos A$ and $\sin A$ respectively.

22. Historical note. The use of tables of the natural functions dates back to the second century A.D. when Hipparchus, a Greek astronomer, computed a table of the lengths of chords subtended by central angles. This table answered the same purpose as a table of sines. Now, instead of using the whole chord, we use the half chord CB and associate it with the half angle COB. The tables of Hipparchus have been lost. The oldest tables now in existence are those of Ptolemaus, sec-



* Buchanan and Sperry, *Logarithmic and Trigonometric Tables.*

ond century A.D., which give the double sines, at intervals of half a degree, with an accuracy equivalent to five decimal places. Ptolemaus divided the degree into sixty equal parts and each of these again into sixty equal parts. The Romans referred to these sixtieths of the first and second kind as *partes minutae primae* and *partes minutae secundae.* Hence the origin of our minutes and seconds.

23. The use of the table of natural functions. It will be observed that the formulas of article **16** enable us to reduce the space necessary for printing the tables by exactly one half. Thus $\sin 32^\circ 12'$ and $\cos 57^\circ 48'$ are found in the same place in the table. Each column is doubly named, the names at the top of the page being used for angles less than 45° and the names at the bottom for angles greater than 45°. We illustrate the use of the tables by two examples.

(1) Find $\sin 32^\circ 12' 20''$. Turn to the page headed 32°. Run down the left column to $12'$; to the right under the column headed *sin* we find .53288. Just below we find $\sin 32^\circ 13'$ to be .53312, which is .00024 larger than $\sin 32^\circ 12'$. Since $32^\circ 12' 20''$ lies $\frac{1}{3}$ of the way from $32^\circ 12'$ to $32^\circ 13'$, we **assume** that $\sin 32^\circ 12' 20''$ lies $\frac{1}{3}$ of the way from $\sin 32^\circ 12'$ to $\sin 32^\circ 13'$. Hence

$$\sin 32^\circ 12' 20'' = .53288 + \tfrac{1}{3}(.00024) = .53296.$$

To save space in the table the decimal points for numbers less than 1 are omitted. When this occurs will be explained in the next article.

(2) Find $\cos 63^\circ 10' 45''$. Turn to the page with 63° at the bottom. Run up the extreme right-hand column to $10'$. On the same horizontal line in the column with *cos* at its foot we find $\cos 63^\circ 10' = .45140$ and just above it $\cos 63^\circ 11' = .45114$. Arrange the work as follows:

$$\cos 63^\circ 10' = .45140$$
$$\cos 63^\circ 10' 45'' = x$$
$$\cos 63^\circ 11' = .45114$$

Since $63^\circ 10' 45''$ is $\frac{3}{4}$ of the way from $63^\circ 10'$ to $63^\circ 11'$, x is placed $\frac{3}{4}$ of the way from .45140 to .45114. This gives $\cos 63^\circ 10' 45'' = .45121$. It will be noticed in the tables that as the angle grows

larger the sine and tangent increase. Hence the correction is added in these cases, while the cosine and cotangent decrease and therefore the correction is subtracted.

The process of finding the functions of angles between those given in the table is called **interpolation.**

24. Placing the decimal point. By definition $\sin A = \frac{a}{c}$ and $\cos A = \frac{b}{c}$. Since the hypotenuse is always greater than either leg, $\sin A$ and $\cos A$ can never be greater than 1. This means that except for the functions of 0° and 90°, which will be explained later, a decimal point belongs at the left of every number in the sine and cosine columns. Since $a \lessgtr b$ according as $A \lessgtr 45°$, it follows that up to 45° $\tan A$ is less than 1, but from 45° to 90° $\tan A$ is greater than 1. A decimal point is to be placed at the left of every number in the tangent column up to 45°.

What can be said about the decimal point in the cotangent column?

25. To find the angle from a given function. The inverse process of finding the angle when one of the functions of the angle is given is best explained by the following examples:

Ex. 1. Find the angle A if $\tan A = 1.8093$. Since $\tan A > 1$, $A > 45°$, we look in the column with *tan* at the bottom of the page and find that 1.8093 is between 1.8090, which is the tangent of 61° 4′ and 1.8103, which is the tangent of 61° 5′. The difference between these two is .0013. The difference between $\tan A$ and $\tan 61° 4'$ is .0003. Therefore $\tan A$ is $\frac{3}{13}$ of the way from $\tan 61° 4'$ to 61° 5′. Hence

$$A = 61° 4' + \tfrac{3}{13} \text{ of one minute}$$

or

$$A = 61° 4' 14''.$$

Ex. 2. Find angle A if $\cos A = .14390$. We look in either of the columns headed *sin* or *cos*, until we find the numbers .14407 and .14378. The difference between the two is .00029. The difference between $\cos A$ and the cosine of *the smaller angle* is .00017. Hence

$$A = 81° 43' 35''.$$

How could we tell by inspection that A is greater than $45°$ and therefore that we should look in the column of cosines for angles at the bottom of the page?

EXERCISES

1. Find the sine, cosine, and tangent of the following angles: $16° 20'$, $37° 15' 10''$, $84° 0' 30''$.

2. From the formulas of article **15** find the secant and cosecant of the angles in exercise 1.

3. Find all the functions of $23° 6'$, $45° 15' 50''$, $75° 59' 25''$, $56° 0' 12''$.

4. The method of interpolation explained above is called **straight line interpolation**. State in words the principle on which the method is based. Draw a diagram illustrating the principle. Is this principle exact?

5. Find all the functions of $8° 18' 18''$, $81° 41' 42''$, $25° 16' 40''$, $64° 73' 20''$, $17° 11' 27''$.

6. Check your results in example 3, article **20**, by comparison with values found from the tables.

7. One form of **protractor** is a graduated semicircle used in measuring angles. Provide yourself with a protractor and construct an angle of 20°. As in the figure in article **14** use it as the common angle of three similar triangles, having the adjacent side, the opposite side, and the hypotenuse successively equal to unity. Two ratios whose denominators are unity may now be read from each triangle. By measuring the lengths of the sides in the numerators construct a table of functions of 20°. Compare your results with the values as given in Table II.

8. Show that the equations in exercises 15 and 16 of article **20** hold if $\theta = 40° 58' 31''$.

9. By examining the tables describe the behavior of the sine and tangent as the angle increases. What can you say from your conclusions as to the behavior of the co-named functions? Check your answers by the tables. How do the secant and cosecant vary as the angle increases?

26. Solution of right triangles by natural functions. We are now able to find all the sides and angles of any right triangle if any two sides are given or if one acute angle and one side are given. This may be done by using three of the five equations,

$$\sin A = \frac{a}{c},\ \cos A = \frac{b}{c},\ \tan A = \frac{a}{b},\ \cot A = \frac{b}{a},\ A + B = 90°,$$

selecting always a formula which has only one unknown in it. To check use a relation involving the three parts found. We illustrate with two examples.

Ex. 1. Given the triangle ABC, right-angled at C with $a = 20$ and $A = 27°$. Solving the equation $\sin A = \frac{a}{c}$ for c, the unknown, we find

$$c = \frac{a}{\sin A}.$$

From the tables $\sin A = \sin 27° = .45399$.

Then by division $c = 44.054$.

Since c is now known, we might use $\cos A = \frac{b}{c}$, for this equation has only one unknown, b.

But since an error in c would make b wrong, it is better to use a formula for b in terms of the parts given, that is, $\tan A = \frac{a}{b}$ or $\cot A = \frac{b}{a}$, and since multiplication is easier than division, the latter is preferable.

From the tables $\cot A = \cot 27° = 1.9626$.

Multiplication by a gives

$$b = 39.252,$$
$$B = 90° - A = 63°.$$

To check we substitute our results in

$$\sin B = \frac{b}{c}.$$

From the tables, $\sin B = .89101$.

By multiplication $c \sin B = 39.253$. (See article **27**.)

But this is the value of b found above. The formula

$$c^2 = a^2 + b^2$$

may also be used as a check, but this does not check angle B. The chances of error in B are, however, slight in this case.

Ex. 2. Given $a = 31$ ft., $b = 19$ ft.

The formula $\tan A = \frac{a}{b}$ has only one unknown in it, namely, the angle A. Let the student fill in the numbers from the tables.

By division $\frac{31}{19}$, $\tan A =$ 1.6311 .

From the tables we find that A lies between 58° 30′ and 58° 31′

Interpolation gives

$$A = 58^\circ 29' 16''.$$

Then $$B = 90^\circ - A = 31^\circ 30' 44''.$$

We may find the side c from either the sine or cosine. We choose to use the sine. (Why?)

$$\sin A = \frac{a}{c}, \quad c = \frac{a}{\sin A}.$$

From the tables $\sin A =$ .852553 .

Then by division $c =$ 36′ 4″ .

Check by substituting in the relation

$$c^2 = a^2 + b^2.$$

1322.04 = 1322

In order to save one multiplication we write $a^2 = c^2 - b^2$ or $a^2 = (c - b)(c + b)$. Substituting the values of a, b, and c, we find $a^2 =$ 961 , $(c - b)(c + b) =$ 935 .

A complete form like the above, including the check, should be made out in every case before starting the solution.

EXERCISES

In the following triangles right-angled at C, two parts are given. Compute the remaining parts and check. Before applying a numerical check the solution should be inspected to see if it is a reasonable one. A rough check may be had by comparing with a triangle drawn to scale from the given data.

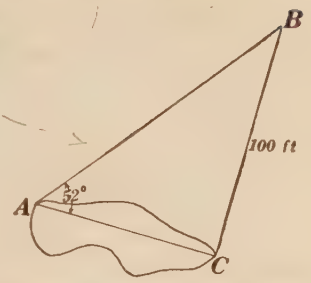
1. $a = 75$ ft., $A = 31°$.
2. $a = 60$ ft., $A = 67° 10'$.
3. $b = 57.65$ ft., $A = 53° 55'$.
4. $a = 19.6$, $b = 36.01$.
5. $a = 681$ cm., $c = 950.2$ cm.
6. $b = 94.27$ in., $c = 124.2$ in.
7. $B = 18° 13' 6''$, $c = 1015$ yds.
8. $c = 187$ miles, $A = 5° 11' 12''$.

9. When the sun was 50° above the horizon, the shadow of a tree was found to be 87 feet long. How tall is the tree?

10. The side of a regular pentagon is 47.2 feet. Find the circumferences of the inscribed and circumscribed circles.

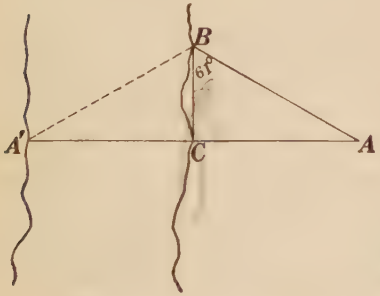
11. The slope of a roof is 3 inches in each horizontal foot. What angle does it make with the horizontal?

12. To measure the distance across a pond a surveyor constructed a right triangle as shown in the figure. Compute AC.



13. In exercise 12 compute AC if $A = 47°$ and $BC = 216$ feet.

14. In the figure below compute the distance across the stream if $BC = 96$ feet, angle $A'BC$ = angle $CBA = 61°$, and angle $C = 90°$. Can the distance $A'C$ be found by measuring CA?



15. A railroad track makes a 12° angle with the horizontal. How many feet does it rise in a thousand feet? What angle would the track make with the horizontal if it rose half as many feet in the same distance?

16. The altitude of an isosceles triangle is 60 and the base 20. Find the length of the equal sides, the vertical and the base angles.

17. The area of an isosceles triangle is 100 square feet and its base is 20 feet. Find its other sides and angles.

18. The vertical angle of an isosceles triangle is 30° and one of its equal sides is 15. Find the other parts.

19. Two men 700 feet apart and facing each other see an aeroplane at angles of elevation of 45° and 60° respectively. How high is the aeroplane?

20. An aeroplane flying at the rate of 80 miles per hour when observed was directly overhead at an altitude of 1000 feet. How far from the observer was it one minute later?

27. Measurement of lines. Significant figures. One of the most common operations of builders, contractors, and tradesmen is the measuring of a distance with some convenient unit. The tradesman usually uses a two-foot rule or a yardstick, the contractor, a 100-foot tape, while a trained engineer, in order to get greater accuracy, uses a steel tape which has been compared with some standard measure.

There are many ways in which errors may be made, but perhaps they are most commonly due to the following:

(*a*) the direction of the tape may not be the same as the direction of the line to be measured;

(*b*) the tape may not be held horizontal all the time or may not be stretched just right;

(*c*) unevenness of the surface on which the measurement is made.

When dealing with absolute numbers, 5. and 5.0 mean the same thing. But this is not the case when dealing with approximate data such as is afforded by measurements. The number of figures represents **the degree of accuracy**. Thus $a = 5$ means accuracy to **the nearest unit**, that is, $4.5 < a < 5.5$. $a = 5.0$ means accuracy to **the nearest tenth** or $4.95 < a < 5.05$, and so on. We say that the first is **accurate to one significant figure** and that the second is **accurate to two significant figures**, that is, the total number of digits including zeros written down represents the number of significant figures with this exception: if the number is integral, any zeros occurring between the decimal point and the first digit must be discarded. Thus 52800. is a number of three significant figures *only*, unless it is specifically stated otherwise, but 5280.0 has five significant figures.

With the utmost care it is not possible to measure a distance of a mile without the possible error of .05 of a foot. That is, the distance measured may be 5280.05 feet or 5279.95 feet or any number in between the two. So that we should not be justified in representing such a measurement by more than five figures as even the fifth is in doubt.

An expert mechanic n making a delicate piece of machinery must often make a measurement accurate to the thousandth of an inch. He would therefore carry out his figures to include the third decimal place.

28. Measurement of angles. The tradesman will not often need to measure an angle, but for the engineer the measurement of angles is equally as important as the measurement of distances.

For this purpose the engineer uses a **transit**. This is a movable circular plate which rotates inside a horizontal graduated circle. There are many refinements on a good transit which enable us to read angles very accurately. But the ordinary instrument which

has no **vernier** does not permit us to regard our readings as accurate to nearer than one minute. This means, for example, that if by measurement we find an angle to be $31^\circ\ 10'$, it may be $31^\circ\ 11'$ or $31^\circ\ 9'$ or any angle in between. Some transits read to the nearest half minute or even to 10 seconds of an arc.

29. Measurement of inaccessible distances. It is obviously impossible to measure the distance from New York to Liverpool or the distance from the earth to the sun. Scientists resort to computations to find such distances, but they must always base their computations on some distance which has been measured.

30. The accuracy of the results. It is easy to see that in general a sum or difference, product or quotient of two approximate numbers can be regarded as accurate to no more places than the less accurate of the two numbers. (It is of course possible sometimes to obtain very accurate results from inaccurate data.) So it is a waste of time to make one measurement involved correct to more significant figures than another. If angles are involved, they must be measured to a degree of accuracy as indicated by the following table:

Number of significant figures in linear measure	1	2	3	4	5
Angles expressed to the nearest unit	1°	$30'$	$5'$	$1'$	$.1'$

We use five-place tables in this book so that angles may be computed with an error usually of no more than five or six seconds. But it must be remembered that the tables are more accurate in some parts than in others. For example, $\sin A = .99999$. Here A may be any angle from $89^\circ\ 42'$ to $89^\circ\ 49'$. In such cases tables of six or seven places are necessary. But care in the selection of a formula will often help to avoid difficulties of this kind.

In the problems in this chapter we assume the accuracy of the last column of the above table for the parts given so that the linear parts found may be assumed correct to five significant figures and the angles to tenths of a minute.

In the next chapter we shall see that it is possible to get a greater degree of accuracy. (See article **44**.)

31. EXERCISES

Solve the following right triangles:

1. $a = 5280.1$ ft., $A = 36°47'$.

2. $a = 5279.9$ ft., $A = 36°45'$.

3. In exercises 1 and 2 of this set what is the possible error in b when a may be incorrect as much as $\frac{1}{10}$ ft. and A may be incorrect $1'$?

4. Given $a = 840 \pm .42$ ft., $A = 63° 10' \pm 1'$. Solve the four right triangles given by taking all possible combinations of the double signs and find the possible variation in b and c.

5. Unless extraordinary care is taken in the measurement of a line, an error of $\frac{1}{20}$ of 1% of its length is not uncommon. The measurement of angles may introduce an error of $\frac{1}{2}$ of a minute. If such is the case in the following triangles, find as in exercise 4 the maximum error in all the parts computed.

(a) $A = 25° 17'$, $b = 274$ ft.
(b) $B = 64° 16'$, $c = 19.2$ cm.

6. A ship sails northeast 17 mi. It is then how far east of the starting point?

7. An isosceles triangle with base 6 in. is inscribed in a circle of radius 15 in. How far is it from the center to each side?

8. Find the length of the side of a regular pentagon inscribed in a circle of radius 60 ft.

9. Solve exercise 8 for a regular octagon.

10. Look up in an encyclopedia, or some other source, articles on the **transit** or **theodolite**, **vernier**, **micrometer**, **reading microscope**, **compass**, and give a report on these subjects to the class.

11. Given an acute angle $A = 26° 35'$ and the hypotenuse $c = 38$ ft. Find the remaining parts.

12. Find the perimeter and area of a regular pentagon inscribed in a circle of radius 10 ft.

13. There are four different cases which may arise in the solution of a right triangle. Show how to solve each case.

14. If $1435 < a < 1445$, and $2765 < b < 2775$, find the numbers between which ab lies. What conclusions can you draw from your results?

CHAPTER III

LOGARITHMS AND APPLICATIONS

32. The need for a new method of computation. We have learned how to solve a right triangle and how to make certain practical applications of it in computing distances and angles. The multiplications and divisions were sometimes laborious. For the longer computations with the oblique triangle this method becomes quite laborious. It is therefore desirable to devote some attention to labor-saving devices in computation.

The ancient peoples had no adequate notation for integers and they were particularly deficient in a notation for fractions. An account of the notations of the Babylonians, Egyptians, Greeks, and Romans may be found in Cajori's *History of Mathematics*. The greatest labor-saving device of all time was the invention of the zero and its use in writing numbers. This is due to the Hindus and was in their literature as early as 600 A.D. Soon after this an adequate notation for common fractions was developed, but neither was used extensively in Europe till after 1400 A.D.

The next step of great importance in abbreviating computations was made by Simon Stevin about 1550. He introduced the decimal fractions, and paved the way for Napier's logarithms. John Napier, a Scotch baron, gave to the world his first table of logarithms in 1614. It was the most remarkable invention since that of the Hindus, for it reduces multiplication to addition, division to subtraction, raising to a power to multiplication, and extraction of a root to simple division. We proceed to develop the theory and use of this remarkable invention.

33. Two related sequences. Consider the two sequences

0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12;
1, 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024, 2048, 4096.

The product of any two numbers in the lower sequence may be found by adding the numbers just above them, and then looking along the upper sequence till the number representing their sum is found; the product is underneath this sum. To find the product of 16 and 64, we add 4 and 6 and underneath 10 find the product, 1024. Similarly to divide, we subtract the number above the divisor from the number above the dividend; underneath the difference we find the quotient. For example, to divide 2048 by 256 we subtract 8 from 11 and under 3 we find the quotient 8. This provides an easy method of multiplying and dividing. A difficulty arises because the lower sequence is not dense enough. Many integers are omitted and all the fractions and irrationals. We notice that the numbers in the upper sequence form an arithmetic progression while the lower form a geometric progression. We can, therefore, make the sequence twice as dense by inserting arithmetic means between each pair of numbers in the upper sequence and geometric means between each pair in the lower sequence. Repeated applications of this principle will enable us to thicken the sequences to any desired degree.

The numbers in the upper sequence are called the **logarithms** of the numbers in the lower sequence. Since the lower numbers are powers of 2, we say that the upper numbers are **logarithms to the base 2.** A table of logarithms is nothing more than a correlation between two such sequences of numbers. By means of logarithms computations involving multiplications, divisions, raising to powers, or extracting roots are made to depend on simpler operations as indicated in the first part of this article.

34. Definition of $\log_a x$. If $a^y = x$, where a is a positive number different from 1, we say that $\log_a x = y$, or *the logarithm of any number* ***x*** *to base* ***a*** *is the exponent of the power that must be given to* ***a*** *to produce* ***x***.

This is in agreement with article **33**, for $2^3 = 8$, then $\log_2 8 = 3$. The base in article **33** is 2 but the base that is ordinarily used is 10 which will be found to be more convenient since our numbers are arranged by multiples of 10. Thus since $10^3 = 1000$, we say $\log_{10} 1000 = 3$.

EXERCISES

1. When 2 is the base, what are the logarithms of 4, 16, 2, 1, 64, $\frac{1}{2}$, $\frac{1}{8}$, $8^{\frac{4}{3}}$?

2. Find the values of x which satisfy

$\log_3 x = 2$	$\log_3 27 = x$	$\log_x 16 = 2$
$\log_3 x = 3$	$\log_9 27 = x$	$\log_x 16 = 4$
$\log_a x = 1$	$\log_{27} 27 = x$	$\log_x 9 = \frac{1}{2}$
$\log_a x = 0$	$\log_{27} 1 = x$	$\log_x 4 = \frac{2}{3}$
$\log_9 x = \frac{1}{2}$	$\log_{\frac{1}{2}} 8 = x$	$\log_x 4 = -\frac{2}{3}$

3. Why do we exclude 1 as a possible base for a system of logarithms?

4. Why do we not use negative numbers for the base?

5. If the base is greater than 1, how does $\log N$ behave as N increases? If the base is a positive proper fraction, what can you say?

35. Theorems. *The logarithm of a product equals the sum of the logarithms of its factors.*

To prove $\log_a xy = \log_a x + \log_a y$.

Proof: Let $\log_a x = m$. Then $a^m = x$ (by the definition).

Let $\log_a y = n$. Then $a^n = y$.

By multiplication $a^{m+n} = xy$.

By definition $\log_a xy = m + n$.

Hence $\log_a xy = \log_a x + \log_a y$.

In a similar manner we may easily prove

$$\log_a \frac{x}{y} = \log_a x - \log_a y.$$

Hence, *the logarithm of a quotient equals the logarithm of the dividend minus the logarithm of the divisor.*

To prove $\log_a x^n = n \log_a x$.

We proceed as follows:

Let $\log_a x = m$. Then $a^m = x$ (by definition).

Now if n is an integer or a fraction or a negative number, we may raise both members of an equation to the nth power without destroying the equality. Hence

$$a^{mn} = x^n.$$

Then by the definition of a logarithm

$$\log_a x^n = mn,$$

or

$$\log_a x^n = n \log_a x.$$

We may state this result in words: *The logarithm of a power of a number equals the product of the logarithm of the number by the index of the power.*

These three theorems are all that are necessary for computation by means of logarithms. Note carefully that they allow us to perform the operations of multiplication, division, raising to powers, and extracting roots. *We cannot use logarithms in performing addition or subtraction.*

36. The characteristic and mantissa. The $\log_{10} 10 = 1$, since $10^1 = 10$. The $\log_{10} 100 = 2$, since $10^2 = 100$. $\text{Log}_{10} 1 = 0$, since $10^0 = 1$. Log $.1 = -1$, since $10^{-1} = .1$. To enable the student to visualize these facts we arrange them in the table that follows. Hereafter when the base is omitted, base 10 is understood.

No.	Log.
10000	4
1000	3
100	2
10	1
1	0
.1	− 1
.01	− 2
.001	− 3
.0001	− 4

If $10^x = y$, it is apparent that as y increases x increases or that the larger the number the larger its logarithm to the base 10. Hence, since $10 < 20 < 100$, $1 < \log 20 < 2$. Similarly $.001 < .002 < .01$, hence $-3 < \log .002 < -2$.

The logarithms of the powers of 10 are integers, but numbers between these powers have decimal fractions for their logarithms. Thus log 20 = 1.30103. The integral part of a logarithm is called the **characteristic**. The fractional part is the **mantissa**. The tables of logarithms are tables of mantissas. The mantissa is *always positive*. The characteristic is positive for numbers larger than 1. It is evident from the table of this article that *for numbers larger than 1 the characteristic is one less than the number of digits to the left of the decimal point.* Thus 934.27 lies between 100 and 1000. Its characteristic is, therefore, 2, which is one less than the number of digits to the left of its decimal point.

Since $.002 = 20 \times .0001$, $\log .002 = \log 20 + \log .0001 = 1.30103 + (-4) = .30103 - 3$. This could of course be written -2.69897, but that form would necessitate a table of *negative* mantissas for the logarithms of numbers less than 1. To avoid this we adopt the following rule: *For numbers less than 1, the charac-*

teristic is negative and numerically one more than the number of zeros between the decimal point and the first significant figure. Thus

$$\log .002 = .30103 - 3 \text{ and } \log .02 = .30103 - 2.$$

To avoid confusion in computations we add and subtract 10, writing such a logarithm, $\log .02 = 8.30103 - 10$.

Sometimes it is convenient to add or subtract some multiple of ten. Thus $\log .02 = 18.30103 - 20$ or $28.30103 - 30$.

Since there are no characteristics given in the table, the student must learn to use these two rules to find the characteristics.

37. Finding logarithms from the table. To find log 3716. Look in Table I in the column headed N for the first three digits. In the row with 371 under the column headed 6 we find *008. Hence the mantissa is .57008. Since there are four digits to the left of the decimal point in 3716, the characteristic is 3. Hence

$$\log 3716 = 3.57008.$$

Since $37160 = 10(3716)$,

$$\log 37160 = \log 10 + \log 3716 = 1 + \log 3716 = 4.57008.$$

Hence, moving the decimal point one place to the right increases the logarithm by one unit, that is, it changes the characteristic but not the mantissa.

Since moving the decimal point to the right or the left is equivalent to multiplying or dividing by a power of 10, it follows that any shift of the decimal point affects the characteristic but not the mantissa. From this we conclude that *the position of the decimal point determines the characteristic* but has no effect on the mantissa.

38. EXERCISES

Find from the tables the logarithms of the following numbers:

1. 621, 6.21, .621, .00621, 27.35, .083, .3138, .002735, 91800, .8089, .07293, 7290000, .516.

2. By the theorems of article **35** find the logarithms of $18.3 \times .9257$, $.0317 \times 8463$, $\frac{83371}{31736}$, $(621)^3$, $(276)^{\frac{1}{2}}$, $(.267)^3$, $\sqrt[3]{.0297}$.

[*Hint.* In finding $\log \sqrt[3]{.0297}$, add and subtract 20 from $\log .0297$ and divide by 3. Why?]

3. If $\log x = 1.81584$, find $\log x^2$, $\log x^{\frac{1}{2}}$, $\log \frac{1}{x}$.

4. If $\log x = 0.78104$ and $\log y = 2.47261$, find $\log xy$, $\log x^2y$, $\log x^2y^2$, $\log \frac{x}{y}$.

5. Find the logarithms of

$$\frac{62.07}{.0813} \times .1627, \sqrt{81.24}, \sqrt{\frac{64.08}{18.91}}, \sqrt[3]{1783}, \sqrt[3]{17.83}, \sqrt[4]{\frac{798.2}{16.83}}.$$

6. If $\log x = 9.81750 - 10$, find $\log x^2$, $\log \sqrt{x}$, $\log \sqrt[3]{x^2}$, $\log x^3$, $\log \frac{100}{x}$, $\log 100x$, $\log \frac{x}{100}$.

7. If $\log x = 1.21714$ and $\log y = 9.10721 - 10$, find $\log xy^3$, $\log \sqrt{xy^3}$, $\log x^{\frac{1}{3}}y^{\frac{2}{3}}$.

8. Prove that if a set of numbers are in geometric progression, their logarithms to any base are in arithmetic progression.

39. Finding the number corresponding to a given logarithm. To find the number whose logarithm is 1.37767. Look in the table for the mantissa .37767. The number corresponding is 23.86. The decimal point is placed between the 3 and 8 because the characteristic is 1 and hence there will be two digits in the number to the left of the decimal point.

The characteristic does not affect the digits. It determines only the position of the decimal point. For example, the number corresponding to the logarithm 3.37767 is 2386. If the logarithm is $7.37767 - 10$, the corresponding number is .002386.

EXERCISES

Find the numbers whose logarithms are:

1. 1.81584, 0.78104, $9.78104 - 10$, 3.42012, $8.42012 - 10$, 2.47261.

2. State rules for placing the decimal point in N, if the characteristic in $\log N$ is (*a*) positive or zero, (*b*) negative.

3. Find x having given:

$\log x = 3.12915$, $\log x = 7.81638$, $\log x = 0.00254$, $\log x = 9.00254 - 10$, $\log x^2 = 4.72173$, $\log \frac{1}{x} = 9.32171 - 10$, $\log x^2 = 9.91752 - 10$, $\log x = 18.61515 - 20$, $\log x = \frac{1}{3}(6.71272)$, $2 \log x = 9.91752 - 10$, $\log \sqrt{x} = 9.21032 - 10$.

40. Interpolation. If N contains more than four digits, it is necessary to interpolate to find $\log N$. We assume that when the differences are small, the mantissas vary directly as the numbers.

We will illustrate the process by an example. Let it be required to find log 47584.

$$\log 47580 = 4.67742 \text{ (from the table)},$$
$$\log 47584 = x \text{ (to be found)},$$
$$\log 47590 = 4.67752 \text{ (from the table)}.$$

Now 47584 is $\frac{4}{10}$ of the way from 47580 to 47590, so that by our assumption x is $\frac{4}{10}$ of the way from 4.67742 to 4.67752. The difference between these two logarithms is .00010 and is called the **tabular difference.***

So that $x = 4.67742 + .4(.00010) = 4.67746.$

In the correction to be added, any digit equal to or greater than 5 in the sixth decimal place is counted as one more in the fifth decimal place. Thus a correction of .000036 would be counted as .00004. Otherwise figures in the sixth place are discarded.

The above method of interpolation works equally well when the logarithm of N is given and we wish to find N.

Let it be required to find N if $\log N = 4.67746$.

$$4.67742 = \log 47580,$$
$$4.67746 = \log N,$$
$$4.67752 = \log 47590.$$

The tabular difference is 10 and the difference between $\log N$ and the logarithm of the smaller number is 4. Hence N lies $\frac{4}{10}$ of the way from 47580 to 47590, so

$$N = 47580 + \tfrac{4}{10} \times 10 = 47584.$$

EXERCISES

Find the value of:

1. $\log 87542$.
2. $\log .074506$.
3. $\log (28532)^2$.
4. $\log 37236$.

Find the number x corresponding to each of the following logarithms:

5. $\log x = 0.29225$.
6. $\log x = 9.69019 - 10$.
7. $\log x = 1.60314$.
8. $\log x = 8.89916 - 10$.
9. $\log x = 3.24715$.

* Usually in speaking of the tabular difference the ciphers preceding are omitted, since hundred-thousandths are understood in a five-place table. Thus here we should say the tabular difference is 10.

Find the value of:

10. $\log \frac{2^2}{(3.1476)^2} = 2(\log 2 - \log 3.1476)$.

[*Hint.* Since the logarithm of the numerator is smaller than that of the denominator, we add 10 to log 2 and then subtract it; that is, log 2 = 10.30103 − 10.]

11. $\log \frac{31.02}{84.716}$.

12. $\log \frac{(43521)^{\frac{1}{3}}}{10^{\frac{1}{2}}(26)^{\frac{2}{3}}}$.

13. $\log \sqrt{20)^3\sqrt{716.23}}$.

14. $\log \pi (11173)^2 \cdot 1000$.

15. $\log \frac{(400)^2(80)^{\frac{1}{2}}}{(25.015)^{\frac{3}{4}}}$.

16. $\log \frac{(.00852)^3(27416)^{\frac{1}{4}}}{(18256)^{\frac{1}{20}}}$.

17. Given log 2 = .30103, log 3 = .47712, log 7 = .84510, find the logarithms of the following numbers without the use of tables:

6, 4, 5, 8, 9, 12, 14, 15, 16, 18, 20, 21.

18. From the above data can you find log 11? log 13? log 17? log 19?

19. Why is $\log_a 1 = 0$?

41. Cologarithms. The **cologarithm** of a number x is the logarithm of its reciprocal, that is, it is defined by the equation,

$$\operatorname{colog}_a x = \log_a \frac{1}{x}.$$

Applying a theorem of article **35** to the right member, we get

$$\operatorname{colog}_a x = \log_a 1 - \log_a x = 0 - \log_a x = 10 - \log_a x - 10.$$

Hence the rule: *To find the cologarithm of a number subtract its logarithm from 10 − 10:*

Thus

$$\begin{aligned} \log 1 &= 10.00000 - 10, \\ \log 68.52 &= \underline{1.83582,} \\ \operatorname{colog} 68.52 &= 8.16418 - 10. \end{aligned}$$

This operation should, however, be performed mentally.

Cologarithms are used when there are more than two logarithms in the computation, so that we may always add instead of sometimes adding and sometimes subtracting. Thus instead of

$$\log \frac{(2)(17)}{175} = \log 2 + \log 17 - \log 175,$$

we write

$$\log \frac{(2)(17)}{175} = \log 2 + \log 17 + \operatorname{colog} 175.$$

This makes it possible to add the cologarithms of factors in the denominator to the logarithms of the factors in the numerator. Cologarithms offer no advantage if the division involves only two numbers.

Ex. Prove that the colog $x^m = m$ colog x.

42. Computation by logarithms. Let it be required to find the value of

$$K = \sqrt{\frac{(28176)(.073)}{\pi^2(68.52)}}.$$

We arrange the computation as follows:

$$\log K = \tfrac{1}{2}[\log 28176 + \log .073 + 2 \operatorname{colog} \pi + \operatorname{colog} 68.52],$$

$$\begin{aligned} \log 28176 &= 4.44988 \\ \log .073 &= 8.86332 - 10 \\ 2 \operatorname{colog} \pi &= 9.00570 - 10 \\ \operatorname{colog} 68.52 &= 8.16418 - 10 \\ 2)30.48308 &- 30 \\ 15.24154 &- 15 \end{aligned}$$

$$\log K = .24154$$

Therefore $$K = \sqrt{\frac{(28176)(.073)}{\pi^2(68.52)}} = 1.7439.$$

It is generally wise to make out a form for the work before using the tables at all. All of the work of interpolation should be done on separate sheets. If some of the factors are negative, we determine the sign of the result before doing any computing and then consider each number as positive.

In the following pages it will be understood that all numbers are correct to 5 places, though for convenience the zeros are omitted. Thus 110 should be written 110.00.

EXERCISES

Find the value of the following to five places:

1. $(8.7415)^3$.

2. $\dfrac{.0743}{.00812}$.

3. $\dfrac{27456(-71.802)}{(-215)^2}$.

4. $\sqrt[3]{\dfrac{3.14161(110)^2}{7.962}}$.

5. $\dfrac{(.8468)^{\frac{1}{7}}(.062815)^{\frac{3}{2}}}{(-.0008912)^3}$.

6. For wrought-iron columns the crushing weight P measured in pounds is given by

$$P = 299610\frac{d^{3.5}}{l^2}$$

where d is measured in inches and l is measured in feet. What weight will crush such a column if the diameter is 4.5 inches and the length is 20 feet?

7. The time of oscillation measured in seconds of a simple pendulum is given by

$$t = 2\pi\sqrt{\frac{l}{g}},$$

where l is the length in feet and $g = 32.16$ feet per second. Find the time of oscillation when $l = 3.5$ feet; when $l = 1.6$ feet.

8. The compound amount of P dollars for n years at rate i per cent per year is

$$S_n = P(1 + i)^n.$$

Find S_n of \$700.20 for $19\frac{1}{2}$ years at 4.25%.

9. By methods similar to those in article **35** show that

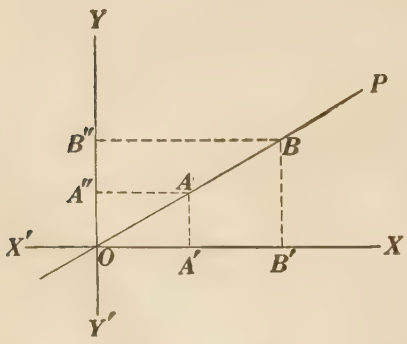
$$\log_b m = \log_a m \log_b a.$$

10. By letting $m = b$ in the last exercise prove that

$$\log_a b = \frac{1}{\log_b a}.$$

11. The Naperian base is $e = 2.7183$. Find $\log_e 243$.

43. Projections. Up to this point we have developed the theory of logarithms and shown how it may be used to shorten certain arithmetic computations. We now apply logarithms to the solution of right triangles and to some practical problems. For this purpose we introduce the notion of projection and define the trigonometric functions in a different manner.



If from the points A and B of the line OP we drop perpendiculars to the line OX, then $A'B'$ is the **projection** of AB on OX. Similarly the projection of AB on OY is $A''B''$. As a special case the projection of OB on OY is OB''. We notice that $OB'' = B'B$. The projections of any radius OB on the axes are therefore the coördinates of the point B.

Draw a set of rectangular axes and any line OB through the origin making an acute angle with the x-axis. The coördinates of B are (x, y). Let $OB = r$ and angle $XOB = \theta$, then

$$\sin\theta = \frac{y}{r}, \text{ or } y = r\sin\theta,$$

$$\cos\theta = \frac{x}{r}, \text{ or } x = r\cos\theta,$$

$$\tan\theta = \frac{y}{x}, \text{ or } y = x\tan\theta,$$

$$\cot\theta = \frac{x}{y}, \text{ or } x = y\cot\theta.$$

We may, therefore, consider $\cos\theta$ and $\sin\theta$ as the *numbers by which r must be multiplied to obtain its projections on the x- and y-axes respectively.*

44. Solution of right triangles by means of logarithms. We are already familiar with the tables of natural trigonometric functions, and have learned to solve right triangles with their aid. Much labor is saved if the computations are performed by means of logarithms. For this purpose Table III, a table of logarithms of the trigonometric functions, is given. Interpolation is based on the same principle as in the preceding pages, but it must be kept in mind that the tabular difference is *negative* in the case of the cosine and cotangent, since those functions decrease as the angles increase, which means that their logarithms decrease as the angles increase. From the tables we can find all the angles and all the sides of a right triangle if any two of its five parts are given, provided one of the given parts is a side. An example will make the method clear.

Given $r = 49.73$ and $x = 32.5$. To find all the sides and angles.

Solution:

$$\cos\theta = \frac{x}{r},$$

$$\log\cos\theta = \log x - \log r,$$

$$\log x = 11.51188 - 10$$

$$-\log r = \underline{\quad 1.69662}$$

$$\log\cos\theta = 9.81526 - 10$$

$$\text{and } \theta = 49^\circ\ 11'\ 32'',$$

$$
\begin{aligned}
90^\circ - \theta &= 40^\circ\,48'\,28''.\\
y^2 = r^2 - x^2 &= (r - x)(r + x).\\
r - x &= 17.23.\\
r + x &= 82.23.\\
\log(r - x) &= \ \ 1.23629\\
+\log(r + x) &= \ \ 1.91503\\
2\log y &= \ \ 3.15132\\
\log y &= \ \ 1.57566.\\
y &= 37.641.
\end{aligned}
$$

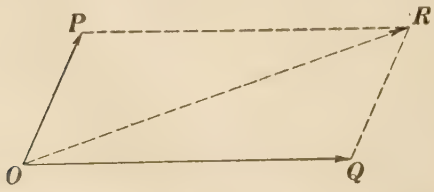
Check:

$$
\begin{aligned}
y &= r \sin\theta\\
\log y &= \log r + \log\sin\theta\\
\log r &= \ \ 1.69662\\
+\log\sin\theta &= \ \ 9.87904 - 10\\
\log y &= 11.57566 - 10
\end{aligned}
$$

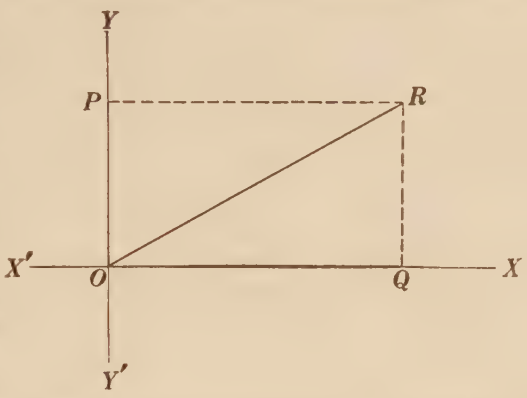
which is log y, as already found. The complete form for the solution should be made out before any logarithms are looked up. If the additions and subtractions are indicated as above by a + or a − sign preceding the left member of the equation, no error will occur through performing the wrong operation.

Angles found from tables of logarithms of the trigonometric functions are usually correct to within three or four seconds. In general, computations with logarithms yield more accurate results than those without, since a sum or difference of approximate numbers is more accurate than a product or quotient. (See article **30**.)

45. The parallelogram law. In certain applications of trigonometry we make use of a law of physics known as the **parallelogram law of forces.** This law is best explained by the adjoining figure. Let two forces be acting on the point O and let the magnitude and direction of these forces be represented by the directed line segments OQ and OP. If we complete the parallelogram, using OQ and OP as adjacent sides, then the two forces are



exactly equivalent to a single force represented in magnitude and direction by OR. Thus the resultant force is represented by the diagonal of the parallelogram. If OQ and OP happen to be perpendicular, then the parallelogram is a rectangle.



Conversely, if a single force OR is acting on the point O, this single force may be broken up into two forces which are represented by OP and OQ. The forces thus obtained are called the **components** of the force OR along the axes.

46. EXERCISES

Solve the right triangles below by logarithms and check all results:

1. $y = .343,\ x = .7215.$
2. $\theta = 28^\circ\ 10',\ x = 6274.$
3. $\theta = 44^\circ\ 28',\ r = 19.22.$
4. $y = 6.27,\ r = 11.64.$
5. $\theta = 63^\circ\ 18',\ y = 374.$

6. What is the height of a tree if its shadow is 37.4 feet long on the level ground when the sun is $63^\circ\ 18'$ high?

7. A wire is stretched from the top of a vertical cliff to the ground. The wire reaches the ground 7320 feet from the foot of the cliff which is 400 feet high. How long is the wire?

8. A flagpole 75 feet high casts a shadow 43.6 feet long on level ground. What is the altitude of the sun?

9. A flagstaff 20 feet long stands on the corner of a building 146.3 feet high. Find the angle subtended by the flagstaff from a point 127 feet from the foot of the building on level ground.

10. Find the projection of a line 62 feet long on another line making an angle of $32^\circ\ 10'$ with it.

11. Two forces of 9 pounds and 23 pounds directed toward the positive ends of the axes, are pulling on the origin. Find by means of the parallelogram law of forces the magnitude and direction of a single force equivalent to both.

12. A plank 18 feet long is inclined to the ground at an angle of 11° 13′. Find the length of its projection on the ground.

13. Find the projections of the sides of an equilateral triangle on the base if each side is 27.4 feet.

14. Find the components along the axes of a force of 136 pounds acting at an angle of 29° 16′ with the x-axis.

15. Find the effective force acting on a boat if a force of 500 pounds is exerted on a tow line which is inclined at an angle of 20° to the line of motion of the boat.

16. Find the length of a side of a regular pentagon inscribed in a circle of radius 500 feet.

17. A rope is attached to the top of a telegraph pole 30 feet above the ground. Two men standing on the ground 36 feet from the foot of the pole exert a pull of 150 pounds on the rope. What are the horizontal and vertical components of this force?

47. Summary and review exercises. This chapter deals with methods of computation. The fundamental theorems are:

$$\log_a xy = \log_a x + \log_a y,$$

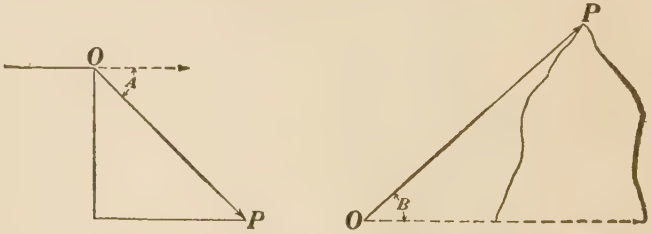
$$\log_a \frac{x}{y} = \log_a x - \log_a y,$$

$$\log_a x^n = n \log_a x.$$

In the last equation n may be positive, negative, integral, or fractional.

What are the rules for finding the characteristic of a logarithm? Define $\operatorname{colog}_a x$. What advantage is gained by the use of cologarithms? How do we calculate with negative numbers?

Define $\sin\theta$, $\cos\theta$. How are they related to the projection of one line on another? Make up from everyday experience some problems like those in article **46**, and solve them.



Isaac Newton said that the invention of logarithms multiplied the life of the astronomer by five. What did he mean?

The angle which the line of sight from an observer to an object makes with the horizontal is called the **angle of elevation or angle of depression** according as the object is above or below the observer. In the figure on page 36 angle A is an angle of depression and angle B is an angle of elevation of an object P observed from O.

EXERCISES

1. At a point 170 feet from the foot of a tower and on the same level with it the angle of elevation of the top was found to be $58° 21' 17''$. Find the height of the tower.

2. From the top of a cliff 120 feet above a lake the angle of depression of a boat on the lake was found to be $22° 16'$. What is the distance from the observer to the boat?

3. In problem 2 how far is it from the boat to a point on the level of the lake directly underneath the observer?

4. A road rises 278.5 feet in half a mile. Another road rises the same amount in 3176 ft. Which road is the steeper and by how much?

5. From a ship, sailing due east 7.6 miles per hour, a headland bears due north at 10 o'clock. At 1 o'clock it bears 37° west of north. How far is it from the ship each time?

6. A building 116.87 feet high casts a shadow 128.32 feet long on level ground. What is the altitude of the sun in degrees?

7. The radius of the earth at the equator is 3963 miles. How far apart, along the equator, are two points whose difference in longitude is .72 radians?

8. A flagpole 20 feet long stands on top of a tower 143.6 feet high. Find the angle subtended by the flagpole from a point 113 feet from the foot of the tower on level ground.

9. The slope of a roof is 35°. From the eaves to the ridge pole is 27.5 feet. Find the height of the ridge pole above the level of the eaves.

10. A circle of radius 38.2 feet is inscribed in a regular octagon. Find the length of a side.

11. What angle does a mountain slope make with the horizontal plane if it rises 190 feet in $\frac{1}{8}$ of a mile?

12. From the top of a hill the angles of depression of two consecutive milestones on a straight level road running due west were found to be 23° and 42°. How high is the hill?

13. A ship is sailing due south at 9 miles per hour. A lighthouse is observed due east at 10 A.M. The bearing of the same lighthouse at 12 : 10 P.M. the same day was N. 46° E. Find the distance from the lighthouse to the ship at 12 : 10 P.M.

14. The angle of elevation of the center of a spherical balloon 25 feet in diameter was found to be 56°. The angle which its diameter subtended at the same time was 2°. What was the height of the balloon?

15. A building 145 feet long is observed from a point directly in front of one corner. The length of the building subtends an angle whose tangent is 3 and the height subtends an angle whose tangent is 2. Find the height of the building.

16. What is the altitude of the sun when a radio pole 71.2 feet high casts a shadow 37.75 feet long?

17. What is the height of a balloon directly overhead when its angle of elevation from a point 8 miles away is 10° 30′?

18. From a point in the street between two buildings the elevation angles of the tops of the buildings are 30° and 60°. From a point 20 feet nearer the first building the angles of elevation are both 45°. Find the width of the street and the height of both buildings.

19. The angle of elevation of the top of a tower due north of an observer is 35°. What will be its angle of elevation from a point 250 feet due west of the first point of observation?

20. The length of string attached to a kite is 300 feet. The kite's angle of elevation is 46°. Find its height.

21. The angle of depression of a ship seen from an airship 2000 feet high is 70°. What is the straight-line distance between the two?

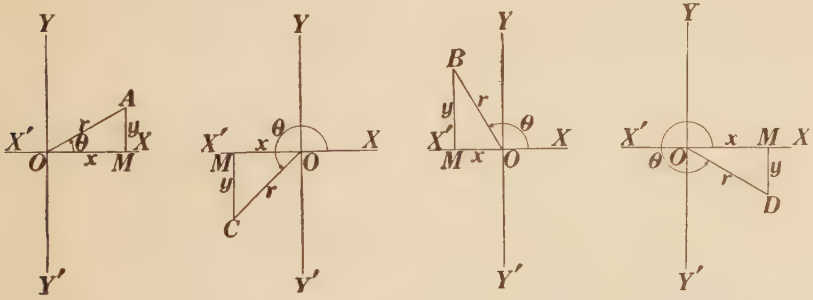
22. A railroad track after fifteen miles of level road takes a downward grade for five miles making an angle of 10° with the horizontal. The company wishes to reduce the grade and so decides to begin the downward grade five miles farther back. How much does this reduce the grade?

CHAPTER IV

FUNCTIONS OF ANY ANGLE

48. Need for an extension of the definitions of the trigonometric functions. In the first chapter we extended our notion of angles so as to include angles of any size either positive or negative. So now we extend our definitions of the trigonometric functions to cover cases when the angle is not acute. Need for this extension arises, for example, when we consider an oblique-angled triangle, or when trigonometry is applied to the solution of certain problems in physics and other sciences. And though we shall make but few such applications, we must pave the way for advancement in other fields.

49. Generalized definitions of the trigonometric functions. Draw any angle θ in standard position such as XOA, XOB, XOC, and XOD.



Take any point other than the origin on the terminal side of each of these angles. Let the coördinates of this point be (x, y) and its distance measured *from* the origin *to* the point be r, which when read in that direction is regarded as positive. The triangle xyr is

called the **triangle of reference**. Then in any quadrant the trigonometric functions of an angle θ are defined as follows:

$$\sin\theta = \frac{y}{r},$$

$$\cos\theta = \frac{x}{r},$$

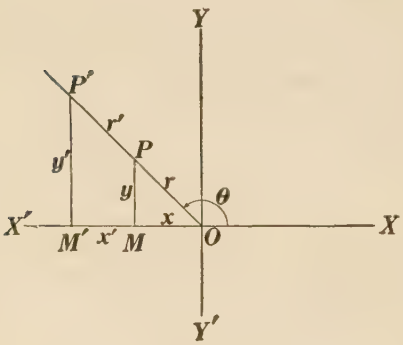
$$\tan\theta = \frac{y}{x},$$

$$\cot\theta = \frac{x}{y},$$

$$\sec\theta = \frac{r}{x},$$

$$\csc\theta = \frac{r}{y}.$$

When θ is acute, these definitions reduce to those already given in Chapter II, hence any relations deduced from these will include those already found for an acute angle as a special case. That the value of the function is the same if any other point is chosen on the terminal side of the angle is easily seen. For select any other point P' on the terminal side of θ. Let its coördinates be (x', y'), and its distance from the origin be r'. The triangle xyr is similar to $x'y'r'$. Therefore the ratios of corresponding sides are equal. Hence



$$\sin\theta = \frac{y'}{r'} = \frac{y}{r},\quad \cos\theta = \frac{x'}{r'} = \frac{x}{r},\quad \tan\theta = \frac{y'}{x'} = \frac{y}{x}.$$

Hence the six trigonometric functions are independent of the position of the point P. They depend only on the size of the angle θ. For any one angle θ there is just one value for each function. It is also evident that two angles having the same initial sides and the same terminal sides, though the angles themselves differ by multiples of 360°, have the same values for the trigonometric

functions. Thus $\sin 390° = \sin (360° + 30°) = \sin 30°$, $\tan (-780°) = \tan (-720° - 60°) = \tan (-60°)$, $\cos (-390°) = \cos (-360° - 30°) = \cos (-30°)$, so that functions of all angles numerically greater than 360° can be expressed as functions of angles numerically less than 360° by simply throwing out the multiples of 360°.

50. The reciprocal relations. From the definitions it is evident that the reciprocal relations for an acute angle given in article **15** hold when θ is any angle whatever. That is,

$$\sin\theta = \frac{1}{\csc\theta} \quad \text{or} \quad \csc\theta = \frac{1}{\sin\theta},$$

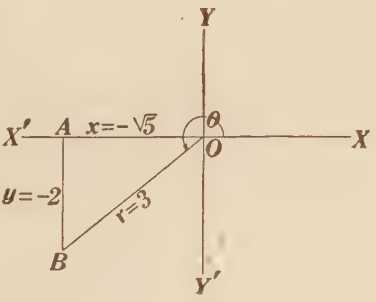
$$\cos\theta = \frac{1}{\sec\theta} \quad \text{or} \quad \sec\theta = \frac{1}{\cos\theta},$$

$$\tan\theta = \frac{1}{\cot\theta} \quad \text{or} \quad \cot\theta = \frac{1}{\tan\theta}.$$

51. Signs of the functions. In the first quadrant x and y are positive. Hence all the functions are positive. In the second quadrant x is negative and y is positive. Hence $\sin\theta$ and its reciprocal are positive, while the other four functions are negative. Verify the signs in the table below.

Quadrant	I.	II.	III.	IV.	
$\sin\theta$	+	+	−	−	$\csc\theta$
$\cos\theta$	+	−	−	+	$\sec\theta$
$\tan\theta$	+	−	+	−	$\cot\theta$

52. Determination of remaining functions if one is known. The six trigonometric functions are not independent, for if any one of them is given, together with the quadrant in which θ lies, we can find all the others. For example, let θ be in the third quadrant and let $\sin\theta = -\frac{2}{3}$. Construct a triangle of



reference, OAB, in the third quadrant with $y = -2$ and $r = 3$. Then $x = -\sqrt{5}$. Hence

$$\cos\theta = \frac{-\sqrt{5}}{3}, \quad \tan\theta = \frac{2}{\sqrt{5}},$$

$$\cot\theta = \frac{\sqrt{5}}{2}, \quad \sec\theta = \frac{-3}{\sqrt{5}},$$

$$\csc\theta = -\tfrac{3}{2}.$$

Any such problem can be solved in a similar fashion.

53. EXERCISES

Construct θ and find the values of the other five trigonometric functions:

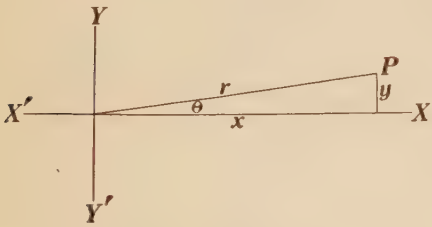
1. θ in the first quadrant and $\sin\theta = \frac{3}{5}$.
2. θ in the second quadrant and $\cos\theta = -\frac{4}{5}$.
3. θ in the fourth quadrant and $\tan\theta = -\frac{3}{4}$.
4. θ in the third quadrant and $\tan\theta = \frac{1}{2}$.
5. θ in the third quadrant and $\sec\theta = -\frac{3}{2}$.
6. θ in the second quadrant and $\csc\theta = \sqrt{3}$.
7. θ in the first quadrant and $\cot\theta = \sqrt{2}$.
8. θ in the third quadrant and $\sin\theta = -\frac{1}{\sqrt{2}}$.
9. θ in the second quadrant and $\tan\theta = -1$.
10. θ in the fourth quadrant and $\cos\theta = \frac{1}{7}$.
11. $\cos\theta = -\frac{1}{2}$ and $\sin\theta$ is positive.
12. $\sin\theta = -\frac{4}{5}$ and $\frac{3\pi}{2} < \theta < 2\pi$.
13. If θ is in the second quadrant and $\sec\theta = -2$, find the value of
$$\frac{\sec\theta + \tan\theta}{\sin\theta - \cos\theta}.$$
14. If θ is in the second quadrant and $\tan\theta = -\sqrt{5}$, find the value of
$$\sec\theta\tan^2\theta + 1.$$
15. Express as functions of an angle numerically less than 360°: $\sin 650°$, $\tan(-340°)$, $\cot\frac{16\pi}{4}$, $\cos(-1020°)$, $\cos\frac{13\pi}{3}$, $\cot 380°$, $\csc 560°$, $\sec\frac{9\pi}{4}$.
16. Show that the equation, $\sin\theta = 2$, is impossible.
17. Is the equation, $\tan\theta = 2$, impossible?
18. Determine the algebraic signs of $\cos 150°$, $\tan 315°$, $\sec 50°$, $\csc\frac{\pi}{4}$, $\cot\frac{3\pi}{4}$, $\sin\frac{5\pi}{6}$, $\sin(-225°)$, $\cot(-120°)$, $\sec(-60°)$.

19. Let θ be an obtuse angle in standard position and let θ_1 be the positive acute angle between the terminal side of θ and the x-axis. Show that the functions of θ are numerically the same as those of θ_1.

20. Make up an exercise similar to exercise 19 for the positive acute angle θ_2 between the terminal side of θ and the y-axis.

54. The functions of 0° and 90°. Some of the definitions of article **49** become meaningless when the terminal side of the angle θ falls on one of the coördinate axes. For in that case one of the numbers, x or y, becomes zero. This happens when θ is an integral multiple of 90° that is, when θ equals one of the so-called **cardinal angles**, $\pm \frac{n\pi}{2}$, where $n = 0, 1, 2\cdots$. We discuss these cases in detail.

Construct a small angle θ, positive or negative. Let us agree to keep r constant and let θ approach 0°. It is clear from the figure that y approaches zero as θ approaches 0°, while x approaches r.



We express this situation in mathematical symbols by the equation

$$\lim_{\theta \to 0} y = 0.$$

This is read " the limit of y as θ approaches 0 is zero."

Also $\lim_{\theta \to 0} x = r.$

In this notation we have

$$\sin 0^\circ = \lim_{\theta \to 0} \sin \theta = \lim_{\theta \to 0} \frac{y}{r} = \frac{0}{r} = 0.$$

Similarly,

$$\cos 0^\circ = \lim_{\theta \to 0} \frac{x}{r} = \frac{r}{r} = 1,$$

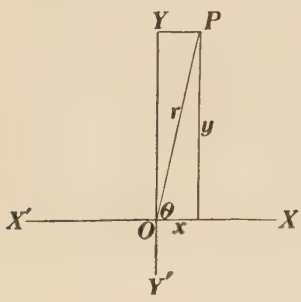
$$\tan 0^\circ = \lim_{\theta \to 0} \frac{y}{x} = \frac{0}{r} = 0,$$

$$\sec 0^\circ = \lim_{\theta \to 0} \frac{r}{x} = \frac{r}{r} = 1,$$

$$\csc 0^\circ = \lim_{\theta \to 0} \frac{r}{y} = \frac{r}{0} = \pm \infty,$$

$$\cot 0^\circ = \lim_{\theta \to 0} \frac{x}{y} = \frac{r}{0} = \pm \infty.$$

The symbol ∞ is read **infinity**. It is a symbol for a "state of affairs" and not a symbol for a number. The statements above mean that as y approaches 0, $\frac{r}{y}$ and $\frac{x}{y}$ grow large without bound.



If θ approaches zero through negative values, the lower signs in the last two equations must be chosen, since in that case y is always negative, while x is positive.

In a similar manner we may construct an angle θ slightly less or greater than 90° and find the functions of 90°. It is obvious from the figure that

$\lim_{\theta \to 90^\circ} x = 0$, $\lim_{\theta \to 90^\circ} y = r$. Hence

$$\sin 90^\circ = \lim_{\theta \to 90^\circ} \frac{y}{r} = 1, \qquad \csc 90^\circ = \lim_{\theta \to 90^\circ} \frac{r}{y} = 1,$$

$$\cos 90^\circ = \lim_{\theta \to 90^\circ} \frac{x}{r} = 0, \qquad \sec 90^\circ = \lim_{\theta \to 90^\circ} \frac{r}{x} = \pm \infty,$$

$$\tan 90^\circ = \lim_{\theta \to 90^\circ} \frac{y}{x} = \pm \infty, \quad \cot 90^\circ = \lim_{\theta \to 90^\circ} \frac{x}{y} = 0.$$

The table below gives a convenient way of remembering the sine and cosine of 0°, 30°, 45°, 60°, 90°. (See articles **18**, **19**.)

Angle	0°	30°	45°	60°	90°
$\sin\theta$	$\frac{1}{2}\sqrt{0}$	$\frac{1}{2}\sqrt{1}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}\sqrt{3}$	$\frac{1}{2}\sqrt{4}$
$\cos\theta$	$\frac{1}{2}\sqrt{4}$	$\frac{1}{2}\sqrt{3}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}\sqrt{1}$	$\frac{1}{2}\sqrt{0}$

55. EXERCISES

1. Show that the functions of 360° are the same as those of 0°.

2. By an argument similar to that in article **54**, find the functions of 180°, 270°.

3. Make a table of values of the functions of all the positive cardinal angles from 0° to 360°.

4. Is $\sin 90^\circ = 2 \sin 45^\circ$?

5. Find the value of $\dfrac{\left(\sec \frac{\pi}{3} - \sin \frac{\pi}{4}\right) \tan \frac{\pi}{4}}{\sec \frac{\pi}{6} + \csc \frac{\pi}{2}}$.

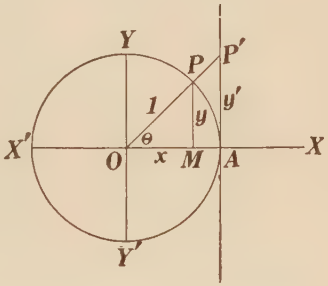
6. Find the value of $\sin 3\pi \cot \frac{3\pi}{2} - \tan \frac{5\pi}{6} \cos 5\pi$.

56. The line representation of the trigonometric functions. Draw a circle about the origin with radius 1 cutting the x-axis at A. Let θ be an acute angle in standard position. In article **49** we found that we could choose any point whatever on the terminal side to form the triangle of reference. Let us choose a point one unit from the origin, that is, on the unit circle.

Then

$$\sin \theta = \frac{y}{1} = y,$$

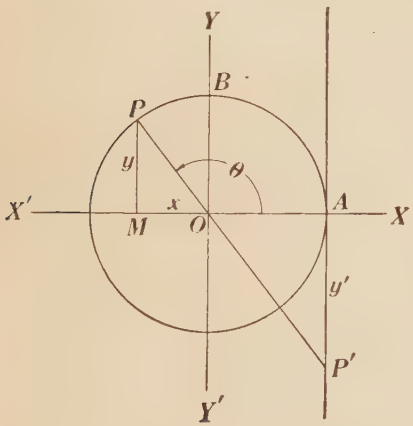
$$\cos \theta = \frac{x}{1} = x.$$



Now draw a tangent line at A. Since $\tan \theta = \dfrac{y'}{OA}$ and $OA = 1$, we have

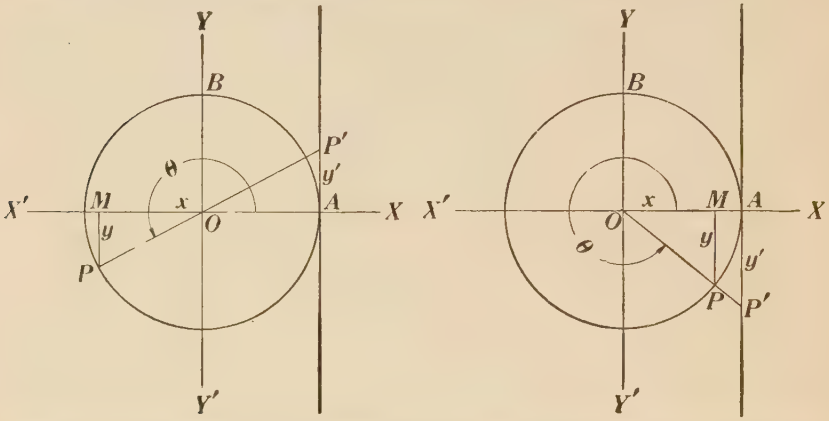
$$\tan \theta = y' \text{ and } \sec \theta = \frac{OP'}{OA} = OP'.$$

Now let θ increase. The point P' moves up the tangent line until $\theta = 90°$, when the terminal side of the angle becomes parallel to the tangent line. When θ becomes obtuse, the terminal side must be produced through the origin in order to meet the tangent line and P' will be below A. Since lines parallel to the axes are considered as having the same direction as the axes, this makes AP' negative, as it should be for the tangent of an angle of the second quadrant. Since OP is always positive, OP' will be negative when θ is between 90° and 180°.



It will be easily seen that the above discussion holds word for

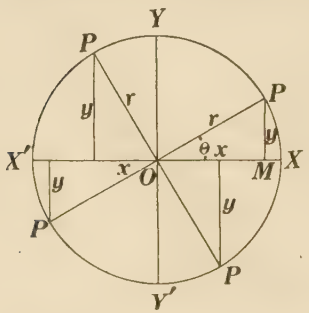
word when applied to the two following figures, that is, for *any* angle θ, $\sin\theta = MP$, $\cos\theta = OM$, $\tan\theta = AP'$, $\sec\theta = OP'$. So the trigonometric functions may be represented by line segments whose lengths in units of radius represent the numerical values of the functions and whose sign is determined by the direc-



tion of the segment. By measuring these line segments a table of the natural functions could be constructed.

These line values are used more or less throughout trigonometry, but they are introduced now so that we may the more easily trace the variation of the functions $\sin\theta$, $\cos\theta$, and $\tan\theta$ as θ increases.

57. The variation of $\sin\theta$, $0 \leqq \theta \leqq 2\pi$. In the adjoining figure let θ start from 0 and increase continuously to 2π. That means OP makes a complete revolution. To get a picture of the way in which $\sin\theta$ varies, we have only to watch the y in the adjoining figure. Evidently $\sin\theta$ starts at 0 and increases to 1 at $\theta = \frac{\pi}{2}$. As OP turns into the second quadrant $\sin\theta$ begins to decrease and continues to decrease till it reaches 0 at $\theta = \pi$. In the third quadrant $\sin\theta$ becomes negative and decreases to -1 at $\theta = \frac{3\pi}{2}$. From this point it begins to increase, reaching 0 at



$\theta = 2\pi$. In this interval sin θ has assumed all values from -1 to $+1$.

EXERCISES

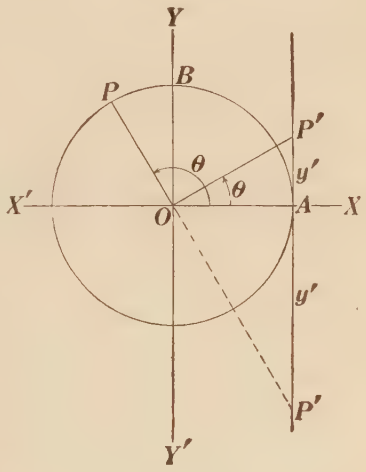
1. Describe the motion of the point M as OP revolves and trace the variation of cos θ by watching x change.

2. What is the least value sin θ may have? What are the greatest and least values of cos θ?

3. In what intervals is cos θ decreasing? sin θ increasing?

58. The variation of tan θ, $0 \leqq \theta \leqq 2\pi$. Since tan θ is measured on the tangent line at point A, we have to watch the variation of y' in the adjoining figure. Tan θ starts with 0 when θ is 0 and as θ increases, tan θ increases, at first slowly and then rapidly till, for

$$\theta = \frac{\pi}{2}, \tan\theta = +\infty.$$



Immediately after θ passes $\frac{\pi}{2}$, tan θ becomes negative and is measured below A. We say that tan θ is **discontinuous** at $\theta = \frac{\pi}{2}$ since it is not defined for that value. As θ increases through the second quadrant tan θ increases, passing through all the values from $-\infty$ to 0. As θ varies from π to 2π, tan θ repeats the values it had from 0 to π.

The name **tangent** was derived from the fact that it is possible to measure the values of the function by line segments on the tangent line.

59. EXERCISES

1. Examine the variations of sec θ for $0 \leqq \theta \leqq 2\pi$.

2. Draw a tangent to the unit circle at B and devise a method of representing csc θ and cot θ by line segments.

3. Discuss the variation of csc θ and cot θ for $0 \leqq \theta \leqq 2\pi$.

4. As θ increases, does the tangent ever decrease?

5. Make use of your conclusions in exercise 4 to describe the behavior of $\cot\theta$ as θ increases.

60. The functions of 120°, 135°, 150°. In figure A, let angle $MOP = 60°$ and angle $MOP' = 120°$. Then it is evident that $MP = M'P'$ and hence

$$\sin 120° = \sin 60° = \frac{\sqrt{3}}{2}.$$

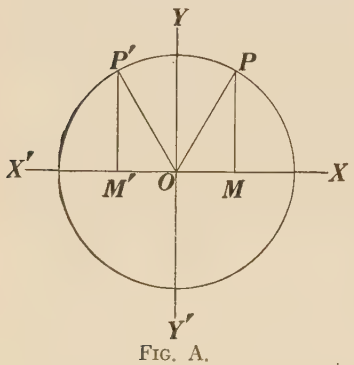


FIG. A.

Since OM' is negative,

$$\cos 120° = -\cos 60° = -\tfrac{1}{2}.$$

Similarly,

$$\tan 120° = -\tan 60° = -\sqrt{3}.$$

In figure B let angle $MOP = 45°$ and angle $MOP' = 135°$. Then, as above, we find

$$\sin 135° = \sin 45° = \frac{\sqrt{2}}{2},$$

$$\cos 135° = -\cos 45° = -\frac{\sqrt{2}}{2},$$

$$\tan 135° = -\tan 45° = -1.$$

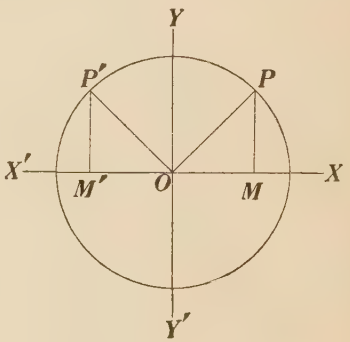


FIG. B.

Draw a figure for the angles 30° and 150° and prove that

$$\sin 150° = \sin 30° = \tfrac{1}{2},$$

$$\cos 150° = -\cos 30° = -\frac{\sqrt{3}}{2},$$

$$\tan 150° = -\tan 30° = -\frac{\sqrt{3}}{3}.$$

61. EXERCISES

Draw a figure and prove:

1. $\sin 210° = -\sin 30°$.
2. $\cos 210° = -\cos 30°$.
3. $\sin 225° = -\sin 45°$.
4. $\cos 225° = -\cos 45°$
5. $\tan 225° = \tan 45°$.

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6. By drawing figures as in the preceding exercises find all the functions of 240°, 300°, 315°, 330°.

7. Draw figures and find all the functions of 390°, 405°, 420°, 450°, 480°.

8. What negative angles have the same initial and terminal sides as those in exercises 6 and 7? What can you say therefore of the values of their trigonometric functions?

9. Determine without the use of the tables the value of the following:

(a) $\sin 120° \cos 30° - \cos 120° \sin 30°$.

(b) $\sin 315° \cos 45° + \cos 315° \sin 45°$.

(c) $\csc^2 240° \sin 300° \tan(-30°) + \sec^2 150° \cot 60° \cos^2 210°$.

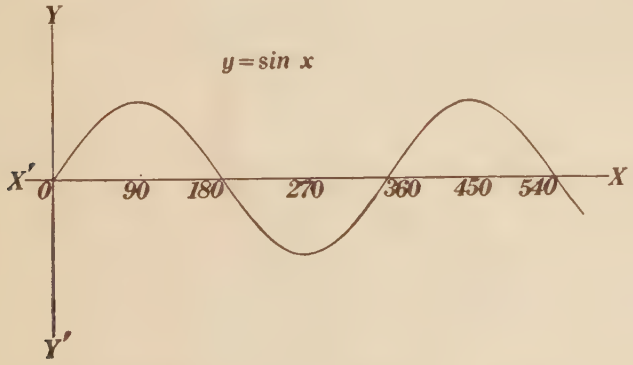
(d) $\dfrac{\tan 240° + \tan 60°}{1 - \tan 240° \tan 60°}$.

62. Graph of $y = \sin x$. To draw the graph of $y = \sin x$, we make a table of values using the results of the preceding articles.

x	0°	30°	45°	60°	90°	120°	135°	150°	180°	210°	225°	240°
$\sin x$	0	.5	.7	.9	1	.9	.7	.5	0	−.5	−.7	−.9

x	270°	300°	315°	330°	360°	390°	405°	420°	450°	480°	
$\sin x$	−1	−.9	−.7	−.5	0	.5	.7	.9	1	.9	

After plotting these values we draw a smooth curve through the points. The result is the wave curve shown below:



Ex. Plot the function $y = \sin x$ for negative values of x.

63. Periodic functions. A function that repeats its values at regular intervals is called a periodic function. Clearly $\sin x$ and

$\cos x$ are periodic. The period of each is $360°$ or 2π radians. All the trigonometric functions are periodic with period 2π, but it can be shown that $\tan x$ and $\cot x$ repeat their values at intervals of π, hence they have also the smaller period. (See article **58**.)

There are many periodic motions in nature that may be represented by trigonometric functions. The motion of waves on the surface of a liquid and the motion of a simple pendulum are examples of periodic motions. They may be discussed by means of sines and cosines.

EXERCISES

Plot the following:

1. $y = \cos x$.
2. $y = \tan x$.
3. $y = \sec x$.
4. $y = \csc x$.
5. $y = \sin x + \cos x$.
6. $y = \sin 2x$.
7. $y = 2 \sin x$.

8. How many of the functions above are continuous? How many are periodic?

9. What is the period of each of the functions? Is there any representation of the period in the graph?

10. Plot $y = \sin x + \sin 2x$.

11. Plot $y = 2 \sin x - \cos x$.

64. Trigonometric identities. In Chapters II and III we worked very largely with the computational side of trigonometry, developing a method of computing unknown distances and angles from certain known distances and angles. The present chapter treats $\sin\theta$, $\cos\theta$, $\tan\theta$, $\cot\theta$, $\sec\theta$, and $\csc\theta$ as functions of θ and exhibits the variation and periodicity of these functions by means of graphs. We now turn our attention to a different aspect of trigonometric analysis, namely, the relations between the functions.

In algebra a **conditional equation** is one that holds only if the unknown has certain special values. For example:

$$2x - 3 = 1$$

holds only if

$$x = 2,$$

and

$$x^2 - 3x + 2 = 0$$

holds only if

$$x = 2 \text{ or } 1.$$

But we found in algebra that there is another type of equation which holds for all values of the unknown, with the possible exception of certain isolated values. For example:

$$x^2 - 4 = (x - 2)(x + 2)$$

holds for all values of x.

Again $$\frac{x-1}{x} + \frac{1}{x+1} = \frac{x^2 + x - 1}{x(x+1)}$$

holds for all values of x except $x = 0$ and $x = -1$. These are called **identical equations** or **identities**. Every time we factor an expression or simplify an expression the sign of equality used expresses an identical equation. Both these types of equations arise in trigonometry. We call them **trigonometric identities** and **equations**. We consider here some of the fundamental trigonometric identities and their applications, reserving the treatment of trigonometric equations for a later chapter.

In article **50** we found that the reciprocal relations hold for any angle θ, that is,

(1) $\sin\theta \csc\theta = 1$,

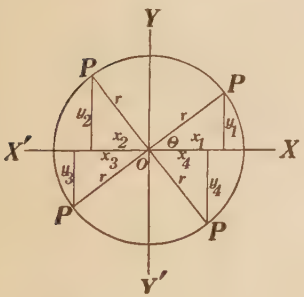
(2) $\cos\theta \sec\theta = 1$,

(3) $\tan\theta \cot\theta = 1$.

In any quadrant, even if x or y is negative, we have,

$$x^2 + y^2 = r^2.$$

Dividing successively by r^2, x^2, and y^2, we get



$$\frac{x^2}{r^2} + \frac{y^2}{r^2} = 1,$$

or (4) $\sin^2\theta + \cos^2\theta = 1$;

$$1 + \frac{y^2}{x^2} = \frac{r^2}{x^2},$$

or (5) $1 + \tan^2\theta = \sec^2\theta$;

and $$\frac{x^2}{y^2} + 1 = \frac{r^2}{y^2},$$

or (6) $\cot^2\theta + 1 = \csc^2\theta$.

Finally from the definitions

$$\frac{\sin\theta}{\cos\theta} = \frac{\frac{y}{r}}{\frac{x}{r}} = \frac{y}{r}\cdot\frac{r}{x} = \frac{y}{x} = \tan\theta,$$

or (7) $$\frac{\sin\theta}{\cos\theta} = \tan\theta.$$

These are seven fundamental identities or formulas, that is, they hold for all values of θ. They will appear in slightly different forms and combinations. For example, (4) may be written

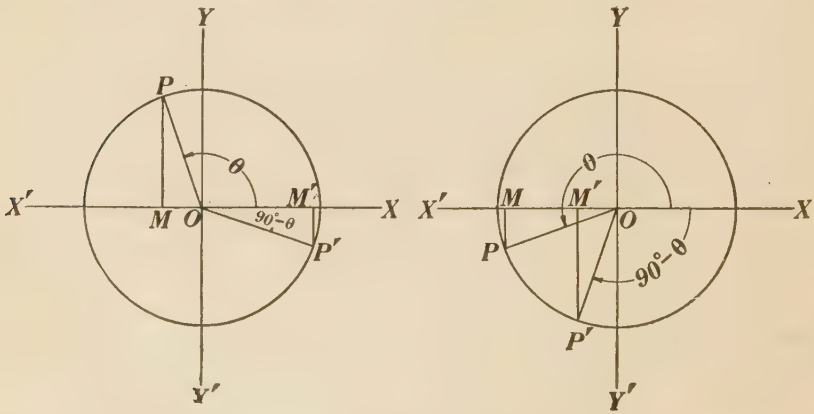
$$\sin^2\theta = 1 - \cos^2\theta \quad \text{or} \quad \sin\theta = \pm\sqrt{1-\cos^2\theta}$$

or $$\cos^2\theta = 1 - \sin^2\theta \quad \text{or} \quad \cos\theta = \pm\sqrt{1-\sin^2\theta}.$$

Again, (7) may be combined with (3) to give

$$\cot\theta = \frac{\cos\theta}{\sin\theta}.$$

65. The functions of $90^\circ - \theta$. In the chapter on the right triangle we derived formulas for the functions of $90^\circ - \theta$ when θ is acute. In the adjoining figures XOP represents angle θ and XOP',

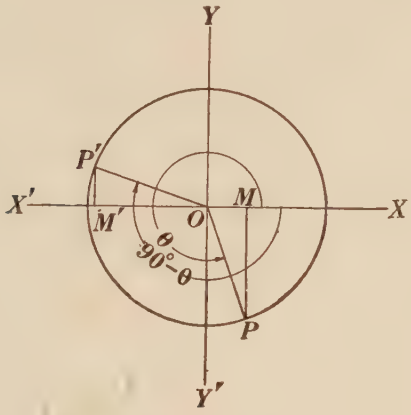


$90^\circ - \theta$, when θ is an angle of the second, third, or fourth quadrants, P and P' being points on the unit circle.

Since the triangles of reference xyr and $x'y'r'$ are congruent, having

the hypotenuse and an acute angle of one equal to the corresponding parts of the other, in each case $OM' = MP$ and $OM = M'P'$ or

$$x' = y, \qquad y' = x.$$



Hence

$$\begin{aligned}
&\sin(90^\circ - \theta) = \cos\theta,\\
&\cos(90^\circ - \theta) = \sin\theta,\\
&\tan(90^\circ - \theta) = \cot\theta,\\
&\csc(90^\circ - \theta) = \sec\theta,\\
&\sec(90^\circ - \theta) = \csc\theta,\\
&\cot(90^\circ - \theta) = \tan\theta.
\end{aligned}$$

The sine and cosine, the tangent and cotangent, the secant and cosecant are called **cofunctions** of each other.

Then the group of formulas above may be stated in words: *Any function of an angle is equal to the cofunction of its complement.*

66. One function in terms of another. By means of the formulas of article **64** it is always possible to express any function in terms of any other. For example:

$$\sec\theta = \frac{1}{\cos\theta} = \pm\frac{1}{\sqrt{1 - \sin^2\theta}}$$

which expresses $\sec\theta$ in terms of $\sin\theta$.

Again $\csc\theta = \pm\sqrt{1 + \cot^2\theta} = \pm\sqrt{1 + \dfrac{1}{\tan^2\theta}} = \pm\dfrac{\sqrt{1 + \tan^2\theta}}{\tan\theta}$

which expresses $\csc\theta$ in terms of $\tan\theta$.

Express each function in terms of the others and complete the following table:

	Sin θ	Cos θ	Tan θ	Cot θ	Sec θ	Csc θ
sin θ	$\sin\theta$					
cos θ	$\pm\sqrt{1-\sin^2\theta}$					
tan θ	$\pm\dfrac{\sin\theta}{\sqrt{1-\sin^2\theta}}$					
cot θ	$\pm\dfrac{\sqrt{1-\sin^2\theta}}{\sin\theta}$					
sec θ	$\pm\dfrac{1}{\sqrt{1-\sin^2\theta}}$					
csc θ	$\dfrac{1}{\sin\theta}$					

Ex. Check each column of this table by constructing a triangle. For example, the first column may be checked by constructing a right triangle with the hypotenuse equal to 1, the opposite side equal to $\sin\theta$, and the adjacent side equal to $\sqrt{1-\sin^2\theta}$.

67. Derived identities. An identity holds for all values of the unknown. We say that two expressions are identical if one can be transformed into the other. Any expression which can be deduced from the seven formulas of article **64** is therefore an identity. For example:

$$\tan\theta+\sec\theta=\frac{\cos\theta}{1-\sin\theta}.$$

Proof:

$$\begin{aligned}\tan\theta+\sec\theta&=\frac{\sin\theta}{\cos\theta}+\frac{1}{\cos\theta}\\&=\frac{\sin\theta+1}{\cos\theta}\\&=\frac{(1+\sin\theta)(1-\sin\theta)}{\cos\theta(1-\sin\theta)}\\&=\frac{1-\sin^2\theta}{\cos\theta(1-\sin\theta)}=\frac{\cos^2\theta}{\cos\theta(1-\sin\theta)}=\frac{\cos\theta}{1-\sin\theta}.\end{aligned}$$

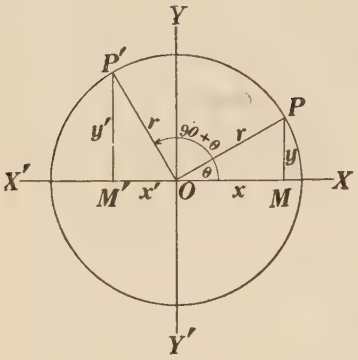
What suggested changing $\tan\theta$ and $\sec\theta$ into expressions involving sines and cosines in the first step? What suggested multiplying the terms of the fraction by $1 - \sin\theta$ in the third step? Notice the form of the work and model your work after it.

EXERCISES

By means of the seven formulas of article **64** and the laws of algebra transform the left members of the following equations into the right members:

1. $\sin\theta\cot\theta = \cos\theta.$
2. $\sin\theta\sec\theta = \tan\theta.$
3. $\cos\theta + \sin\theta = \cos\theta\,(1 + \tan\theta).$
4. $\dfrac{\sec^2 x}{\sin^2 x} = \csc^2 x + \sec^2 x.$
5. $\dfrac{\cos a}{1 - \sin a} = \dfrac{1 + \sin a}{\cos a}.$
6. $\sec^2 a - \csc^2 a = \tan^2 a - \cot^2 a.$
7. $\cos^2 a\,(1 + \tan^2 a) = 1.$
8. $\sec^2 a - 1 = \dfrac{\sin^2 a}{\cos^2 a}.$
9. $\sin(90^\circ - x)\sec x = \dfrac{1}{\cos x \sec x}.$
10. $(\tan\theta - \sin\theta)^2 + (1 - \cos\theta)^2 = (1 - \sec\theta)^2.$
11. $(\sin 30^\circ - \cos 60^\circ)(\sin 30^\circ + \cos 30^\circ) = 0.$
12. $(\cos 30^\circ - \sin 45^\circ)(\cos 30^\circ + \cos 45^\circ) = \sin^2 30^\circ.$
13. $\dfrac{\sin 90^\circ \cos 60^\circ + \tan 45^\circ \cos 0^\circ}{\sin 30^\circ \cos 30^\circ} = ?$
14. $(\cot x + \csc x)^2 = \dfrac{1 + \cos x}{1 - \cos x}.$
15. $(\sin x + \cos x)(\tan x + \cot x) = \sec x + \csc x.$
16. $\cos(90^\circ - \theta)\sec\theta = \tan\theta.$
17. Use the definitions of the trigonometric functions and the theorem of Pythagoras to verify the identities in the first eight exercises.

68. Functions of $90^\circ + \theta$. Construct the angles θ and $90^\circ + \theta$ with their triangles of reference. As in article **65** the triangles of reference MOP and $M'OP'$ are congruent. Here $x' = -y$ and $y' = x$.

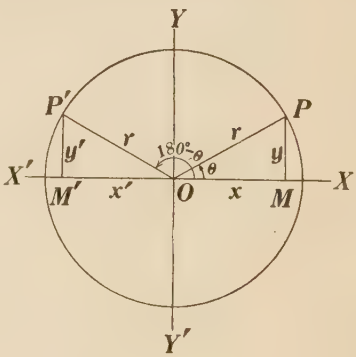


Hence $\cos(90° + \theta) = \frac{x'}{r} = -\frac{y}{r} = -\sin\theta,$

$\sin(90° + \theta) = \frac{y'}{r} = \frac{x}{r} = \cos\theta,$

$\tan(90° + \theta) = \frac{y'}{x'} = -\frac{x}{y} = -\cot\theta.$

69. Functions of $180° - \theta$. Construct the angles θ and $180° - \theta$ with their triangles of reference. Triangles MOP and $M'OP'$ are congruent as in the preceding section, but here $x' = -x$ and $y' = y$.



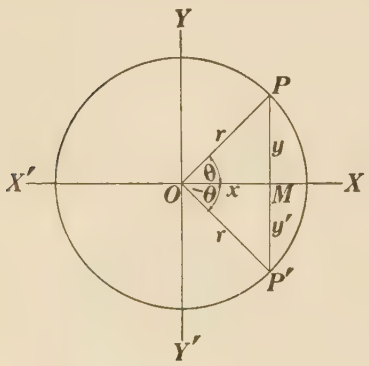
Hence

$\sin(180° - \theta) = \frac{y'}{r} = \frac{y}{r} = +\sin\theta,$

$\cos(180° - \theta) = \frac{x'}{r} = \frac{-x}{r} = -\cos\theta,$

$\tan(180° - \theta) = \frac{y'}{x'} = \frac{y}{-x} = -\tan\theta.$

70. Functions of $-\theta$. Construct the angles θ and $-\theta$ with their triangles of reference. Triangles MOP and MOP' are congruent. Here $y' = -y$ and $x' = x$. Hence



$\sin(-\theta) = \frac{y'}{r} = -\frac{y}{r} = -\sin\theta,$

$\cos(-\theta) = \frac{x}{r} = \cos\theta,$

$\tan(-\theta) = \frac{y'}{x'} = -\frac{y}{x} = -\tan\theta.$

The solution of any question of the type of those in this article and the two preceding it depends on one's ability to draw the angles and their triangles of reference, and to pair the sides with proper attention to signs. The formulas in the last three sections were proved for the case when θ is acute. It can be easily shown, by constructing figures for the cases when θ is an angle of the second, third, or fourth quadrant and suitably labeling them, that the above proofs hold word for word for any angle θ.

Ex. Use the reciprocal relations to derive three other formulas from each set given above.

The tables of trigonometric functions give the values of the functions for acute angles only. Hence it is necessary to make use of one of the above formulas to find the functions of angles greater than 90°.

For example:

$$\sin (116°) = \sin (180° - 64°) = \sin 64°.$$

71. EXERCISES

1. Draw figures and derive the formulas expressing the functions of $(180° + \theta)$ as functions of θ.

2. Draw figures and derive formulas expressing the sine and cosine of $270° - \theta$ as functions of θ. Use the fundamental identities to derive four other formulas from them.

3. Derive the formulas for the sine and cosine of $270° + \theta$.

4. Find the numerical values of the following from the tables:

$\sin 158° 20'$,	$\cos 158° 20'$,
$\log \sin 117°$,	$\log \sin 124° 50'$,
$\tan 137° 36'$,	$\cos 145\ 10'$.

Prove the following:

5. $\sin 150° = \sin 30° = \frac{1}{2}$.

6. $\cos 150° = -\frac{\sqrt{3}}{2}$.

7. $\tan 110° = -\tan 70° = -\cot 20°$.

8. $\sec 140° = -\csc 50° = -\sec 40°$.

9. $\sin x \cos (90° - x) - \cos x \sin (90° - x) = 2 \sin^2 x - 1$.

Using the formulas of the preceding articles, find the values of all the functions of the angles in the following exercises:

10. 120°, − 120°.
11. 135°, − 135°.
12. 150°, − 150°.
13. 210°, − 210°.
14. 225°, − 225°.
15. 300°, − 300°.
16. 315°, − 315°.
17. 330°, − 330°.

18. By logarithms find the value of

$$\frac{\sin 218^\circ\ 10'\cos 113^\circ}{\cos(-27^\circ\ 41')}.$$

19. Use the logarithm tables to evaluate

$$\frac{\tan(153^\circ\ 16')\sin(-18^\circ\ 43')}{\cos(161^\circ\ 31')}.$$

20. Check the results in exercises 18 and 19 by using the natural functions.

21. Find the numerical values of sin 247° 16′, cos 193° 19′, tan 195 56′, cot 216° 46′.

22. Derive formulas for the functions of 360° − θ.

23. Find the numerical values of tan 328° 10′ 30″, sin 293° 11′ 25″, cos 346° 37′.

24. By logarithms find the value of

$$\frac{\sin 192^\circ\ 10'\sec 283^\circ}{\tan(-28^\circ\ 10')}.$$

25. Summarize the quadrantal formulas, (articles **65** to **68**) giving two rules, one for odd cardinal angles $\pm\theta$, and one for even cardinal angles $\pm\theta$.

72. Summary and review exercises. The results obtained in this chapter may be grouped under three heads.

I. The definition of the six trigonometric functions in terms of rectangular coördinates and the discussion of their variation and periodicity by means of graphs.

II. The development of certain fundamental identities and their use in deriving other identities.

III. The development of the quadrantal formulas and the use of these formulas in finding the values of the functions of angles larger than 90°.

EXERCISES

1. Prove the formulas for sin (90° + θ) and cos (90° + θ) for all values of θ. (See article **65**.) Which formulas of this chapter have not been proved for all possible values of θ?

2. What is a periodic function? Do you know of any in nature?

3. What formulas make it unnecessary to have our tables extend beyond 90°? What formulas enable us to stop the table at 45°?

4. What is an identity? Give an example.

5. Are the six trigonometric functions dependent, that is, if the value of one of them is known, is it possible to find the others? Is the sign determined?

6. Define the word "function" as it is used in mathematics.

7. What is a continuous function? Are all the trigonometric functions continuous for all values of the angle?

8. What is the period of $\sin\theta$? $\sin 2\theta$? $\sin\frac{\theta}{2}$? $\sin a\theta$?

9. In articles **62** and **63** we plotted the first two of the functions in exercise 8. Plot the remaining two.

Prove the following identities by means of any of the formulas of this chapter:

10. $\cos^2\theta(1+\tan^2\theta)=1$.

11. $\cos\theta+\tan\theta\sin\theta=\sec\theta$.

12. $\sin\theta+\cos\theta=\sin\theta\cos\theta(\sec\theta+\csc\theta)$.

13. $(1+\tan\theta)(1+\cot\theta)\sin\theta\cos\theta=(\sin\theta+\cos\theta)^2$.

14. $\csc^2(90°+x)+\cos^2 x=2+\sin^2 x\tan^2(180°-x)$.

15. $(\cot\theta+\csc\theta)^2=\dfrac{1+\cos\theta}{1-\cos\theta}$.

16. $\sec^4\theta-\tan^2\theta=\sec^2\theta+\tan^4\theta$.

17. $(\cos^2\theta-1)(\cot^2\theta+1)+1=0$.

18. $\tan A(1-\cot^2 A)+\cot A(1-\tan^2 A)=0$.

19. $(\sin A+\sec A)^2+(\cos A+\csc A)^2=(1+\sec A\csc A)^2$.

20. $\tan^2 w+\cot^2 w=\sec^2 w\csc^2 w-2$.

21. $\cos^2 t+1=2\cos^3 t\sec t+\sin^2 t$.

22. $\sin A+\cos A=-(1+\tan A)\cos(180°+A)$.

23. $\dfrac{\cot x}{\cos x}=\sqrt{1+\cot^2 x}$.

24. $\tan^2 p=\left(\dfrac{1}{\cos^2 p}-1\right)(\sin^2 p+\cos^2 p)$.

25. $\sec^2\theta+\csc^2\theta=\sec^2\theta\csc^2\theta$.

26. $\cos^2 m+2\sin^2 m=1+\sin^2 m$.

27. $\tan B+\cot B=\sec B\csc B$.

28. $\dfrac{1+\sin\theta}{1-\sin\theta}=(1+\sin\theta)^2\sec^2\theta$.

29. $\cos(90°+x)\sec(180°-x)=\tan(180°+x)$.

30. $\cos(180°-\theta)+\tan(180°+\theta)\sin(-\theta)=\sec(180°+\theta)$.

31. Express $\tan(90°-x)\sec x$ in terms of $\sin x$.

32. Express $\cos(270° - a) \sin a + \tan(90° + a)$ in terms of $\cos a$.

33. Construct the graph of $y = 2 \sin x + \cos x$.

34. Construct the graph of $y = \tan x - \sin x$.

35. Express $\dfrac{\tan(180° + x)}{\tan(360° - x)}$ in terms of $\sin x$.

36. Express $\cos(180° - x) \tan(90° - x)$ in terms of $\sec x$.

37. Express $\dfrac{\sin(270° + \theta) + \sec(90° - \theta)}{\cot(180° - \theta)}$ in terms of $\cos \theta$.

38. Express in terms of sines and cosines of angles less than 90°

$$\frac{\tan 310° - \sin 261°}{\cos 189° \sec 156°}.$$

39. Show how use may be made of the unit circle and the line representation of the values of the sine to construct the sine curve mechanically.

40. Show that if you replace θ by $\theta - \alpha$ in $\sin \theta$, the curve is moved α units to the right if α is positive, and α units to the left if α is negative.

41. Replace θ by $90° + \theta$ in $\sin \theta$ and show how the same graph may be made to represent the cosine curve by simply drawing a new y-axis.

42. If a function is unchanged when θ is replaced by $-\theta$, it is called an *even function* — otherwise *odd*. Which are the even trigonometric functions? Which are the odd?

43. Show that an even function is symmetric to the y-axis.

44. Show that the graph of the cotangent may be obtained from that of the tangent by sliding the tangent curve 90° to the left and reflecting it over the x-axis. Show that this is justified because $\cot \theta = -\tan(90° + \theta)$.

45. Show how to obtain the graph of $\csc \theta$ from that of $\sec \theta$ by sliding the curve in a certain direction or, what is the same thing, sliding the y-axis through the same distance in the opposite direction.

46. By using arguments similar to those above, tell how to obtain the graphs of the following from the graph of $\sin \theta$:

$\cos(90° - \theta)$, $\cos(180° - \theta)$, $\cos(\theta - 180°)$, $\cos(270° - \theta)$, $\cos(270° + \theta)$, $\cos(-\theta)$.

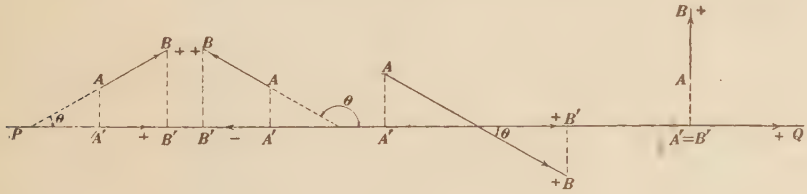
CHAPTER V

THE ADDITION THEOREMS

73. The angle between two lines. Projection. When two lines intersect, two pairs of vertical angles are formed and the angles in each pair are equal. In order to tell certainly which angle is meant when we speak of the angle between two lines it is necessary to assign a positive direction on each line. Then the angle between them is defined as the angle between their positive directions. In the adjoining figures the arrows indicate the positive directions on the lines and the angles between the lines are marked.

In article **43** we considered the projection of a line segment upon another line for the case when the angle of projection was acute. It is now necessary to extend our notion of projection so as to include the case when the angle is not acute. Let PQ be any line whatever with the positive direction as indicated from P towards Q. Let AB be any line segment on a line making with PQ an angle θ. Drop perpendiculars from the initial point A and the terminal point B to the line PQ, cutting it in A' and B'. Then $A'B'$ is the projection of AB. That is, *to project a line segment on another line project the ends of the line segment and read from the projection of the initial point to the projection of the terminal point.*

In the adjoining figures AB is positive and its projection is positive when B' is in the positive direction on PQ from A'.

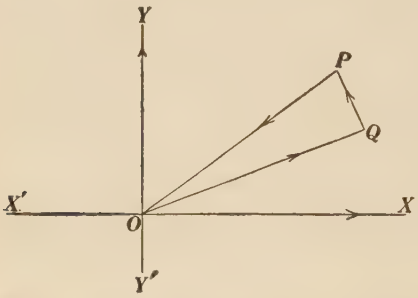


EXERCISES

1. In each of the four figures above prove that $A'B' = AB \cos \theta$.

2. In the figures above prove that the projections of the segments AB on any line perpendicular to PQ are given by $A'B' = AB \sin \theta$.

3. In the triangle OQP the positive directions of the sides are indicated by the arrows. Prove that the sum of the projections of the sides on the x-axis is zero.



4. In exercise 3 prove that the sum of the projections of the sides on the y-axis is zero.

5. Show that if θ is the angle of projection of a line segment on the x-axis and R the length of the projection, then its projection on the y-axis is $R \tan \theta$.

6. Determine the length of the projection on the x-axis of a segment 10 inches long on a line making an angle of 45° with the x-axis. Solve the problem if the angle made with the x-axis is 135°, 60°, 150°.

7. What is the length of the projection of a line segment parallel to the x-axis on the x-axis? On the y-axis?

8. Solve problem 6 when the projection is made on the y-axis.

74. The projection of a polygon. Let $ABCDE$ be any closed polygon. Assign to each side a positive direction by beginning at any vertex and going around the polygon, keeping the area on the left. It is evident that the sum of the projections of all the sides on the x-axis is zero. It is equally evident that the sum of the projections of all sides on the y-axis is zero. This principle may be stated thus:

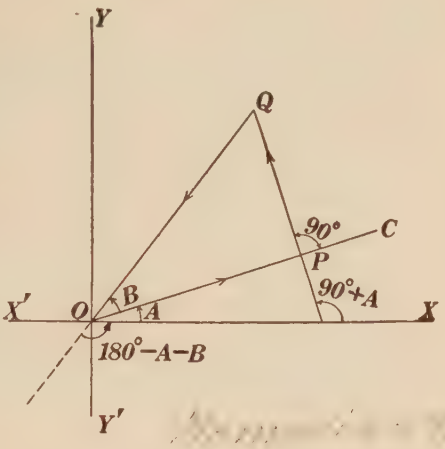
The sum of the projections of all the sides of any closed polygon on any straight line is zero, provided the sides are taken in order.

75. Functions of the sum of two angles. We are now able to develop formulas for expressing $\cos (A + B)$ and $\sin (A + B)$ in terms of the functions of A and B.

Construct a set of rectangular axes and let angle $XOC = A$, and angle $COQ = B$. Then angle $XOQ = A + B$. From any point Q on the terminal side of angle B drop a perpendicular to OC, cutting

it at P. Assign positive directions to the sides of the triangle OPQ, keeping the area to the left. The angles which these sides make with the x-axis are shown in the adjoining figure. Now, projecting on the x-axis, we have, by article **74**,

$QO \cos (180° - A - B) + OP \cos A + PQ \cos (90° + A) = 0.$



But by the quadrantal formulas

$\cos (180° - A - B) = - \cos (A + B)$ and $\cos (90° + A) = - \sin A.$

Hence $- QO \cos (A + B) + OP \cos A - PQ \sin A = 0,$

or $$\cos (A + B) = \frac{OP}{QO} \cos A - \frac{PQ}{QO} \sin A,$$

and

$$\cos (A + B) = \cos A \cos B - \sin A \sin B.$$

One can easily verify that the line QO makes an angle $90° + A + B$ with the y-axis; the line PQ makes an angle A with the y-axis; and the line OP makes the angle $90° - A$ with the y-axis. Then, projecting on the y-axis, we find

$QO \cos (90° + A + B) + OP \cos (90° - A) + PQ \cos A = 0,$

or, since $\cos (90° + A + B) = - \sin (A + B)$ and $\cos (90° - A) = \sin A$

$$\sin (A + B) = \frac{OP}{QO} \sin A + \frac{PQ}{QO} \cos A.$$

Hence

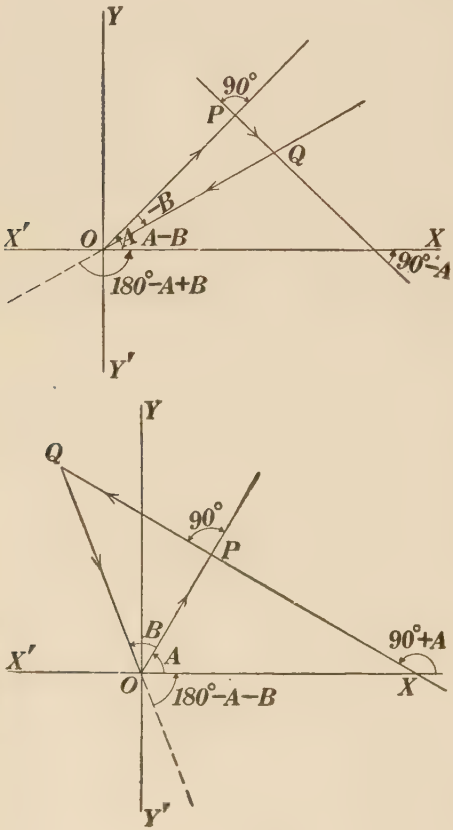
$$\sin (A + B) = \sin A \cos B + \cos A \sin B.$$

These are the desired formulas. Such formulas are called **addition formulas**.

It should be carefully observed that the sine of the sum of two angles *is not the sum of their sines*; that is, the sine operation is not distributive. This is an error the careless or inexperienced student is apt to make.

76. EXERCISES

1. From the adjacent figure, following the method of article **75** develop formulas for sin $(A - B)$ and cos $(A - B)$ where A and B are acute angles. (Notice how the arrows run.)



2. Find the value of sin 75° by using the formula for sin $(A + B)$ where $A = 30°$ and $B = 45°$.

3. Find the value of cos 75°.

4. Prove sin 90° = 1 by using the formula for sin $(A + B)$ where $A = 30°$ and $B = 60°$.

5. Prove cos 90° = 0 from the formula for cos $(A + B)$.

6. From the lower figure prove the formulas for sin $(A + B)$ and cos $(A + B)$ where $A + B$ is obtuse but both A and B are acute.

7. Find the values of sin 15° and cos 15° by making $A = 45°$ and $B = 30°$ in the formulas for sin $(A - B)$ and cos $(A - B)$.

8. Find cos 60° from

$$\cos (30° + 30°).$$

9. Find sin 60° from

$$\sin (30° + 30°).$$

10. Find sin 60° from

$$\sin (90° - 30°).$$

11. Use one of the addition formulas to get sin 120°, cos 135°, sec 150°.

12. Use one of the addition formulas to get cos 120°, sin 150°.

13. Do the addition and subtraction formulas hold if A or B is 0°? 90°? 180°? $\frac{n\pi}{2}$?

77. The addition formulas for any values of the angles. The *method* of proof of the addition formulas given in article **75** is general. But to show that the formulas hold for any values of A and B, however great, we use a method of proof to cover all possible cases. This process is called **mathematical induction**. It may be likened

to the making of a chain. The first step consists of making the first link, the second consists of showing that having any link it is possible to fasten another to it, and the third consists of fastening a second link to the first, a third to the second, and so on. For our purposes here the steps are as follows:

1. We show that the formulas hold for all values of A and B less than 90°.

2. We show that *if* the formulas hold for any *particular* values of A and B, they will hold when either or both are increased by 90°.

3. We choose as our particular values in step 2 the acute angles of step 1 and extend the formulas to the case when one or both of the angles are obtuse, then choose these angles as the particular angles in step 2 and extend the formulas still further. By repeating this process we show that the formulas hold for angles of any size.

The details of the proof follow.

1. Step 1 has already been taken in article **75** and in exercise 6 above.

2. **Assume** that the formulas

$$\sin (A + B) = \sin A \cos B + \cos A \sin B$$

and $$\cos (A + B) = \cos A \cos B - \sin A \sin B$$

hold for certain particular values of A and B not necessarily acute.

Let A be increased by 90°, that is, $A' = 90° + A$.

Then $$\sin(A' + B) = \sin (A + 90° + B) = \cos (A + B)$$
$$= \cos A \cos B - \sin A \sin B.$$

But $$A = A' - 90°,$$

hence $$\sin(A' + B) = \cos (A' - 90°) \cos B - \sin (A' - 90°) \sin B$$
$$= \sin A' \cos B + \cos A' \sin B.$$

But this is the desired formula when A has been increased by 90°. Owing to the symmetry of the formula, it follows at once that it holds when B is increased by 90° and hence when both are increased by 90°. In the same way we may show that if the formula for $\cos (A + B)$ holds for particular values of A and B, it holds if one or both angles are increased by 90°.

3. Since the formulas hold for acute angles, they will hold for one obtuse and one acute or for both obtuse. Since they hold for this

case, they will hold when one or both of the angles are angles of the third quadrant. By repeating this line of reasoning we show that the formulas hold whatever the size of the angles.

78. Formulas for $\sin(A - B)$ and $\cos(A - B)$. In the formula

$$\sin(A + B') = \sin A \cos B' + \cos A \sin B'$$

let $B' = 90^\circ - B$ where B is acute. Then

$$\sin(A - B + 90^\circ) = \sin A \cos(90^\circ - B) + \cos A \sin(90^\circ - B)$$
$$= \sin A \sin B + \cos A \cos B.$$

Hence

$$\mathbf{\cos(A - B) = \cos A \cos B + \sin A \sin B.}$$

In the formula

$$\cos(A + B') = \cos A \cos B' - \sin A \sin B'$$

let $B' = 90^\circ - B$. Then

$$\cos(A - B + 90^\circ) = \cos A \cos(90^\circ - B) - \sin A \sin(90^\circ - B),$$

or

$$-\sin(A - B) = \cos A \sin B - \sin A \cos B,$$

or

$$\mathbf{\sin(A - B) = \sin A \cos B - \cos A \sin B.}$$

It may be shown as in article **77** that these formulas hold whatever values A and B may have.

EXERCISES

1. Show by mathematical induction that the formulas for the $\sin(A - B)$ and $\cos(A - B)$ hold for all values of A and B.

2. If $\sin A = -\frac{1}{3}$ and $\cos B = \frac{3}{5}$ where A and B are in the same quadrant, find $\sin(A \pm B)$ and $\cos(A \pm B)$.

3. Derive formulas for $270^\circ - \theta$ from the above subtraction formulas.

4. Determine all the angles less than 360° whose functions may be found from those of 30° and 45° without the use of tables.

5. Prove that if the addition formulas hold for any particular values of A and B, they still hold when either A or B or both are diminished by 90°.

79. Formulas for $\tan(A + B)$ and $\tan(A - B)$. From the formulas for $\sin(A + B)$ and $\cos(A + B)$ we may derive a formula for $\tan(A + B)$ as follows:

$$\tan(A + B) = \frac{\sin(A + B)}{\cos(A + B)} = \frac{\sin A \cos B + \cos A \sin B}{\cos A \cos B - \sin A \sin B}.$$

Dividing numerator and denominator by $\cos A \cos B$,

$$\tan (A + B) = \frac{\dfrac{\sin A \cos B}{\cos A \cos B} + \dfrac{\cos A \sin B}{\cos A \cos B}}{\dfrac{\cos A \cos B}{\cos A \cos B} - \dfrac{\sin A \sin B}{\cos A \cos B}}$$

or
$$\tan (A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}.$$

Similarly it may be proved that

$$\tan (A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}.$$

80. EXERCISES

1. Determine tan 75°, cot 75°, tan 15°, cot 15°, tan 105°, cot 105°.

2. Could we use 90° and 15° in the addition formula for the tangent to find tan 105°? to find cot 105°?

3. If tan $x = 3$ and tan $y = \frac{2}{3}$, find tan $(x + y)$, tan $(x - y)$, cot $(x + y)$, cot $(x - y)$. Check your results from the tables.

4. Tan $A = 4$ and A is in the third quadrant, sin $B = \frac{4}{5}$ and B is in the second quadrant. Find tan $(A \pm B)$ and cot $(A \pm B)$. In what quadrant does the sum angle lie? the difference angle?

5. Derive a formula for csc $(A - B)$ in terms of c c A and csc B.

6. Prove $\tan \left(\frac{\pi}{4} + x\right) = \frac{1 + \tan x}{1 - \tan x}$.

7. Prove $\sin \left(\frac{\pi}{4} + x\right) = \frac{\cos x + \sin x}{\sqrt{2}}$.

8. Prove $\sin (45° + \theta) \cos (45° - \theta) + \cos (45° + \theta) \sin (45° - \theta) = 1$.

9. Prove $\tan (x + 45°) = \frac{1 + \cot x}{1 - \cot x}$.

10. Since the addition and subtraction formulas for sine and cosine hold if one of the angles is zero or if one is 90°, explain how these formulas may be used to show that the quadrantal formulas for sine and cosine are true for any angle θ. Do all the other quadrantal formulas hold therefore?

Using the formulas developed in articles **77**, **78**, **79**, prove the following:

11. $\sin (90° + \theta) = \cos \theta$.

12. $\cos (90° + \phi) = -\sin \phi$.

13. $\cos (45° + x) = \frac{\cos x - \sin x}{\sqrt{2}}$.

14. $\sin (180° - \theta) = \sin \theta$.

15. $\cos (180° - \phi) = -\cos \phi$.

16. Given $\tan a = \frac{1}{3}$ and $\tan b = 2$, find $\tan (a + b)$.

17. Given $\sin z = \frac{2}{5}$ and $\cos w = \frac{1}{4}$, find $\sin (z - w)$.

Prove the following identities:

18. $\cot (x + y) = \dfrac{\cot x \cot y - 1}{\cot x + \cot y}$.

19. $1 + \tan A \tan B = \dfrac{\cos (A - B)}{\cos A \cos B}$.

20. $\cos (n + 1)\, x \cos x + \sin (n + 1)\, x \sin x = \cos nx$.

21. $\tan (A + B) \tan (A - B) = \dfrac{\tan^2 A - \tan^2 B}{1 - \tan^2 A \tan^2 B}$.

22. $\cos (\theta + \phi) \cos (\theta - \phi) = \cos^2 \theta - \sin^2 \phi$.

23. $\tan (45^\circ - \theta) = \dfrac{1 - \tan \theta}{1 + \tan \theta} \left(\dfrac{1}{\sec^2 \theta} + \dfrac{1}{\csc^2 \theta} \right)$.

24. $\cos^4 \theta - \sin^4 \theta = 2 \cos^2 \theta - 1$.

[*Hint.* Let $A = B = \theta$ in an addition formula.]

25. $\sec^4 \theta - \tan^4 \theta = 2 \sec^2 \theta - 1$.

26. $\sin^3 \theta + \cos^3 \theta = (\sin \theta + \cos \theta)(1 - \sin \theta \cos \theta)$.

27. $\cos^6 A + \sin^6 A = 1 - 3 \cos^2 A \sin^2 A$.

28. $\cos^6 x + 2 \cos^4 x \sin^2 x + \cos^2 x \sin^4 x + \sin^2 x = 1$.

81. Functions of 2 A. The formulas of articles **75** and **79** are true for all values of A and B. A case of special interest is that in which $A = B$. In this case we have

$$\sin(A + A) = \sin A \cos A + \cos A \sin A,$$

or
$$\mathbf{\sin 2A = 2 \sin A \cos A.}$$

From
$$\cos(A + B) = \cos A \cos B - \sin A \sin B,$$

by letting $B = A$ we easily find

$$\begin{aligned} \mathbf{\cos 2A} &= \mathbf{\cos^2 A - \sin^2 A} \\ &= \mathbf{2 \cos^2 A - 1} \\ &= \mathbf{1 - 2 \sin^2 A.} \end{aligned}$$

From the formula for $\tan(A + B)$ by letting $B = A$ we get

$$\tan(A + A) = \frac{\tan A + \tan A}{1 - \tan A \tan A},$$

or
$$\mathbf{\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}.}$$

It is necessary to be able to recognize these formulas no matter how the angle is expressed. For example:

$$\sin 3x = 2 \sin \frac{3x}{2} \cos \frac{3x}{2},$$

$$\cos \frac{4p}{3} = \cos^2 \frac{2p}{3} - \sin^2 \frac{2p}{3}$$

$$= 2 \cos^2 \frac{2p}{3} - 1 = 1 - 2 \sin^2 \frac{2p}{3}$$

are different ways in which the formulas for $\sin 2A$ and $\cos 2A$ may appear.

EXERCISES

1. Express the following in terms of $\cos x$:

$$\sin 2x + \cos x; \quad \cos 4x; \quad \csc 2x + \csc x.$$

2. Find the functions of $2A$ if A is in the second quadrant and $\cos A = -\frac{3}{5}$.

3. Derive a formula for cosecant $2A$ in terms of sec A and csc A.

82. The functions of $\frac{\theta}{2}$ in terms of θ. Let the formula for $\cos 2A$ be written in the forms

$$\cos \theta = 1 - 2 \sin^2 \frac{\theta}{2}$$

and

$$\cos \theta = 2 \cos^2 \frac{\theta}{2} - 1.$$

Solving these for $\sin \frac{\theta}{2}$ and $\cos \frac{\theta}{2}$, we find:

$$\sin \frac{\theta}{2} = \pm \sqrt{\frac{1 - \cos \theta}{2}},$$

$$\cos \frac{\theta}{2} = \pm \sqrt{\frac{1 + \cos \theta}{2}}.$$

Dividing the first of these equations by the second,

$$\tan \frac{\theta}{2} = \pm \sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}}.$$

Why are the double signs before the radicals necessary?

These formulas express the functions of any angle in terms of its double.

Ex. In the formulas above let $\theta = 30°$, then

$$\sin 15° = \sqrt{\frac{1 - \cos 30°}{2}} = \sqrt{\frac{1}{2} - \frac{\sqrt{3}}{4}} = .25882,$$

$$\cos 15° = \sqrt{\frac{1 + \cos 30°}{2}} = \sqrt{\frac{1}{2} + \frac{\sqrt{3}}{4}} = .96593,$$

$$\tan 15° = \sqrt{\frac{1 - \cos 30°}{1 + \cos 30°}} = \sqrt{\frac{2 - \sqrt{3}}{2 + \sqrt{3}}} = .26795.$$

83. EXERCISES

1. Work out the values of the functions of $22\frac{1}{2}°$, and check your results from the table of natural functions.

2. If $\cos x = .6$, find $\sin \frac{x}{2}$ and $\cos \frac{x}{2}$. Is $\sin \frac{x}{2}$ the same as $\frac{1}{2} \sin x$? Is $\sin 2x$ the same as $2 \sin x$? Is $\cos 3x$ the same as $3 \cos x$?

3. Prove $\sin 2x = 2 \sin x \cos x$ from a figure like that in article **75**.

4. Determine the value of the functions of $67\frac{1}{2}°$ by the formulas of article **82**, and check your results from the tables.

5. Prove that $\sec \frac{\theta}{2} = \pm \sqrt{\frac{2 \sec \theta}{1 + \sec \theta}}$.

6. Express the formulas for $\tan \frac{\theta}{2}$ and $\cot \frac{\theta}{2}$ rationally in terms of $\sin \theta$ and $\cos \theta$.

7. Express $\sin 3\theta$ in terms of $\cos 6\theta$.

84. The factoring formulas. Often a sum or difference of the form $\sin \theta \pm \sin \phi$ or $\cos \theta \pm \cos \phi$ will arise. Such expressions may be transformed into products as follows:

$$\sin (x + y) = \sin x \cos y + \cos x \sin y,$$
$$\sin (x - y) = \sin x \cos y - \cos x \sin y.$$

By addition $\sin (x + y) + \sin (x - y) = 2 \sin x \cos y$, and by subtraction $\sin (x + y) - \sin (x - y) = 2 \cos x \sin y$.

Now let $x + y = \theta$,

and $x - y = \phi$.

Then $2x = \theta + \phi$ and $2y = \theta - \phi$.

Hence $x = \frac{\theta + \phi}{2}$ and $y = \frac{\theta - \phi}{2}$.

Substituting these values of x and y, we get

$$\sin\theta + \sin\phi = 2\sin\frac{\theta+\phi}{2}\cos\frac{\theta-\phi}{2},$$

$$\sin\theta - \sin\phi = 2\cos\frac{\theta+\phi}{2}\sin\frac{\theta-\phi}{2}.$$

In like manner, starting with

$$\cos(x+y) = \cos x\cos y - \sin x\sin y,$$
$$\cos(x-y) = \cos x\cos y + \sin x\sin y,$$

by addition $\cos(x+y) + \cos(x-y) = 2\cos x\cos y$, and
by subtraction $\cos(x+y) - \cos(x-y) = -2\sin x\sin y$.

Substituting

$x + y = \theta$ and $x - y = \phi$, as above, we find

$$\cos\theta + \cos\phi = 2\cos\frac{\theta+\phi}{2}\cos\frac{\theta-\phi}{2},$$

$$\cos\theta - \cos\phi = -2\sin\frac{\theta+\phi}{2}\sin\frac{\theta-\phi}{2}.$$

85. EXERCISES

1. State in words each of the formulas derived in the last article.

Change the following into products:

2. $\sin 15° - \sin 45°$, $\sin 85° + \sin 15°$.
3. $\cos 60° + \cos 30°$, $\cos 36° - \cos 54°$.
4. $\sin 2A + \sin 4A$, $\cos 3a - \cos(3a - b)$.
5. $\cos(3x + 4y) + \cos(3x - 4y)$.

Change the following products into sums or differences:

6. $2\sin 30°\cos 50°$, $2\sin 40°\cos 20°$.
7. $\sin 25°\cos 55°$, $6\cos 35°\cos 40°$.
8. $2\cos(30° + A)\cos(60° - A)$.

Prove the following identities:

9. $\sin\frac{x}{2}\cos\frac{y}{2} + \cos\frac{x}{2}\sin\frac{y}{2} = \sin\frac{x+y}{2}$.

10. $1 + \tan 2a \tan a = \sec 2a$.

11. $\cos 6\theta = 1 - 2\sin^2 3\theta$.

12. $\cos^2\theta + \cos^2\left(\frac{\pi}{3} + \theta\right) + \cos^2\left(\theta - \frac{\pi}{3}\right) = \frac{3}{2}$.

13. $\dfrac{\sin 4A + \sin 2A}{\sin 4A - \sin 2A} = \dfrac{3 - \tan^2 A}{1 - 3\tan^2 A}$.

14. $\tan^2\left(\frac{\pi}{4} - A\right) = \dfrac{1 - \sin 2A}{1 + \sin 2A}$.

15. $\dfrac{1}{1 + \cos^2\theta} = \dfrac{\sec^2\theta}{2 + \tan^2\theta}$.

16. $\dfrac{\tan P}{\tan P - \tan 3P} + \dfrac{\cot P}{\cot P - \cot 3P} = 1$.

17. $\cos 5\theta \cos 2\theta = \frac{1}{2}(\cos 7\theta + \cos 3\theta)$.

18. $\dfrac{\sin(x+y) + \sin(x-y)}{\sin(x+y) - \sin(x-y)} = \dfrac{\tan x}{\tan y}$.

19. $\sin 35° + \cos 65° = \cos 5°$.

20. $\tan\dfrac{x}{2} = \csc x - \cot x$.

21. $\sin x = \dfrac{2}{\cot\frac{x}{2} + \tan\frac{x}{2}}$.

86. Summary and review exercises. This chapter is wholly concerned with trigonometric analysis. The object has been to build up theorems (formulas) involving the trigonometric functions of the sum and difference of two angles, the double angle and the half angle. The most important formulas are given below.

(1) $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$.

(2) $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$.

(3) $\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$.

(4) $\sin 2A = 2\sin A \cos A$.

(5) $\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$.

(6) $\tan 2A = \dfrac{2\tan A}{1 - \tan^2 A}$.

(7) $\sin\dfrac{x}{2} = \sqrt{\dfrac{1 - \cos x}{2}}$.

$$(8) \qquad \cos\frac{x}{2} = \sqrt{\frac{1+\cos x}{2}}.$$

$$(9) \qquad \tan\frac{x}{2} = \sqrt{\frac{1-\cos x}{1+\cos x}}.$$

$$(10) \quad \sin x + \sin y = 2\sin\frac{x+y}{2}\cos\frac{x-y}{2}.$$

$$(11) \quad \sin x - \sin y = 2\cos\frac{x+y}{2}\sin\frac{x-y}{2}.$$

$$(12) \quad \cos x + \cos y = 2\cos\frac{x+y}{2}\cos\frac{x-y}{2}.$$

$$(13) \quad \cos x - \cos y = -2\sin\frac{x+y}{2}\sin\frac{x-y}{2}.$$

These formulas are important not only for later developments in trigonometry but also in more advanced mathematical subjects such as calculus, harmonic analysis, and mathematical physics.

As a rule we have written the shorter member of these identities on the left so that reading them from left to right they seem to expand, but they should be used equally as readily from right to left. In the latter event they usually contract the size of the expression. For example, by (10) we may change $\sin 2\phi + \sin\phi$ into a product, viz., $2\sin\frac{3\phi}{2}\cos\frac{\phi}{2}$. But we may also use (10) to change a product of a sine and cosine into a sum of sines, *e.g.*

$$\sin 45^\circ \cos 15^\circ = \frac{\sin 60 + \sin 30}{2}$$
$$= \frac{\sqrt{3}+1}{4}.$$

EXERCISES

Prove the following identities:

1. $\cot A - \cot 2A = \csc 2A.$

2. $$\frac{\tan\left(\frac{\pi}{4} - \frac{\phi}{2}\right)}{\tan\left(\frac{\pi}{4} + \frac{\phi}{2}\right)} = \left\{\frac{1-\tan\frac{\phi}{2}}{1+\tan\frac{\phi}{2}}\right\}^2.$$

3. $\dfrac{\tan x}{\tan (x+\phi)} = 1 - \dfrac{2 \sin \phi}{\sin (2x+\phi) + \sin \phi}$.

4. $\cos^2 x + \cos^2 y - 1 = \cos (x+y) \cos (x-y)$.

5. $\sqrt{(\cos a - \cos b)^2 + (\sin a - \sin b)^2} = 2 \sin \dfrac{a-b}{2}$.

6. $\cos (A+B) \cos (A-B) = \cos^2 A - \sin^2 B$.

7. $\tan A + \tan B = \dfrac{\sin (A+B)}{\cos A \cos B}$.

8. If $A + B = Z$, prove $\cos^2 A + \cos^2 B - 2 \cos A \cos B \cos Z = \sin^2 Z$.

9. $\sin (A+B) \cos A - \cos (A+B) \sin A = \sin B$.

10. $\cos (A+B) \cos (A-B) + \sin (A+B) \sin (A-B) = \cos 2B$.

11. $\sin (2\phi - \theta) \cos (\phi - 2\theta) - \cos (2\phi - \theta) \sin (\phi - 2\theta) = \sin (\theta + \phi)$.

12. $1 - \tan^2 x \tan^2 y = \dfrac{\cos^2 y - \sin^2 x}{\cos^2 x \cos^2 y}$.

13. $\sin 3A = 3 \sin A - 4 \sin^3 A$.

14. $\cos 3A = 4 \cos^3 A - 3 \cos A$.

15. $\dfrac{\tan x + \tan y}{\cot x + \cot y} = \tan x \tan y$.

16. $\tan^2 (45^\circ - A) = \dfrac{1 - 2 \sin A \cos A}{1 + \sin 2A}$.

In each of the following transform all the sums and differences into products and all the products into sums or differences.

17. $\cos 3a + \cos a$.

18. $\sin x - \sin 5x$.

19. $2 \sin 5x \cos x$.

20. $\cos 2A - \cos 4A$.

21. $\cos 7x \cos 2x$.

22. $\sin 75^\circ \sin 15^\circ$.

23. $\sin 36^\circ + \sin 54^\circ$.

24. $\cos (2a + 3b) \sin (2a - 3b)$.

25. $\cos (30^\circ + \phi) + \cos (30^\circ - \phi)$.

26. $\sin (2a + b) + \sin (2a - b)$.

27. $\sin \phi + \sin \dfrac{\phi}{2}$.

28. $\sin \dfrac{3\phi}{2} + \sin \dfrac{\phi}{2}$.

29. $\cos \dfrac{5\phi}{2} - \cos \dfrac{\phi}{2}$.

Prove:

30. $\dfrac{\sin A + \sin B}{\sin A - \sin B} = \tan \dfrac{A+B}{2} \cot \dfrac{A-B}{2}$.

31. $\dfrac{\cos x + \cos 2y}{\cos x - \cos 2y} = -\cot \dfrac{x+2y}{2} \cot \dfrac{x-2y}{2}$.

32. $\dfrac{\sin p + \sin q}{\cos p + \cos q} = \tan \dfrac{p+q}{2}$.

33. Simplify $\dfrac{\sin A + \sin 2A + \sin 3A}{\cos A + \cos 2A + \cos 3A}$.

34. If A, B, and C are the angles of a triangle, prove

$$\sin A + \sin B + \sin C = 4 \cos \frac{A}{2} \cos \frac{B}{2} \cos \frac{C}{2}.$$

35. If $\cos^2 x + \cos^2 2x + \cos^2 3x = 1$, prove that

$$\cos x \cos 2x \cos 3x = 0.$$

36. Solve for x and y:

$$x \sin a + y \cos a = b.$$
$$x \cos a - y \sin a = c.$$

37. Prove that $\tan nA = \dfrac{\tan (n-1)A + \tan A}{1 - \tan (n-1)A \tan A}$.

38. $\cos cx = 2 \cos (c-1)x \cos x - \cos (c-2)x$.

39. If $\alpha + \beta + \gamma = 90°$, prove that

$$\tan \alpha \tan \beta + \tan \beta \tan \gamma + \tan \gamma \tan \alpha = 1.$$

40. Prove that

$$\cot (x + 15°) - \tan (x - 15°) = \frac{4 \cos 2x}{2 \sin 2x + 1}.$$

41. The side of a right triangle is $b = c \sin B$. Show from the addition and subtraction formulas that if an error of $\pm e$ is made in measuring the angle B, the true length b' of the side is given by $b' = c (\sin B \cos e \pm \cos B \sin e)$. What is the error in b if $c = 10$, $B = 30°$, and $e = 4°$?

CHAPTER VI

THE GENERAL TRIANGLE

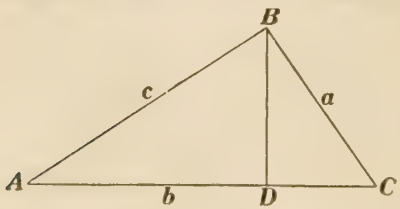
87. The solution of a triangle. When we speak of solving a triangle, we mean the finding of all its angles and sides from the given parts. These data must include a distance. Usually we are given some of the sides and angles and it is required to find the other sides and angles. The problem is analogous to the construction of a triangle in plane geometry. We denote the angles of a triangle by A, B, and C and the sides opposite these angles by a, b, and c. The following cases arise:

1. Given a, b, c.
2. Given A, b, c.
3. Given A, a, b.
4. Given A, B, a.
5. Given A, B, c.

A triangle cannot be solved if only the three angles are given. (Why not?)

We proceed to derive formulas for the solution of the triangle in each of the five cases.

88. The law of cosines. In the triangle ABC let AD be the projection of AB on AC. Then if A is acute a theorem of plane geometry gives,



$$a^2 = b^2 + c^2 - 2\,b \cdot AD.$$

Again, let A be obtuse, as in the figure below. Then

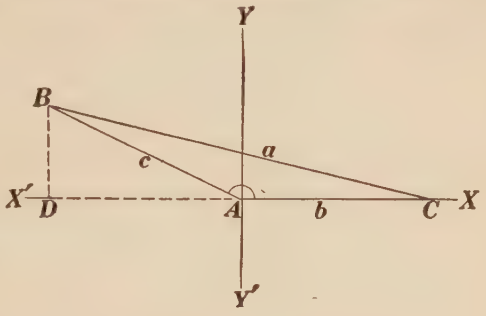
$$a^2 = b^2 + c^2 + 2\,b \cdot AD.$$

Consider angle CAB placed in standard position. Then it is evident in both the figures that

$$\cos A = \frac{AD}{c},$$

or $$AD = c \cos A.$$

Hence both of these theorems of geometry may be expressed in the one equation:



$$a^2 = b^2 + c^2 - 2\,bc \cos A.$$

For if the angle A is obtuse, $\cos A$ becomes negative and automatically takes care of the sign.

Applying the same theorem to each side in turn, we find by a cyclic permutation of the letters

$$b^2 = a^2 + c^2 - 2\,ac \cos B$$

and $$c^2 = b^2 + a^2 - 2\,ab \cos C.$$

Solving for $\cos A$, $\cos B$, and $\cos C$,

(1) $$\cos A = \frac{b^2 + c^2 - a^2}{2\,bc},$$

(2) $$\cos B = \frac{a^2 + c^2 - b^2}{2\,ac},$$

(3) $$\cos C = \frac{a^2 + b^2 - c^2}{2\,ab}.$$

These formulas constitute the **law of cosines**. To what do they reduce if an angle is 90°?

Ex. How will the cosine and therefore the angle be affected if two sides remain fixed but the side opposite increases and the angle is acute? obtuse?

Formulas (1), (2), and (3) are sufficient to find the angles of a triangle when the three sides are given, but they are not adapted to logarithmic computation. (Why not?) We show, in the next paragraph, how to derive from them formulas in which logarithms may be used.

89. The half-angle formulas. We may replace the left member of (1) by $1 - 2\sin^2\frac{A}{2}$. Then

$$1 - 2\sin^2\frac{A}{2} = \frac{b^2 + c^2 - a^2}{2bc}.$$

Solving for $\sin^2\frac{A}{2}$, we find

$$\sin^2\frac{A}{2} = \frac{a^2 - b^2 + 2bc - c^2}{4bc} = \frac{a^2 - (b - c)^2}{4bc}.$$

Factoring the numerator,

$$\sin^2\frac{A}{2} = \frac{(a - b + c)(a + b - c)}{4bc}.$$

We may equally well replace the left member of (1) by $2\cos^2\frac{A}{2} - 1$. Then

$$2\cos^2\frac{A}{2} - 1 = \frac{b^2 + c^2 - a^2}{2bc},$$

or

$$2\cos^2\frac{A}{2} = 1 + \frac{b^2 + c^2 - a^2}{2bc} = \frac{(b + c)^2 - a^2}{2bc}.$$

Hence

$$\cos^2\frac{A}{2} = \frac{(b + c - a)(b + c + a)}{4bc}.$$

Dividing $\sin^2\frac{A}{2}$ by $\cos^2\frac{A}{2}$, we get

$$\tan^2\frac{A}{2} = \frac{(a - b + c)(a + b - c)}{(b + c - a)(b + c + a)}.$$

Now let

$$2s = a + b + c.$$

Then by subtracting $2a$ from both sides of the equation, we get

$$2(s - a) = b + c - a.$$

Subtracting $2b$ and $2c$ we find

$$2(s - b) = a - b + c,$$

and

$$2(s - c) = a + b - c.$$

Substituting these above and extracting the square root,

$$\tan\frac{A}{2} = \sqrt{\frac{(s - b)(s - c)}{s(s - a)}}.$$

In a similar manner we may prove

$$\tan\frac{B}{2} = \sqrt{\frac{(s-a)(s-c)}{s(s-b)}}.$$

$$\tan\frac{C}{2} = \sqrt{\frac{(s-a)(s-b)}{s(s-c)}}.$$

Now if we multiply numerator and denominator of the three radicands by $s-a$, $s-b$, and $s-c$ respectively and denote the common factor $\sqrt{\frac{(s-a)(s-b)(s-c)}{s}}$ by r, we obtain the following:

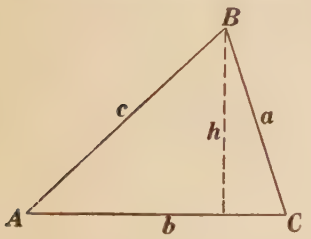
$$\tan\frac{A}{2} = \frac{r}{s-a},$$

$$\tan\frac{B}{2} = \frac{r}{s-b},$$

$$\tan\frac{C}{2} = \frac{r}{s-c}.$$

The last three formulas are the ones commonly used to find the angles when the sides are given.

90. The law of sines. Let ABC be any triangle. Drop a perpendicular h from vertex B to the opposite side. Then



$$\sin A = \frac{h}{c} \text{ or } h = c \sin A,$$

$$\sin C = \frac{h}{a} \text{ or } h = a \sin C. \text{ Hence}$$

$$c \sin A = a \sin C.$$

Dividing by ac, we get

$$\frac{\sin A}{a} = \frac{\sin C}{c}.$$

If a perpendicular be dropped from C to AB, we find

$$\frac{\sin A}{a} = \frac{\sin B}{b}.$$

Combining the two equations, we get

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}.$$

These formulas are known as the **law of sines.**

EXERCISES

1. Show that these formulas hold when one of the angles is obtuse.
2. What does the law of sines become if the angle C is a right angle?

91. The law of tangents. The first equality in the formula of article **90** may be written

$$\frac{\sin A}{\sin B} = \frac{a}{b}.$$

Adding 1 to both members,

$$\frac{\sin A}{\sin B} + 1 = \frac{a}{b} + 1,$$

or

$$\frac{\sin A + \sin B}{\sin B} = \frac{a + b}{b}.$$

By article **84**,

$$(1) \qquad \frac{2 \sin \frac{A + B}{2} \cos \frac{A - B}{2}}{\sin B} = \frac{a + b}{b}.$$

Similarly, by subtracting 1 from both members of $\frac{\sin A}{\sin B} = \frac{a}{b}$ and reducing the numerator of the left-hand member to a product, we find

$$(2) \qquad \frac{2 \cos \frac{A + B}{2} \sin \frac{A - B}{2}}{\sin B} = \frac{a - b}{b}.$$

Dividing equation (2) by equation (1),

$$\frac{2 \cos \frac{A + B}{2} \sin \frac{A - B}{2}}{2 \sin \frac{A + B}{2} \cos \frac{A - B}{2}} = \frac{a - b}{a + b},$$

or

$$\tan \frac{A - B}{2} \cot \frac{A + B}{2} = \frac{a - b}{a + b}.$$

Hence

$$\tan \frac{A - B}{2} = \frac{a - b}{a + b} \tan \frac{A + B}{2}.$$

But in any triangle $A + B = 180° - C$. Hence

$$\frac{A + B}{2} = 90° - \frac{C}{2} \text{ and } \tan \frac{A + B}{2} = \cot \frac{C}{2}.$$

Then we have

$$\tan\frac{A-B}{2} = \frac{a-b}{a+b}\cot\frac{C}{2}.$$

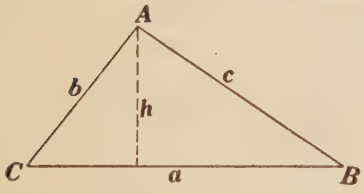
In a similar manner we may prove

$$\tan\frac{B-C}{2} = \frac{b-c}{b+c}\cot\frac{A}{2},$$

$$\tan\frac{C-A}{2} = \frac{c-a}{c+a}\cot\frac{B}{2}.$$

These three formulas constitute the **law of tangents.**

92. The area of a triangle. Let h be the altitude of triangle ABC drawn from vertex A. Then the area is $\frac{ah}{2}$. But $h = b\sin C$.



Hence $\text{Area} = \frac{ab}{2}\sin C$.

The altitude may be drawn from any vertex so we have the theorem:

The area of a triangle equals one half the product of two sides times the sine of the included angle.

There are other formulas for the area, but this is probably the most useful one.

93. Illustrative examples. (1) Solve the triangle having given $a = 31$, $b = 25$, $c = 42$, using the formulas of article **89**.

$s = 49$, $s - a = 18$, $s - b = 24$, $s - c = 7$.

$$\tan\frac{A}{2} = \frac{r}{s-a}.$$

$$\begin{aligned}
\log(s-a) &= 1.25527\\
\log(s-b) &= 1.38021\\
\log(s-c) &= .84510\\
\operatorname{colog} s &= 8.30980 - 10.\\
\log r^2 &= 11.79038 - 10,\\
\log r &= 10.89519 - 10,\\
-\log(s-a) &= 1.25527.\\
\log\tan\frac{A}{2} &= 9.63992 - 10
\end{aligned}$$

$$\frac{A}{2} = 23^\circ\ 34'\ 41'',$$

$$A = 47^\circ\ 9'\ 22''.$$

$$\tan\frac{B}{2} = \frac{r}{s-b}.$$

$$\log r = 10.89519 - 10$$

$$-\log(s-b) = 1.38021$$

$$\log\tan\frac{B}{2} = 9.51498 - 10$$

$$\frac{B}{2} = 18^\circ\ 7'\ 29'',$$

$$B = 36^\circ\ 14'\ 58''.$$

$$\tan\frac{C}{2} = \frac{r}{s-c}.$$

$$\log r = .89519$$

$$-\log(s-c) = .84510$$

$$\log\tan\frac{C}{2} = .05009.$$

$$\frac{C}{2} = 48^\circ\ 17'\ 48'',$$

$$C = 96^\circ\ 35'\ 36''$$

$$A = 47^\circ\ 9'\ 22''$$

$$B = 36^\circ\ 14'\ 58''$$

Check: $A + B + C = 179^\circ\ 59'\ 56''.$

(2) Solve the triangle having given $A = 48^\circ$, $b = 47$, $c = 26$.

$$\tan\frac{B-C}{2} = \frac{b-c}{b+c}\cot\frac{A}{2}$$

$$\log(b-c) = 1.32222$$

$$\operatorname{colog}(b+c) = 8.13668 - 10$$

$$\log\cot\frac{A}{2} = .35142$$

$$\log\tan\frac{B-C}{2} = 9.81032 - 10$$

$$\frac{B-C}{2} = 32^\circ\ 52'\ 4'',$$

$$\frac{B+C}{2} = 66^\circ.$$

$$B = 98^\circ\ 52'\ 4'',$$
$$C = 33^\circ\ 7'\ 56''.$$
$$\frac{\sin A}{a} = \frac{\sin B}{b},$$
$$a = \frac{b \sin A}{\sin B}.$$
$$\log b = 1.67210$$
$$\log \sin A = 9.87107 - 10$$
$$\operatorname{colog} \sin B = 0.00522$$
$$\log a = 11.54839 - 10$$
$$a = 35.3$$

To check the angles use $c \sin B = b \sin C$. To check the side a use $a \sin C = c \sin A$.

$$\log c = 1.41497 \qquad \log b = 1.67210$$
$$+\log \sin B = 9.99478 - 10 \qquad +\log \sin C = 9.73765 - 10$$
$$\log c \sin B = 1.40975 = \log b \sin C = 1.40975$$
$$\log a = 1.54839 \qquad \log c = 1.41497$$
$$+\log \sin C = 9.73765 - 10 \qquad +\log \sin A = 9.87107 - 10$$
$$\log a \sin C = 1.28604 = \log c \sin A = 1.28604$$

(3) Solve the triangle, given $A = 37^\circ$, $b = 186$, $a = 147$

$$\frac{\sin A}{a} = \frac{\sin B}{b},$$
$$\sin B = \frac{b \sin A}{a}.$$
$$\log b = 2.26951$$
$$\log \sin A = 9.77946 - 10$$
$$\operatorname{colog} a = 7.83268 - 10$$
$$\log \sin B = 9.88165 - 10$$

There are two supplementary values of B which satisfy this equation, hence there are two triangles which have the given parts.

$$B = 49^\circ\ 35'\ 38'' \qquad B' = 130^\circ\ 24'\ 22''$$
$$C = 93^\circ\ 24'\ 22'' \qquad C' = 12^\circ\ 35'\ 38''$$
$$\frac{\sin B}{b} = \frac{\sin C}{c}, \qquad \frac{\sin B'}{b} = \frac{\sin C'}{c'}.$$
$$c = \frac{b \sin C}{\sin B}. \qquad c' = \frac{b \sin C'}{\sin B'}.$$

$\log b =$	2.26951	$\log b =$	2.26951
$\log \sin C =$	9.99923 − 10	$\log \sin C' =$	9.33853 − 10
$\operatorname{colog} \sin B =$	0.11835	$\operatorname{colog} \sin B' =$	0.11835
$\log c =$	12.38709 − 10	$\log c' =$	11.72639 − 10
$c =$	243.83.	$c' =$	53.259.

Check:

$$\tan \frac{C - B}{2} = \frac{c - b}{c + b} \tan \frac{C + B}{2}$$

$$\tan \frac{B' - C'}{2} = \frac{b - c'}{b + c'} \tan \frac{B' + C'}{2}$$

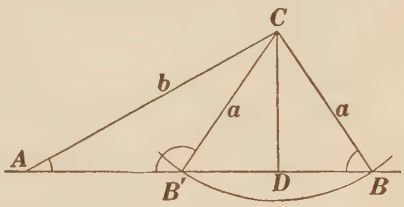
$\log (c - b) =$	1.76215
$\operatorname{colog} (c + b) =$	7.36670 − 10
$\log \tan \frac{C + B}{2} =$	0.47548
$\log \frac{c - b}{c + b} \tan \frac{C + B}{2} =$	9.60433 − 10
$\log \tan \frac{C - B}{2} =$	9.60435 − 10
$\log (b - c') =$	2.12300
$\operatorname{colog} (b + c') =$	7.62113 − 10
$\log \tan \frac{B' + C'}{2} =$	.47548
$\log \frac{b - c'}{b + c'} \tan \frac{B' + C'}{2} =$	.21961
$\log \tan \frac{B' - C'}{2} =$	.21962

94. How to choose a formula. The three examples above illustrate sufficiently the method of solving triangles. There are nine formulas to choose from: three from the law of sines, three from the tangents of the half-angles, and three from the law of tangents. The first thing to look for is a formula which has only one unknown in it. In the first example above we were given the three sides, therefore the only formulas involving one unknown were those for $\tan \frac{A}{2}$, $\tan \frac{B}{2}$, and $\tan \frac{C}{2}$. Any one of these could have been used first. The relation $A + B + C = 180°$ may always be used to

find the third angle, but it is better, when the sides are given, to save it for a check.

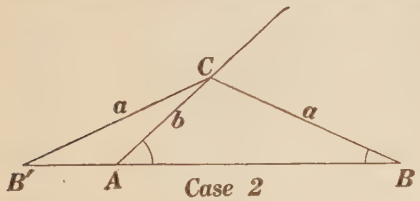
If two sides and the included angle are given, then the law of tangents must be used first and we must choose that one which involves the known angle. There is no other formula which has only one unknown in it.

95. The ambiguous case. The case when two sides and the angle opposite one of them are given is the most troublesome of all the cases. It is known as the **ambiguous** case, because there may be two solutions, as in the example above. This situation is not new, however, for we have met it before in plane geometry. The adjoining figures explain the various possibilities. Obviously the following cases arise.



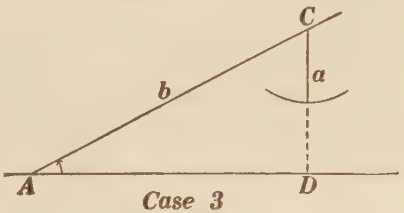
Case 1

CASE 1. **Angle A acute and side a < b** but a long enough to cut the side c of angle A in two points B' and B, both on the same side of point A. This will happen when a is greater than the perpendicular CD, and less than b, that is, when $b > a > b \sin A$. Then there are two distinct solutions, triangle $AB'C$ and ABC, both of which must be completely solved and checked.



Case 2

CASE 2. **Angle *A* acute and $a \geqq b$.** Then the triangle $AB'C$ does not fulfill the conditions, for the angle $B'AC$ is not equal to the given angle A. There is only one triangle in this case, namely, ABC.



Case 3

CASE 3. **Angle *A* acute and $a \leqq b \sin A$.** Since the distance $CD = b \sin A$, it follows that if $a < b \sin a$, there is no triangle which will satisfy the conditions. It is clear that if $a = b \sin A$, the resulting triangle is a right triangle.

CASE 4. **Angle A, obtuse.** If $a \leqq b$, there will be no solution otherwise one.

EXERCISES

1. In which of the above cases may the number of solutions be determined by inspection?

2. Show that there will be two solutions, one, or none according as $\log \sin B \gtreqless 0$.

96. Checks. A check is absolutely necessary, for no mathematician is so perfect that he never makes a mistake. The only way to be sure that we have solved a triangle correctly is to check it. The selection of a check formula is therefore as important as the selection of the proper formulas for the solution. What conditions must a good check formula fulfill?

Examine the three illustrative examples and verify that the check formulas satisfy the conditions.

97. Accuracy to be expected. If no errors are made and if the angles involved are not near 0° or 90°, the results should check to within $10''$ when the comparison of two angles is made. But if the check is made by comparing two logarithms, they should not differ by more than five or six units in the fifth decimal place.

An angle less than 3° or an angle differing from 90° by less than 3° may introduce errors in excess of the above limits. If great accuracy is desired, then a special table giving the values of the functions of small angles and angles near 90° at smaller intervals must be used. Such a table is given in the tables which accompany this text.* But the ordinary tables are accurate enough for all except the most exacting computations.

98. Types of errors. The errors made in computation may sometimes be detected without applying a check. Such errors are called **gross errors.** They often are detected by the appearance of a positive logarithm for the sine or cosine, or by the appearance of angles whose sum is greater than 180°. They are most often due to the use of a wrong formula, an error in the characteristic of a logarithm, taking the mantissa from the wrong column, or perform-

* Table IV.

ing the wrong arithmetic operation on the logarithms found. The latter may be rendered less likely by indicating the addition or subtraction by a + or − sign as was done in the illustrative examples.

Errors of interpolation are usually small and are detected only by applying the check. If such an error is not detected by going over the work carefully once, it is usually better to throw away all work and start over again.

99. EXERCISES

Find all the unknown parts and the area of the following triangles:

1.	$a = 647$ ft.	$A = 15° 18'$	$B = 72° 15'$
2.	$a = 62.71$ ft.	$A = 24° 17'$	$B = 55° 42'$
3.	$c = 116.2$ ft.	$A = 37° 36'$	$C = 101° 23'$
4.	$b = 63.27$ ft.	$B = 19° 15'$	$C = 68° 12'$
5.	$a = 784.10$ ft.	$b = 597.3$ ft.	$C = 46° 45'$
6.	$b = 117.61$ ft.	$c = 96.37$ ft.	$A = 73° 16'$
7.	$a = 210$ mi.	$b = 271$ mi.	$c = 241.6$ mi.
8.	$a = 91.62$ yds.	$b = 84.71$ yds.	$c = 96.90$ yds.
9.	$A = 16° 32'$	$a = 131.6$ cm.	$b = 157.7$ cm.
10.	$A = 37° 16'$	$a = 64.17$ ft.	$b = 47.81$ ft.
11.	$b = 3.1416$ m.	$A = 63° 10'$	$B = 46° 58'$
12.	$c = 43.76$ ft.	$A = 47° 12' 15''$	$B = 37° 40' 15''$
13.	$a = 64.8$ ft.	$b = 94.71$ ft.	$c = 81.64$ ft.
14.	$a = 17.19$ ft.	$b = 22.64$ ft.	$c = 30.78$ ft.

15. In the adjoining figure $AB = 500$ ft., $DBA = 73° 9'$, $CBD = 15° 1' 30''$, $CAB = 72° 35'$, $CAD = 15° 46' 13''$. Find CD.

16. When the altitude of the sun is 42°, a pole standing on a slope whose inclination is 20° casts a shadow down the slope 49.6 feet. How long is the pole?

17. From two stations in the same horizontal line with the base and on the same side of a tower the angles of elevation of its top are 28° and 56°. The distance between the two stations is 400 feet. Find the height of the tower.

18. The distances from a point to three objects are 217 feet, 384 feet, 516 feet, respectively. The angles included by the lines of sight of the objects taken in order are 64° and 34°. Find the distances between the objects.

19. Two straight railroad tracks intersect at an angle of 42°. How far apart will two hand cars be at the end of one hour if they start from the crossing at the same instant, their speeds being 12 and 14 miles per hour respectively?

20. An observer on top of a hill 100 feet above a lake finds the angle of elevation of a mountain top across the lake to be 33°. He also finds the angle of depression of the reflection of the mountain top in the surface of the lake to be 47°. Find the height of the mountain top above the lake in a single computation.

21. In surveying a harbor a submerged rock, D, was located by sighting three objects A, B, C on shore from a boat immediately above the rock. If $AB = 300$ ft., $CA = 500$ ft., $CDB = 23° 25'$, $CDA = 30° 50'$, $ACB = 62° 37'$, find the distance from the rock to C and the angle this line makes with AC.

22. A yacht sails northeast until it reaches a lightship. It then sails east 30° south (its course makes an angle of 30° with the east-west line in a southerly direction). It follows that course for 10 miles, then heads for its starting point, which it reaches after 15 miles. In what direction was it sailing on the last tack?

23. A triangular piece of land measuring 100 yards, 80 yards, and 60 yards along its sides is to be staked off from a large piece of land. How is this to be accomplished if the first line is determined in position and direction?

100. Impossible triangles. In order to center the student's attention on the principal problem we have purposely avoided impossible triangles. If the data are not sufficient to determine a triangle, this fact may usually be discovered by making a fairly careful figure before proceeding to the solution. It is advisable to subject the data to the following three rules:

(1) *If the three sides are given, the sum of any two must be greater than the third;*

(2) *If two angles are given, their sum must be less than 180°;*

(3) *If A, b, a, are given, then a must be greater than or equal to $b \sin A$ if A is acute and greater than b if A is obtuse.* (See ambiguous case, article **95**.)

If one attempts to solve an impossible triangle by logarithms, there will arise an impossible situation in the computation. This usually appears in the form of a positive logarithm for a sine or cosine.

101. Summary and review exercises. This chapter has been directed toward one end, namely, toward developing and learning

how to use the formulas necessary for solving any triangle. We have avoided all digressions and have derived only the minimum number of formulas. There are ten of these formulas including the area formula. They are collected here for reference.

(1) $$\tan\frac{A}{2} = \frac{r}{s-a}$$

(2) $$\tan\frac{B}{2} = \frac{r}{s-b}$$

(3) $$\tan\frac{C}{2} = \frac{r}{s-c}$$

where $$r = \sqrt{\frac{(s-a)(s-b)(s-c)}{s}}$$

(4), (5), (6) $$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

(7) $$\tan\frac{A-B}{2} = \frac{a-b}{a+b}\cot\frac{C}{2}$$

(8) $$\tan\frac{A-C}{2} = \frac{a-c}{a+c}\cot\frac{B}{2}$$

(9) $$\tan\frac{B-C}{2} = \frac{b-c}{b+c}\cot\frac{A}{2}$$

(10) $$\text{Area} = \tfrac{1}{2}bc\sin A$$

The relation $A + B + C = 180°$ is useful in connection with the above formulas.

EXERCISES

1. Prove the following formula for the area of a triangle

$$\text{Area} = bc\sin\frac{A}{2}\cos\frac{A}{2}.$$

2. Show that if a circle is inscribed in a triangle ABC, then $s - a$ is the distance from A to the points of tangency on the sides b and c.

3. Show that the radius of the circle inscribed in the triangle ABC is

$$r = \sqrt{\frac{(s-a)(s-b)(s-c)}{s}}.$$

4. Prove that the area of any triangle in terms of its sides is

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)} = rs.$$

Which of the following are possible triangles?

5. $a = 100$, $b = 60$, $c = 170$.

6. $A = 161° 10'$, $c = 17.8$ ft., $B = 30° 15'$.

7. $A = 142° 18'$, $b = 28.17$ ft., $C = 13° 27'$.

8. Attempt to solve the three triangles above by the formulas and find where the difficulty arises.

9. From the top and bottom of a vertical column the elevation angles of the summit of a tower 225 feet high and standing on the same horizontal plane are 41° and 53°. Find the height of the column.

10. A man standing due south of a tower 150 feet high finds its elevation to be 70°. He walks due west to a street where its elevation is 40°; walking farther in the same direction one block, he finds its elevation to be 20°. What is the length of the block?

11. Two sides of a triangle including an acute angle are 5 and 7 feet. The area is 14. Find the other side.

12. If A, B, and C are the angles of a triangle, prove

$$\cos A + \cos B + \cos C = 1 + 4 \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}.$$

13. If x, y, and z are the angles of a triangle, prove

$$\sin 2x + \sin 2y + \sin 2z = 4 \sin x \sin y \sin z.$$

14. Show that in any triangle

$$a^2 + b^2 + c^2 = 2(bc \cos A + ac \cos B + ab \cos C).$$

15. Compute the radius of the inscribed circle and the lengths of the segments into which it divides the sides of the triangle $a = 6$, $b = 10$, $c = 13$.

16. In any triangle prove that

$$\sin \frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}},$$

and

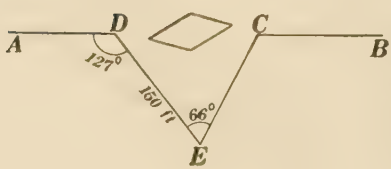
$$\cos \frac{A}{2} = \sqrt{\frac{s(s-a)}{bc}}.$$

17. Show that the radius of the circle circumscribing the triangle ABC is

$$R = \frac{a}{2 \sin A} = \frac{b}{2 \sin B} = \frac{c}{2 \sin C}.$$

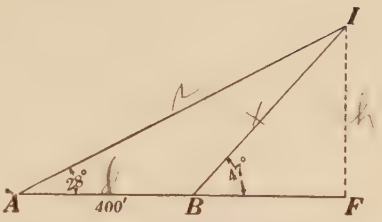
18. Using the results of exercise 17, show that the area of the triangle ABC is

$$\text{Area} = \frac{abc}{4R}.$$



19. Since an obstacle prevented running a straight line from A to B, an engineer measured the angles and distance shown in the adjoining figure. Find EC and the angle ECB so that $ADCB$ shall lie on the same straight line.

20. In the adjoining figure the angles of elevation of the top of the tower from A and B are 28° and 47° respectively. $AB = 400$ feet. A and B are in the same horizontal plane with the foot of the tower. Find the height of the tower.



21. In exercise 20 let the angles of elevation at A and B be α and β respectively and let $AB = d$. Show that the height of the tower is given by

$$h = \frac{d \sin \alpha \sin \beta}{\sin (\beta - \alpha)}.$$

22. Find the radius of the largest cylindrical gas tank which can be built on a triangular lot with sides 28 feet, 37 feet, and 44 feet. Locate the center of the base of the tank.

23. A tower is situated on a hillside the inclination of which is 20°. At a distance of 90 feet, measured from the foot of the tower down the slope, the tower subtends an angle of 20°. Find the height of the tower.

24. In example 23 let the inclination of the hillside be i and let t represent the angle the tower subtends at a distance d down the slope. Show that the height of the tower is

$$h = \frac{d \sin t}{\cos (t + i)}.$$

25. Prove that the area S of a triangle may be given by the formula

$$S = \frac{a^2 \sin B \sin C}{2 \sin A}.$$

26. Write two other forms of the formula in the last exercise.

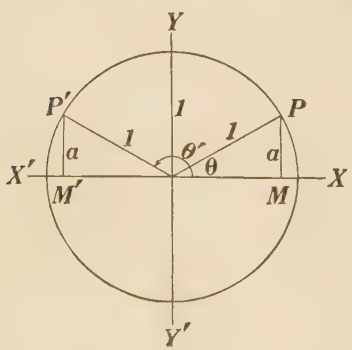
CHAPTER VII

THE INVERSE FUNCTIONS AND TRIGONOMETRIC EQUATIONS

102. To find the angle when a function is given. So far in our study of trigonometry we have with a few exceptions considered the problem of finding the functions when the angle is given. We proceed now to study the inverse problem: given the value of one of the functions to find the corresponding angles.

We have had occasion to find the angle θ from a given value of some function of θ in the solution of triangles. In particular in the solution of the general triangle in Chapter VI we found that there are sometimes two angles which may be taken when $\sin\theta$ is given. This illustrates a fundamental difference between the direct problem and inverse problem. We have seen that there is but one value of any one of the functions for a given angle but if the value of one of the functions is given, we shall find that there are many angles corresponding to it.

103. To find θ when sin θ is given. In the solution of the general triangle the angle to be found was limited to the first or second quadrant. If $\sin\theta$ is given and no limitation placed on the size of θ, then θ may have an unlimited number of values positive or negative.



Let $\sin\theta = a$. If a is greater than 1 or less than -1, the problem has no solution. (Why?) If a lies between 0 and $+1$, then there certainly are two angles, one in the first quadrant and one in the second, such that the sine of each is a, for in the figure $\theta' = 180^\circ - \theta$.

Hence

$$\sin\theta' = \sin(180^\circ - \theta) = \sin\theta = a.$$

But we may add or subtract 360° or any integral multiple of 360° to either of these angles without changing the sine, therefore we may write $\sin(180^\circ - \theta + n \cdot 360) = \sin(\theta + n \cdot 360^\circ) = a$ where n is any positive or negative integer.

Since

$$180^\circ - \theta + n \cdot 360^\circ = (2n + 1)\,180^\circ - \theta$$

and

$$\theta + n \cdot 360^\circ = 2n \cdot 180^\circ + \theta,$$

we have

$$(1) \qquad \sin[(2n + 1)\,180^\circ - \theta] = \sin(2n \cdot 180^\circ + \theta) = a.$$

This equation means that *all angles obtained either by subtracting θ from an odd multiple of 180° or by adding θ to an even multiple of 180° have the same sine.*

We may abbreviate these results by the following ingenious formula. Let θ_1 be the smallest positive angle which satisfies $\sin\theta = a$, then all the angles θ which satisfy the same equation are given by

$$\theta = m \cdot 180^\circ + (-1)^m \theta_1$$

or in radian measure

$$\theta = m\pi + (-1)^m \theta_1.$$

The m must be an integer, but it may be either even or odd, positive or negative. If it is even, then θ_1 has a $+$ sign, but if m is odd, θ_1 has a minus sign so that the results of (1) are embodied in a simple formula.

If a lies between -1 and 0, the above reasoning holds except that θ_1 lies in the fourth quadrant instead of the first and is negative. With this agreement the above formula holds where m may have any integral value and a any value such that $-1 \leqq a \leqq +1$.

EXERCISES

Without using the tables find all the angles which will satisfy the equations below:

1. $\sin\theta = \frac{1}{2}$, $\sin\theta = 0$.

2. $\sin\theta = \frac{\sqrt{3}}{2}$, $\sin\theta = \frac{1}{2}$.

3. $\sin\theta = -\frac{\sqrt{2}}{2}$, $\sin\theta = -\frac{\sqrt{3}}{2}$.

Use the tables to find all the angles which satisfy.

4. $\sin\theta = .6716$, $\sin\theta = -.3841$.

5. Show that if $\csc\theta = a$, $\theta = m \cdot 180^\circ + (-1)^m\theta_1$. What values may a have?

6. From the graph of $y = \sin x$ explain how you would find the angles whose sine is $\frac{1}{2}$, whose sine is $-\frac{1}{2}$.

104. Inverse formulas for $\cos\theta = a$ and $\tan\theta = a$. Arguments similar to those used in article **103** lead us to the following formulas for finding all values of θ when $\cos\theta$ or $\tan\theta$ is given.

$$\theta = 2m \cdot 180^\circ \pm \theta_1,$$

or $$\theta = 2m\pi \pm \theta_1, \qquad \text{when } \cos\theta = a,$$

and

$$\theta = m \cdot 180^\circ + \theta_1,$$

or $$\theta = m\pi + \theta_1, \qquad \text{when } \tan\theta = a.$$

The symbol $(-1)^m$ is not needed in these formulas since only one angle less than 180° is determined when $\cos\theta$ or $\tan\theta$ is given.

EXERCISES

Determine, without the table when possible, all the angles which satisfy the equations below.

1. $\cos\theta = \frac{1}{2}$, $\cos\theta = -\frac{1}{2}$.
2. $\tan\theta = \frac{1}{3}$, $\tan\theta = -\frac{1}{3}$.
3. $\cos\theta = .1923$, $\tan\theta = 1.6$.
4. $\tan\theta = -1.82$.

105. The inverse trigonometric functions. In the equation

$$x = \sin y$$

we have, heretofore, regarded x as a function of y. Now, we propose to regard y as a function of x. We express this fact by saying **y is an angle whose sine is x**. We abbreviate this statement by the equation

$$y = \arcsin x \text{ * (read } \textbf{arc sine } x\text{)}.$$

The function $\arcsin x$ is called an **inverse trigonometric function.**

In the same way we define $\arccos x$ to mean **the angle whose cosine is x**. Similarly $\arctan x$, $\operatorname{arc\,cot} x$, $\operatorname{arc\,sec} x$, $\operatorname{arc\,csc} x$ are

* The notation $y = \sin^{-1} x$ is in use also. The above notation is to be preferred since it avoids confusion with the exponent notation. Thus $\sin^2 x$ is $(\sin x)^2$, $\sin^{-1} x$ might be interpreted to mean $\frac{1}{\sin x}$. It also has the advantage of emphasizing the fact that y is an *angle*.

the angles whose tangent, cotangent, secant, and cosecant respectively are equal to x.

The function arc sin x is not defined for values of x numerically larger than 1 since there is no angle whose sine is numerically greater than 1. On the other hand, if $|x| < 1$, there are infinitely many angles whose sine is x. Unless the contrary is specifically stated by arc sin x we mean the *numerically smallest angle* whose sine is x. Similar statements hold for arc tan x, arc cot x, and arc csc x. By arc cos x we mean the *smallest positive angle* whose cosine is x. Similarly for arc sec x. These are sometimes called the **principal values** of the angle and are denoted by capital letters; thus, Arc sin x, etc. The notation though desirable is not yet generally in use.

106. Method of dealing with the inverse functions. By the definition of arc sin x the two equations

$$y = \text{arc sin } x$$

and

$$x = \sin y$$

mean exactly the same thing. It is therefore possible to turn the inverse functions into the direct functions. That is what is usually done when an identity between them is to be established. So that it is not necessary to develop a set of formulas for the inverse functions.

In working with inverse functions we must keep in mind that arc sin x, arc cos x, etc., are angles, whereas the direct functions sin x, cos x, etc., are pure numbers like $\frac{1}{2}$ and .7. For example, arc sin $\frac{1}{2} = 30°$, arc cos $\frac{1}{2} = 60°$.

From the definition of the inverse functions it follows that

$$\sin(\text{arc sin } x) = x, \quad \cos(\text{arc cos } x) = x, \quad \tan(\text{arc tan } x) = x,$$

with similar statements for arc cot x, arc sec x, and arc csc x.

EXERCISES

Find the number of degrees and radians in each of the following angles:

1. arc sin $\frac{\sqrt{3}}{2}$, arc cos $\frac{1}{\sqrt{2}}$.
2. arc tan 1, arc tan $\frac{\sqrt{3}}{3}$.
3. arc cot $(-\sqrt{3})$, arc tan (-1).
4. arc sin (-1), arc cos (-1).
5. arc cos 0, arc sin 0.

Find the values of the functions:

6. $\sin\left(\arctan\frac{\sqrt{3}}{3}\right)$.

7. $\tan\left(\arccos\frac{\sqrt{2}}{2}\right)$.

8. $\cos(\arctan a)$.

9. $\sin[\arctan(-1)]$.

10. $\cos(\arccos\frac{3}{4})$.

11. $\sin(\arccos\frac{3}{4})$.

12. $\cos(\arcsin\frac{1}{2}+\arccos\frac{1}{2})$.

13. $\csc(\arccsc 3)$.

14. $\cos(2\arccos\frac{1}{3})$.

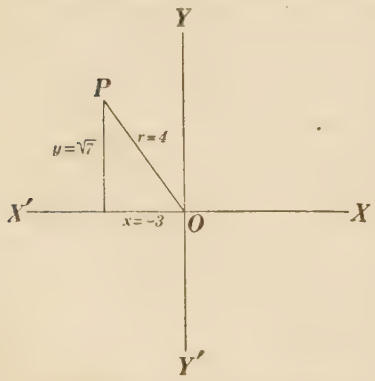
15. $\cos(\frac{1}{2}\arccos\frac{1}{3})$.

16. $\tan(2\arcsin\frac{3}{5})$.

17. Since $y=\sin x$ and $x=\arcsin y$ are equivalent equations, they are represented by the same graph. Show that the graph of $y=\arcsin x$ may be obtained from that of $y=\sin x$ by renaming the axes.

18. Draw graphs of the other inverse trigonometric functions by the method of the last exercise.

107. To contsruct an inverse function. Let $\theta=\arccos(-\frac{3}{4})$. Construct the right triangle xyr with $x=-3$ and $r=4$, then the $\angle XOP=\theta$. The θ might also be an angle of the third quadrant.



It is clear that θ might equally well be defined as $\arcsin\frac{\sqrt{7}}{4}$ but in this case only one angle, $0\leqq\theta\leqq 2\pi$, can be constructed.

From the figure it is apparent that $\sin(\arccos-\frac{3}{4})=\frac{\sqrt{7}}{4}$ and $\sec(\arccos-\frac{3}{4})=-\frac{4}{3}$.

108. Addition formulas for inverse functions. To reduce the sum of two inverse functions to a single inverse function we proceed as follows:

Let $x=\arcsin a,\ y=\arcsin b,$

then $\sin x=a,\quad \sin y=b$

and $\arcsin a+\arcsin b=x+y.$

Taking the sine of both members of this equation, we find

$$\begin{aligned}\sin(\arcsin a + \arcsin b) &= \sin(x + y)\\ &= \sin x \cos y + \cos x \sin y\\ &= a\sqrt{1 - b^2} + b\sqrt{1 - a^2}\end{aligned}$$

Hence

$$\arcsin a + \arcsin b = \arcsin(a\sqrt{1 - b^2} + b\sqrt{1 - a^2}).$$

In a similar manner any problem of this type may be discussed.

109. EXERCISES

Prove that:

1. $\arcsin a + \arccos b = \arcsin(ab + \sqrt{1 - a^2}\sqrt{1 - b^2})$.
2. $\arcsin \frac{1}{2} + \arccos \frac{1}{2} = \frac{\pi}{2}$.
3. $\arctan a + \arctan b = \arctan \frac{a + b}{1 - ab}$.
4. $2 \arcsin a = \arcsin 2a\sqrt{1 - a^2}$.
5. $2 \arccos a = \arccos(2a^2 - 1)$
6. $\arctan \sqrt{\frac{1 - \cos x}{1 + \cos x}} = \frac{x}{2}$.
7. If $y = \arcsin \frac{x}{\sqrt{1 + x^2}}$, prove $\tan y = x$.
8. $\arcsin \sqrt{\frac{x}{a + x}} = \arctan \sqrt{\frac{x}{a}}$.

110. Trigonometric equations. The equation

$$2 \sin^2 x - 3 \sin x + 1 = 0$$

holds for but two values of $\sin x$, $\sin x = \frac{1}{2}$, and $\sin x = 1$. There are an unlimited number of values of the angle x which satisfy this equation, for $x = m \cdot 180° + (-1)^m 30°$ and $x = m \cdot 180° + (-1)^m 90°$ will satisfy it for every integral value of m. It is usual, however, to limit the size of the angle to lie between 0° and 360°. In this event the only permissible values of the angle are $x = 30°, 150°, 90°$. Since the equation holds for isolated values of the unknown and not for all values, it is called a **conditional equation**.

111. Method of attack. The problem in the previous paragraph is relatively easy, for it is simply a quadratic equation in $\sin x$. Not all trigonometric equations are so easily handled. For example,

$$2\cos^2 x + 3\sin x = 0.$$

This may be changed into a quadratic equation in $\sin x$ by replacing $\cos^2 x$ by $1 - \sin^2 x$:

$$2(1 - \sin^2 x) + 3\sin x = 0,$$
$$2\sin^2 x - 3\sin x - 2 = 0,$$
$$\sin x = \frac{3 \pm \sqrt{9 + 16}}{4} = 2 \text{ or } -\tfrac{1}{2}.$$

The value 2 must be rejected. (Why?)
Therefore $x = 210°$ or $330°$.

Again the equation $2\sin\theta\sin 3\theta - \sin^2 2\theta = 0$ needs considerable simplification since different angles are involved. First by the factoring formulas of article **84** we find that

$$\cos 2\theta - \cos 4\theta = 2\sin\theta\sin 3\theta,$$

so that the equation may be written

$$\cos 2\theta - \cos 4\theta - \sin^2 2\theta = 0.$$

But
$$\cos 4\theta = \cos^2 2\theta - \sin^2 2\theta.$$

Hence the equation reduces to

$$\cos 2\theta - \cos^2 2\theta = 0$$

which is a quadratic in $\cos 2\theta$. The solution is now obvious.

In every case we must express all the functions in terms of a *single* function (usually the sine or cosine) of a *single* angle. The equation then becomes algebraic with this function as the variable. As usual, checks should be applied to insure accuracy.

EXERCISES

Solve for all values of the angle between 0° and 360°.

1. $\sin^2\theta + \cos\theta = 1$.
2. $\sin 2\theta + \sin\theta = 0$.
3. $\sin 2\theta + \cos 2\theta + 1 = 0$.
4. $\tan\theta + \cot\theta = 0$.
5. $(\cos x + \sin x)^2 = \sin^2 x$.
6. $\sin 2x + 1 = \cos 2x + 2\sin x$.
7. $\sin 2\theta + \sin\theta = \cos 2\theta + \cos\theta$.
8. $\cot 2\theta = \tan\theta - 1$.

112. Literal equations. The equation $\sin\theta = a$ is solved when we express θ as arc sin a.

Ex. $$a\sin^2\theta + \cos\theta = 1,$$
$$a(1 - \cos^2\theta) + \cos\theta = 1,$$
$$a\cos^2\theta - \cos\theta + (1 - a) = 0,$$
$$\cos\theta = \frac{1 \pm \sqrt{1 - 4a(1-a)}}{2a}$$
$$= \frac{1 \pm (1 - 2a)}{2a},$$
$$\cos\theta = \frac{1-a}{a} \text{ or } 1.$$

Hence $$\theta = \arccos\frac{1-a}{a} \text{ or } \theta = 0°.$$

Ex. Solve for θ, $a\sin\theta + b\cos\theta = c$.

This equation may be solved by an ingenious device.

Let (1) $a = r\sin x$ and (2) $b = r\cos x$. Then substituting these values in the above equation, we get

$$r\sin x\sin\theta + r\cos x\cos\theta = c,$$

or $$r\cos(\theta - x) = c.$$

Then $$\cos(\theta - x) = \frac{c}{r}.$$

But since $\sin^2 x + \cos^2 x = 1$, we have $r^2 = a^2 + b^2$.

Hence $$\cos(\theta - x) = \frac{c}{\sqrt{a^2 + b^2}},$$

or $$\theta - x = \arccos\frac{c}{\sqrt{a^2 + b^2}}.$$

Now dividing (1) by (2), $\tan x = \frac{a}{b}$, or $x = \arctan\frac{a}{b}$.

Hence $$\theta = \arctan\frac{a}{b} + \arccos\frac{c}{\sqrt{a^2 + b^2}}.$$

The use of logarithms is advisable if a, b, and c contain several digits. Such problems occur, for example, in surveying.

113. EXERCISES

Solve the following for θ:

1. $a\cos 2\theta + \cos\theta = -a$.
2. $\sin^2\theta + \cos\theta = c$.
3. $a\sin\theta + \cos 2\theta = c$.
4. $a\cos\theta + b\sin\theta = 0$.

Solve:

5. $2.1271 \sin\theta - 3.1566 \cos\theta = 1$.

6. $-3.4271 \sin\theta + 6 \cos\theta = -5.8469$.

7. Solve $a \sin\theta + b \cos\theta = c$ by setting $\sin\theta = \sqrt{1 - \cos^2\theta}$ and verify that the solution obtained is identical with that obtained in article **112**.

114. Summary and review exercises. The present chapter is concerned with two topics which are closely related.

1. The definitions of the inverse trigonometric functions and the method of using them.

2. The solution of various types of trigonometric equations.

There are very few new formulas needed for the discussion of these subjects, for we are able to change the inverse functions into direct functions and so to make use of the formulas of the previous chapters.

The trigonometric equations are reduced to algebraic equations in a single function of a single angle. There is no general rule for making the necessary changes, but they may always be made by means of the formulas for the direct functions.

The connection between inverse functions and trigonometric equations lies in the fact that the former are solutions of the latter.

For example, if

$$\sin\theta = a,$$

then

$$\theta = \arcsin a.$$

EXERCISES

1. Construct $\arcsin(-\frac{4}{5})$, $\arctan\frac{3}{5}$.

2. Prove that $\arcsin\frac{2}{5} = \arctan\frac{2}{\sqrt{21}}$.

3. Prove that $\arccos\frac{a}{\sqrt{a^2+b^2}} = \arctan\frac{b}{a}$.

4. If $\arccos(2x^2 - 2x) = \frac{2\pi}{3}$, find x.

5. If $\arctan(3x^2 - x + 1) = \frac{\pi}{4}$, find x.

6. Is $\arcsin(x^2 + 2x + 3) = \frac{\pi}{2}$ a possible equation?

7. Solve the simultaneous system

$$\arccos(2x - y) = \frac{\pi}{3},$$

$$\arctan(3x + 2y - 3) = \frac{\pi}{4}.$$

8. Prove that $\arccos a + \arctan b = \arcsin\left(\frac{ab + \sqrt{1 - a^2}}{\sqrt{1 + b^2}}\right)$.

Find all values of θ between 0 and 2π which satisfy the equations below.

9. $\cos^2\theta - \sin^2\theta = \frac{3}{4}$.

10. $\sin 5\theta + \sin 3\theta = \cos\theta$.

11. $\cos 3\theta + \cos 2\theta + \cos\theta = 0$.

12. $6\tan^2\theta - 4\sin^2\theta = 1$.

13. $\frac{\sin\theta}{1 - \cos\theta} = 2 + \cot\theta$.

14. $\sin\theta + \sin 2\theta = \sqrt{3}\cos\frac{\theta}{2}$.

15. $\sin\theta + \sin 2\theta + \sin 3\theta = 1 + \cos\theta + \cos 2\theta$.

16. $2\sin 3\theta\cos\theta = 2\sin\theta\cos 3\theta - 1$.

17. $\tan\theta + 2\cot\theta = \frac{5}{2}\csc\theta$.

18. $\cos 2\theta + \sin\theta = 2(1 - \cos 2\theta)$.

19. $\sec^2\theta + \csc^2\theta = \frac{16}{3}$.

20. Prove that $\arcsin\frac{\sqrt{5}}{5} + \operatorname{arccot} 3 = \frac{\pi}{4}$.

21. Prove that $x + \arctan(\cot 2x) = \arctan(\cot x)$.

22. Determine all angles such that the cotangent of each is twice its cosine.

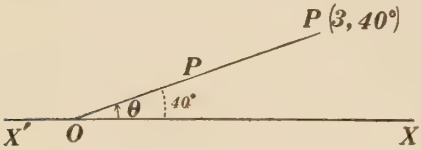
23. Determine all angles such that the secant of each is twice the reciprocal of its cotangent.

24. Determine all angles such that the sum of the sine and cosine of each is zero.

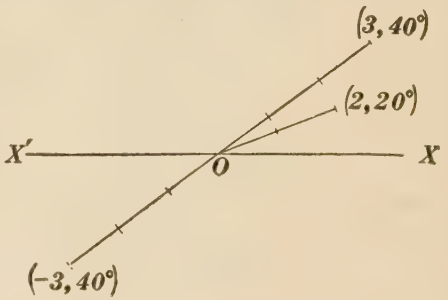
CHAPTER VIII

POLAR COÖRDINATES. WAVE MOTION

115. Polar coördinates. We have learned how to plot points in rectangular coördinates. In fact we defined the trigonometric functions in terms of the rectangular coördinates of a point. We now learn to locate points in another way and to make use of this method in studying a certain type of curve.



Let $X'O$ be a straight line and O any point on it. Let P be any point in the plane. Join P to O. The position of P is known if the angle XOP and the distance OP are given. We denote angle XOP by θ. It is always taken as positive if generated by rotating OP from an initial position along the line OX in a counter-clockwise sense about O. If the rotation is clockwise, θ is negative. The distance OP is denoted by ρ and is positive when measured from O along the terminal side of θ and negative when measured in the opposite direction. The numbers (ρ, θ) are the **polar coördinates** of P. The line $X'OX$ is called the **initial line** and O the **pole** of a system of **polar coördinates**. It is apparent that every point may be represented by a pair of numbers in four different ways and no more, provided that we limit the size of the angle to less than 360°. They are (ρ, θ), $(\rho, -360° + \theta)$, $(-\rho, 180° + \theta)$, and $(-\rho, -180° + \theta)$. Thus in the first figure the coördinates of P may be written (3, 40°), (3, − 320°), (− 3, 220°), and (− 3, − 140°). To locate a point when its polar coördinates are given



first turn off the angle θ by rotating OP from an initial position on OX in a counter-clockwise or clockwise sense according as the angle θ is positive or negative. Then measure ρ along the terminal side of θ if ρ is positive, but if ρ is negative, measure it in the opposite direction. The second figure shows the points $(3, 40°)$, $(2, 20°)$, and $(-3, 40°)$.

EXERCISES

Plot the points:

1. $(2, 60°)$, $\left(2, \frac{\pi}{2}\right)$, $\left(-2, \frac{\pi}{2}\right)$. 2. $(-3, 30°)$, $(5, 65°)$, $(4, 150°)$.

3. $(3, 210°)$, $\left(2, \frac{3\pi}{2}\right)$, $(7, 0)$.

4. Write the coördinates of each of the points in the first three exercises in three other ways.

5. Write the coördinate of a point symmetric to $(3, 40°)$, (a) with respect to the pole, (b) with respect to the initial line, (c) with respect to the perpendicular to the initial line through the pole (called the 90° line).

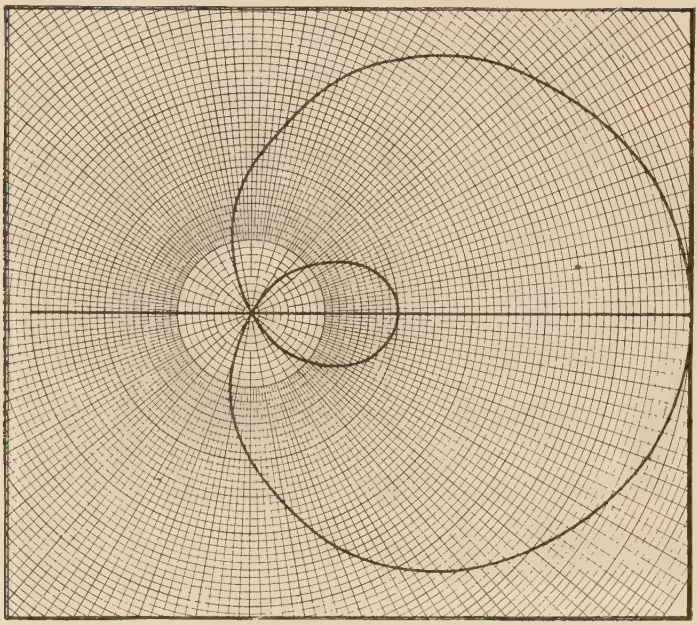
116. Plotting in polar coördinates. Let us take the equation

$$\rho = 1 + 2 \cos \theta.$$

From the table of natural functions construct a table of values at intervals of 10°. Plotting these points we obtain the curve shown in the adjoining figure. Any curve whose equation is given in polar coördinates may be plotted by this method.

ρ	θ	ρ	θ	ρ	θ
3.0	0	− 0.7	150	2.0	300
3.0	10	− 0.9	160	2.3	310
2.9	20	− 1.0	170	2.5	320
2.7	30	− 1.0	180	2.7	330
2.5	40	− 1.0	190	2.9	340
2.3	50	− 0.9	200	3.0	350
2.0	60	− 0.7	210	3.0	360
1.7	70	− 0.5	220		
1.3	80	− 0.3	230		
1.0	90	0.0	240		
0.7	100	0.3	250		
0.3	110	0.7	260		
0.0	120	1.0	270		
− 0.3	130	1.3	280		
− 0.5	140	1.7	290		

It will be observed from the table that two angles θ and $360° - \theta$ give the same value of ρ due to the fact that $\cos(360° - \theta) = \cos \theta$. This fact is evidenced by the symmetry of the curve with respect to the initial line.



EXERCISES

Plot the curves given by the following equations:

1. $\rho = \sin\theta$.
2. $\rho = 1 + \sin\theta$.
3. $\rho = \sin 2\theta$.
4. $\rho = \cos 2\theta$.
5. $\rho^2 = \cos 2\theta$.
6. $\rho^2 = \sin\theta$.
7. $\rho = 1 - \sin 2\theta$.
8. $\rho^2 = 1 + \sin\theta$.
9. $\rho = \cos^2\theta$.
10. $\rho^2 = 1 + \sin\theta$.
11. $\rho = \tan\theta$.
12. $\rho = \sec^2\frac{\theta}{2}$.
13. $\rho = \sin 3\theta$.
14. $\rho = \cos 3\theta$.

15. Show that if in $f(\rho, \theta) = 0$ you can replace θ by $-\theta$ without changing the function, the curve is symmetric to the initial line; if you can replace ρ by $-\rho$ without changing the function, it is symmetric with respect to the pole; if you can replace θ by $\pi - \theta$ without changing the function, it is symmetric to the 90° line.

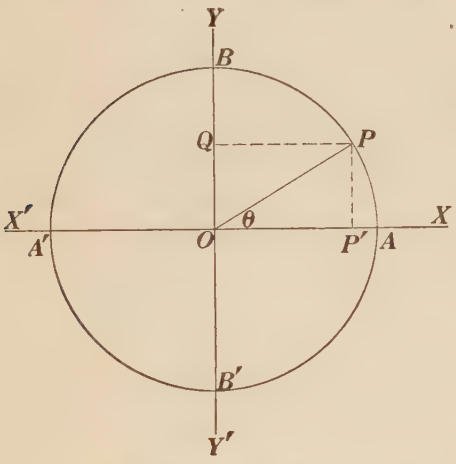
16. Since there are four ways of representing a point in polar coördinates, the tests given in exercise 15 are not the only ones which may be devised. What are some others?

17. Apply the tests in the last two exercises to the curves in (1)–(14)

117. Simple harmonic motion. The trigonometric functions are of great importance, not only because they enable us to compute distances and angles, but also on account of their periodicity. Many natural phenomena are periodic in character, and whenever an attempt is made to get a graphic representation of such phenomena the trigonometric functions are generally used.

This periodicity property has been discussed several times already. In the last set of exercises it appeared with especial clearness in the loops of the curves in exercises 3 and 13.

Perhaps the simplest example of periodically recurring motion is that of a uniformly rotating body. Let the point P describe a circular path of radius r about the point O as a center and let θ be the angle OP makes with the x-axis. Let us start measuring time when P is at A so that $\theta = 0$, when $t = 0$. We further assume that P moves with uniform angular velocity, that is, θ increases uniformly or $\theta = at$, where a is a constant. If P moves in the counter-clockwise sense, a is positive. As P moves around the circle its projection on the x-axis, P', moves to and fro between A and A', but P' does not move with uniform speed. The position of P' is given by the equation



$$OP' = OP \cos \theta$$

or

$$x = r \cos \theta = r \cos at. \tag{1}$$

Similarly the point Q moves according to the law

$$y = r \sin \theta = r \sin at. \tag{2}$$

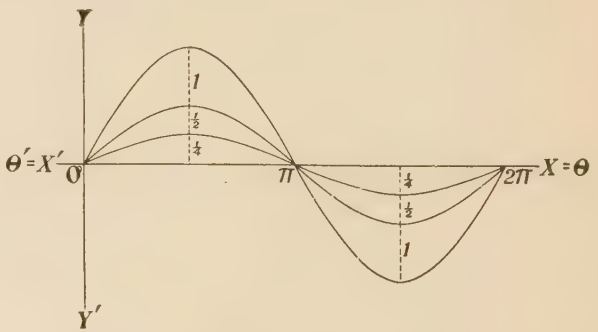
Such a motion is called a **simple harmonic motion**. It is evident that the motion of Q on the y-axis is exactly like the motion of P' on the x-axis. Since $\sin \theta$ is a little more simple to handle than $\cos \theta$, we shall study equation (2).

118. The period and amplitude. Evidently the point Q starts at the origin, moves up to B, turns and moves down through O to B' and back to O. It then repeats the same motion. The time taken, from the instant Q leaves O till it reaches finally O again, is the **period**. We are able to find the period from equation (2). The period of $\sin\theta$ is 2π, but $\theta = at$ or $t = \frac{\theta}{a}$; therefore, the period in t is $\frac{2\pi}{a}$. This means that if θ is measured in radians and t in seconds, it takes $\frac{2\pi}{a}$ seconds for Q to move from O up to B, down to B', and up to O again.

The largest value $\sin\theta$ has is 1. Hence the largest value y attains is r. This constant, r, is called the **amplitude** of the motion.

119. Graphical representation of the motion. In article **62** we plotted the curve $y = \sin x$. From the table of values given there we may easily construct a table of values of $y = r\sin\theta$. The values in the adjoining table are taken from that article. The graph depends on the value assigned to r. The three curves shown are found by assigning the values $\frac{1}{4}$, $\frac{1}{2}$, and 1 to r. The amplitude is clearly the height of the crest of the wave above the θ-axis. The period is represented by the distance along the θ-axis from the origin to the point at which $\theta = 2\pi$ or 360°. Notice that we may make a curve of any height whatever by varying r.

θ	
0	.0
30	.5 r
45	.7 r
60	.9 r
90	r
120	.9 r
135	.7 r
150	.5 r
180	0
210	− .5 r
225	− .7 r
240	− .9 r
270	− r
300	− .9 r
315	− .7 r
330	− .5 r
360	0



EXERCISES

Draw the graphs of the following:

1. $y = 3 \sin x$, $y = \frac{1}{3} \sin x$.

2. $y = 4 \sin x$, $y = \frac{1}{4} \sin x$.

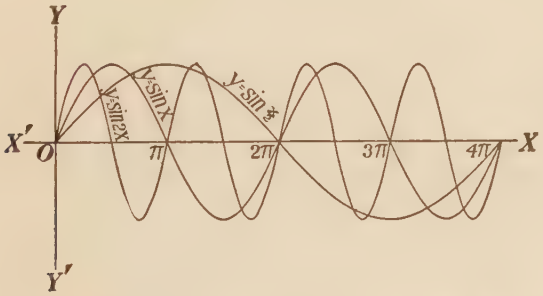
3. Determine graphically the period and amplitude of each of the functions in exercises 1 and 2.

4. Explain how the curves in exercises 1 and 2 may be represented by a single graph by choosing a suitable scale for each on the y-axis.

120. Functions with various periods. We have found how to write the equation of a periodic function which shall have any desired amplitude by assigning a proper value to r in the equation

$$y = r \sin \theta.$$

It is equally possible to find a function with any period we please. By assigning values of x and computing the corresponding y, we easily plot the curves $y = \sin \frac{x}{2}$, $y = \sin x$, $y = \sin 2x$. The resulting curves are shown on the same set of axes. The graphs show that the function $y = \sin \frac{x}{2}$ has a period 4π; $y = \sin x$ has a period 2π; $y = \sin 2x$ has a period π. These results are in accord with article **118**, where we found that the period of $\sin at$ was $\frac{2\pi}{a}$. All the functions of this article have the same amplitude, which is equal to 1.



The equation $y = \sin ax$ is a generalization of the equations above, for we get these equations by making $a = \frac{1}{2}$, 1, and 2 respectively.

EXERCISES

Construct the graphs and determine the periods of the following:

1. $y = \sin 4x$.
2. $y = \sin \frac{x}{3}$.
3. $y = \sin \frac{3x}{2}$.
4. $y = \sin ax$.
5. $y = \cos 2x$.
6. $y = \cos \frac{x}{2}$.
7. $y = \cos ax$.

8. Explain how the three preceding curves may be represented by a single graph by a suitable choice of scale on the x-axis.

121. Functions with any period and any amplitude. By combining the results of the previous articles we may write a function which has any desired period and any desired amplitude. For this purpose it is only necessary to assign proper values to a and r in the equation

$$y = r \sin ax.$$

If we want a large amplitude, we make r large. If we want a small period, we make a large, and conversely.

Thus we are able by means of the trigonometric functions to construct functions whose graphs approximate very closely any wave motion that may occur in nature.

EXERCISES

Construct the graphs and determine the periods and amplitudes of the following functions:

1. $y = 2 \sin 2x$.
2. $y = 3 \sin 3x$.
3. $y = \frac{1}{2} \sin 2x$.
4. $y = \frac{1}{10} \sin 4x$.
5. $y = \frac{2}{3} \sin \frac{x}{2}$.
6. $y = 5 \cos \frac{x}{3}$.

Construct functions which have the following periods and amplitudes. Plot the functions.

7. $\frac{2\pi}{3}$, 2.
8. π, $\frac{1}{3}$.
9. $\frac{6\pi}{5}$, 3.
10. $\frac{\pi}{10}$, $\frac{1}{2}$.
11. 1.57, .6.

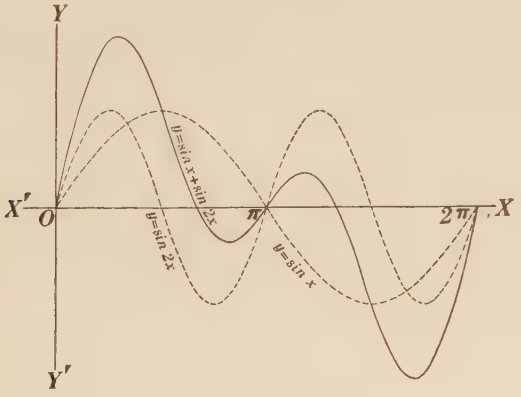
122. The graph of the sum of two functions. It is easy to get the graph of

$$y = \sin x + \sin 2x$$

from the graphs of $y = \sin x$ and $y = \sin 2x$. All that is necessary, after the $\sin x$ and $\sin 2x$ curves have been drawn, is to add the ordinates which correspond to any abscissa. This may be done rapidly by placing a ruler on the y-axis and moving it to the right (and left), measuring the ordinates and plotting their sum. The period of such a sum is the least common multiple of the periods of the parts. The approximate amplitude is found by measuring the distance from the x-axis up to the highest point on the curve.

Any function composed of sums or differences may be plotted by plotting its parts and adding algebraically the ordinates. For an example we show the graph of

$$y = \sin x + \sin 2x.$$



EXERCISES

Plot and find the periods and amplitudes if the function is periodic.

1. $y = \sin x + \cos x$.
2. $y = \sin 2x + \cos x$.
3. $y = \sin x - \cos x$.
4. $y = \sin 2x + \cos 2x$.
5. $y = x + \sin x$.
6. $y = 2 - \frac{\sin x}{2}$.
7. $y = \sin x - \sin 2x$.
8. $y = x^2 + \sin x$.

123. Retrospection and review. Every chapter of this book has had a definite aim. In order to get a good idea of the "rivets

which hold the chapter together" there is a summary at the end of each. Now we pause and look back over the whole course, endeavoring to get a grasp of the fundamentals.

The first three chapters were preparatory. In them we learned how to generalize the notion of an angle, to measure angles in a new unit, to compute unknown distances and angles by means of the sine, cosine, and tangent, and finally how to shorten computations by the use of logarithms. It is possible to solve any triangle by means of these three ratios without going through the developments of chapters IV, V, and VI. But to do so would deprive the student of much of the beauty and much of the usefulness of Trigonometry. We arrived at the idea of $\sin\theta$, $\cos\theta$, $\tan\theta$ as functions of the variable θ through our general definitions of these ratios in the beginning of chapter IV. Such general definitions were necessary in order to study the functions for all values of the angles and to derive their properties and illustrate these properties graphically.

By means of the definitions of article **49** we were able to express the secant, cosecant, and cotangent in terms of the sine, cosine, and tangent. Thus the latter three functions assume a rôle of greater importance, for the first three may always be replaced by them. The tangent was also expressed in terms of the sine and cosine by the formula

$$\tan\theta = \frac{\sin\theta}{\cos\theta}. \tag{1}$$

The theorem of Pythagoras gave us three more formulas:

$$\sin^2\theta + \cos^2\theta = 1, \tag{2}$$

$$\sec^2\theta = 1 + \tan^2\theta, \tag{3}$$

$$\csc^2\theta = 1 + \cot^2\theta. \tag{4}$$

These four formulas together with the three reciprocal formulas enable us to find all the functions when any one is given.

The angles $0°$, $90°$, $180°$, and $360°$ are important for all the functions. There is a group of formulas connected with each of these angles.

$$\sin(-\theta) = -\sin\theta, \qquad \cos(-\theta) = \cos\theta, \tag{5}$$

$$\cos(90 \pm \theta) = \mp\sin\theta, \qquad \sin(90 \pm \theta) = \cos\theta, \tag{6}$$

$$\sin(180 \pm \theta) = \mp\sin\theta, \qquad \cos(180 \pm \theta) = -\cos\theta. \tag{7}$$

These formulas made it unnecessary to continue the tables for angles greater than 45°.

The angle 360° (or 2π) is most interesting of all, for after θ passes 360° all the functions repeat their values. Thus 360° or 2π radians is the common period of all the trigonometric functions. This periodicity property was illustrated several times by graphs. In the present chapter it was evidenced by the loops in the polar curves as well as by the waves in the curves in rectangular coordinates.

The addition formulas for the sine and cosine are:

(8) $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B,$

(9) $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B.$

Out of these, by making $B = A$, we obtained the important formulas

(10) $\sin 2A = 2 \sin A \cos A,$

(11) $\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A.$

These six are indispensable in trigonometric analysis. Without them many trigonometric equations could not be solved and they aid us in transforming certain sums into products.

By addition and subtraction and renaming of angles we obtained from (8) and (9) the relations

(12) $$\sin x + \sin y = 2 \sin \frac{x+y}{2} \cos \frac{x-y}{2},$$

(13) $$\sin x - \sin y = 2 \cos \frac{x+y}{2} \sin \frac{x-y}{2},$$

(14) $$\cos x + \cos y = 2 \cos \frac{x+y}{2} \cos \frac{x-y}{2},$$

(15) $$\cos x - \cos y = -2 \sin \frac{x+y}{2} \sin \frac{x-y}{2}.$$

In the sets of formulas represented by (5)–(11) above we may always find the corresponding formula for the tangent from

$$\tan \theta = \frac{\sin \theta}{\cos \theta}.$$

The above formulas also enabled us to derive convenient formulas for solving the general triangle. From the law of cosines,

(16) $$a^2 = b^2 + c^2 - 2bc \cos A,$$

by using (11) we found

$$\tan\frac{A}{2} = \frac{r}{s-a} \text{ where} \tag{17}$$

$$r = \sqrt{\frac{(s-a)(s-b)(s-c)}{s}}.$$

From the law of sines,

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}, \tag{18}$$

by using (12) and (13), it followed that

$$\tan\frac{A-B}{2} = \frac{a-b}{a+b}\cot\frac{C}{2}. \tag{19}$$

There are two other formulas which may be written at once by simply changing the letters in (16), (17), and (19). These with the formulas for the area

$$\text{Area} = \frac{bc}{2}\sin A, \tag{20}$$

and

$$\text{Area} = rs, \tag{21}$$

enable us to solve any triangle.

Thus the whole of trigonometry is grouped about these 21 formulas. The number of combinations of them is endless and the variety of uses found for them is truly amazing. We conclude with a set of exercises which use them in many ways. The mastery of trigonometry is certain when the ability to handle such exercises is attained.

GENERAL EXERCISES

1. Find six angles between $-360°$ and $+540°$ whose sine is .27871.
2. Express the following angles in degrees

$$\frac{3\pi}{2},\ 1,\ 2.5\pi,\ -2,\ \frac{9\pi}{4},\ \frac{\pi}{12},\ -3.4\pi.$$

3. Prove that $\arccos\frac{3}{\sqrt{10}} + \arcsin\frac{1}{\sqrt{5}} = \frac{\pi}{4}$.
4. Draw the graph of $\sec 2x$.

5. Construct arc cos $\frac{2}{3}$. Find tan (arc cos $\frac{2}{3}$).

6. Plot $y = \csc x$.

7. What is the period of the following functions: $\sin 3x$, $\cos 2x$, $\tan 2x$, $\cos \frac{x}{4}$?

8. Prove that $\sin x + \sin 3x + \sin 5x = \frac{\sin^2 3x}{\sin x}$.

9. Prove that $\tan (x + y) = \frac{\sin 2x + \sin 2y}{\cos 2x + \cos 2y}$.

10. If $A + B + C = 90°$, prove that

$$\tan A \tan B + \tan B \tan C + \tan C \tan A = 1.$$

11. Through how many radians do the minute hand and hour hand of a clock turn in 30 seconds? In 20 minutes?

12. The front wheel of a wagon is 2 ft. in diameter, the back wheel 3 ft. Through how many radians will the back wheel turn while the front is turning through 390°?

13. Prove that $\sin 105° + \cos 105° = \cos 45°$.

14. Prove that

$$\cos (45° - x) \cos (45° + x) - \sin (45° - x) \sin (45° + x) = 0.$$

15. Prove that

$$\sin x \cos (90° - x) - \cos x \sin (90° - x) = -\cos 2x.$$

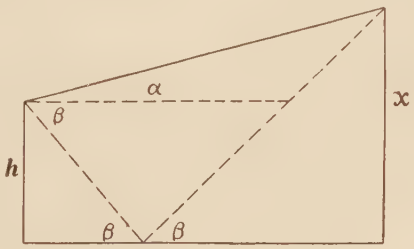
16. Prove that

$$\cos (30° - 45°) - \cos (30° + 45°) = \sin 45°.$$

17. Given $\sin 1° = .01745$, find $\sin 2°$. Show how to compute a table of sines for intervals of 1°.

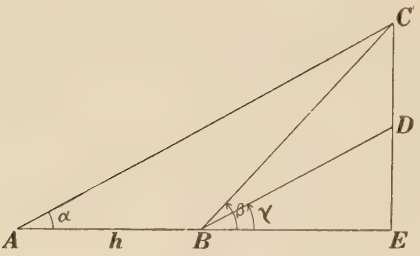
18. A man standing on a hill h feet above a lake observes the angle α of elevation of the top of a mountain across the lake. He also observes β, the angle of depression of the reflection of the top of the mountain in the lake. Show that the height of the mountain above the lake is given by (see exercise 20, article **99**)

$$x = h \frac{\sin (\beta + \alpha)}{\sin (\beta - \alpha)}.$$



19. Discuss problem 18 for various values of α and β. Particularly when $\alpha = \beta$ and $\alpha > \beta$.

20. Solve problem 18 when $h = 50$ feet, $\alpha = 10°$, $\beta = 20°$.



21. In the adjoining figure show that CD may be found from the formula

$$CD = h\frac{\sin\alpha\sin(\beta-\gamma)}{\cos\gamma\sin(\beta-\alpha)}.$$

22. Discuss problem 21 for various values of α, β, γ, particularly $\beta = \gamma$, and $\beta = \alpha$, $\gamma = 0$.

23. Solve problem 21 if $h = 500$ feet, $\alpha = 27°$, $\beta = 42°$, $\gamma = 25°$.

24. Prove that

$$\tan(2\arctan x) = \frac{2x}{1-x^2}.$$

25. Prove $\arcsin x + \arcsin\sqrt{1-x^2} = \frac{\pi}{2}$.

26. Prove $3\arctan x = \arctan\frac{3x-x^3}{1-3x^2}$.

27. Prove $2\arccos x = \arccos(2x^2-1)$.

28. Prove $\arctan\frac{1}{3} + \arctan\frac{1}{5} = \arctan\frac{4}{7}$.

Solve the following equations for all values of the unknown between 0° and 360°:

29. $\sin 2x = 2\cos x$.

30. $\sin x + \cos x = 1$.

31. $\sin x + \cos 2x = 4\sin^2 x$.

32. $\sec x = 4\csc x$.

33. $\cot\frac{\theta}{2} = 2 - \csc\theta$.

34. $\sin x + \sin 2x = \sin 3x$.

35. $\tan x + \cot x = \tan 2x$.

36. $\arcsin x + 2\arccos x = \frac{2\pi}{3}$.

37. $\arctan x + 2\operatorname{arccot} x = 135°$.

38. $\sin(x+12°) + \sin(x-8°) = \sin 20°$.

39. $2\sin x - \cos x = .1$.

Prove the following identities:

40. $(1+\cot A+\tan A)(\sin A-\cos A) = \frac{\sec A}{\csc^2 A} - \frac{\csc A}{\sec^2 A}$.

41. $\sin\theta + \sin\phi + \sin(\theta+\phi) = 4\cos\frac{\theta}{2}\cos\frac{\phi}{2}\sin\frac{\theta+\phi}{2}$.

42. $\cot^2 x - \cos^2 x = \cot^2 x\cos^2 x$.

43. $\cot^4 x + \cot^2 x = \csc^4 x - \csc^2 x$.

44. $(\sin 2x - \sin 2y) \tan (x + y) = 2(\sin^2 x - \sin^2 y)$.

45. $\sin x + \sin\left(x - \frac{2\pi}{3}\right) + \sin\left(\frac{\pi}{3} - x\right) = 0$.

46. $\cos x \sin (y - z) + \cos y \sin (z - x) + \cos z \sin (x - y) = 0$.

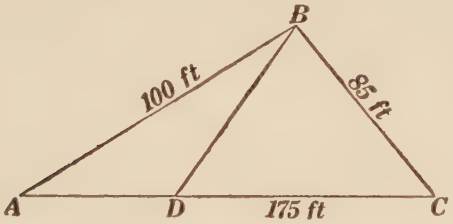
47. To find the height of a mountain a north-and-south base line 1000 yards long is measured. From one end of this base line the summit bears N. 80° E. and has an angle of elevation of 13° 14′. From the other end the summit bears N. 43° 30′ E. Find the height.

48. Discuss problem 47 for literal values of the angles and distances and work out a formula that will apply in all such cases.

49. To find the width of the Mississippi at New Orleans a base line 500 ft long was laid out along the levee. The angles which this line makes with the lines to a well-defined object across the river were found to be 90° and 85°. Find the width.

50. If ABC is any triangle prove that

$$\frac{a - b}{c} = \frac{\sin \frac{1}{2}(A - B)}{\cos \frac{1}{2} C}.$$ (Mollweide's Identity.)



51. In the above triangle find the length of BD so as to make the area of $ADB = \frac{1}{3}\ ABC$, if $DC = 175$ feet.

FORMULAS

Algebra

1. If $ax^2 + bx + c = 0$, then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

2. Arithmetic progression: $l = a + (n-1)d$, $s = \frac{n}{2}(a + l)$.

3. Geometric progression: $l = ar^{n-1}$; $s = \frac{a(1 - r^n)}{1 - r}$.

4. Binomial theorem. $(a + h)^n = a^n + na^{n-1}h + \frac{n(n-1)}{2!}a^{n-2}h^2 + \cdots$
$$+ \frac{n(n-1)(n-2)\cdots(n-r+1)}{r!}a^{n-r}h^r + \cdots + h^n.$$

5. Permutations of n things taken r at a time:
$${}_nP_r = n(n-1)(n-2)\cdots(n-r+1).$$

6. Combinations of n things taken r at a time: ${}_nC_r = \frac{{}_nP_r}{r!}$.

Plane Geometry

1. Area of a square $= a^2$.
2. Area of a parallelogram $= bh$.
3. Area of a triangle $= \frac{1}{2}bh$.
4. Area of a trapezoid $= \frac{1}{2}h(b_1 + b_2)$.
5. Area of a circle $= \pi r^2$.
6. Circumference of a circle $= 2\pi r$.

Solid Geometry

1. Volume of a prism $= bh$.
2. Volume of a pyramid $= \frac{1}{3}bh$.
3. Volume of a right circular cylinder $= \pi r^2 h$; its total surface $= 2\pi r(r + h)$.
4. Volume of a right circular cone $= \frac{1}{3}\pi r^2 h$; its total surface $= \pi r(r + l)$.
5. Volume of a sphere $= \frac{4}{3}\pi r^3$; its surface $= 4\pi r^2$.

Logarithms

1. Log ab = log a + log b.

2. $\log \frac{a}{b} = \log a - \log b$.

3. $\log a^n = n \log a$.

4. $\log \sqrt[n]{a} = \frac{\log a}{n}$.

5. $\log \frac{1}{a} = \operatorname{colog} a = (10 - \log a) - 10$.

LOGARITHMIC AND TRIGONOMETRIC TABLES

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Explanation of Table I

This table gives the mantissas of the common logarithms of numbers from 1 to 10,000. The characteristics must be supplied in accordance with the following rules:

If ***N*** *is greater than* ***1****, the characteristic of log* ***N*** *is one less than the number of places to the left of the decimal point.*

If ***N*** *is less than* ***1****, the characteristic is negative and is found by subtracting the number of zeros to the right of the decimal point, before the first significant figure, from* ***9–10****.*

The use of the table will be illustrated by the following examples:

Ex. 1. Find log 2023.7. This logarithm will be larger than the logarithm of 2023. To find the correction, find the tabular difference, which is the difference between log 2023 and log 2024 or .00021. Multiply this difference by .7. This gives .000147 as the correction to be added to log 2023. But since we are using five-place tables, we call this .00015. Hence log 2023.7 = 3.30615. We use the table of proportional parts to perform this multiplication. Look in the column headed Prop. Parts for 21. In this column are given the tenths of 21. Opposite 7 is the figure 14.7 which is taken as 15 units of the fifth decimal place as above.

Ex. 2. Log N = 9.10031 − 10. In the column headed 0 find the figure 10. Then since 031 lies between 003 and 037 the correction is 28 and the tabular difference is 34. The first four figures of N are 1259. The fifth figure is found, by dividing 28 by 34, to be 8. Hence N = .12598. The figure in the fifth place may be read from the table of proportional parts by hunting in the column headed 34 for the figures nearest to 28. These are found to be 27.2 and correspond to 8 tenths. Hence 8 is the figure in the fifth place.

TABLE I

FIVE-PLACE COMMON LOGARITHMS OF NUMBERS

FROM

1 TO 10,000

N.	0	1	2	3	4	5	6	7	8	9
100	00 000	043	087	130	173	217	260	303	346	389
101	432	475	518	561	604	647	689	732	775	817
102	860	903	945	988	*030	*072	*115	*157	*199	*242
103	01 284	326	368	410	452	494	536	578	620	662
104	703	745	787	828	870	912	953	995	*036	*078
105	02 119	160	202	243	284	325	366	407	449	490
106	531	572	612	653	694	735	776	816	857	898
107	938	979	*019	*060	*100	*141	*181	*222	*262	*302
108	03 342	383	423	463	503	543	583	623	663	703
109	743	782	822	862	902	941	981	*021	*060	*100
110	04 139	179	218	258	297	336	376	415	454	493
111	532	571	610	650	689	727	766	805	844	883
112	922	961	999	*038	*077	*115	*154	*192	*231	*269
113	05 308	346	385	423	461	500	538	576	614	652
114	690	729	767	805	843	881	918	956	994	*032
115	06 070	108	145	183	221	258	296	333	371	408
116	446	483	521	558	595	633	670	707	744	781
117	819	856	893	930	967	*004	*041	*078	*115	*151
118	07 188	225	262	298	335	372	408	445	482	518
119	555	591	628	664	700	737	773	809	846	882
120	918	954	990	*027	*063	*099	*135	*171	*207	*243
121	08 279	314	350	386	422	458	493	529	565	600
122	636	672	707	743	778	814	849	884	920	955
123	991	*026	*061	*096	*132	*167	*202	*237	*272	*307
124	09 342	377	412	447	482	517	552	587	621	656
125	691	726	760	795	830	864	899	934	968	*003
126	10 037	072	106	140	175	209	243	278	312	346
127	380	415	449	483	517	551	585	619	653	687
128	721	755	789	823	857	890	924	958	992	*025
129	11 059	093	126	160	193	227	261	294	327	361
130	394	428	461	494	528	561	594	628	661	694
131	727	760	793	826	860	893	926	959	992	*024
132	12 057	090	123	156	189	222	254	287	320	352
133	385	418	450	483	516	548	581	613	646	678
134	710	743	775	808	840	872	905	937	969	*001
135	13 033	066	098	130	162	194	226	258	290	322
136	354	386	418	450	481	513	545	577	609	640
137	672	704	735	767	799	830	862	893	925	956
138	988	*019	*051	*082	*114	*145	*176	*208	*239	*270
139	14 301	333	364	395	426	457	489	520	551	582
140	613	644	675	706	737	768	799	829	860	891
141	922	953	983	*014	*045	*076	*106	*137	*168	*198
142	15 229	259	290	320	351	381	412	442	473	503
143	534	564	594	625	655	685	715	746	776	806
144	836	866	897	927	957	987	*017	*047	*077	*107
145	16 137	167	197	227	256	286	316	346	376	406
146	435	465	495	524	554	584	613	643	673	702
147	732	761	791	820	850	879	909	938	967	997
148	17 026	056	085	114	143	173	202	231	260	289
149	319	348	377	406	435	464	493	522	551	580
150	609	638	667	696	725	754	782	811	840	869
N.	0	1	2	3	4	5	6	7	8	9

PROP. PARTS

	44	43	42
2	8.8	8.6	8.4
3	13.2	12.9	12.6
4	17.6	17.2	16.8
5	22.0	21.5	21.0
6	26.4	25.8	25.2
7	30.8	30.1	29.4
8	35.2	34.4	33.6
9	39.6	38.7	37.8

	41	40	39
2	8.2	8.0	7.8
3	12.3	12.0	11.7
4	16.4	16.0	15.6
5	20.5	20.0	19.5
6	24.6	24.0	23.4
7	28.7	28.0	27.3
8	32.8	32.0	31.2
9	36.9	36.0	35.1

	38	37	36
2	7.6	7.4	7.2
3	11.4	11.1	10.8
4	15.2	14.8	14.4
5	19.0	18.5	18.0
6	22.8	22.2	21.6
7	26.6	25.9	25.2
8	30.4	29.6	28.8
9	34.2	33.3	32.4

	35	34	33
2	7.0	6.8	6.6
3	10.5	10.2	9.9
4	14.0	13.6	13.2
5	17.5	17.0	16.5
6	21.0	20.4	19.8
7	24.5	23.8	23.1
8	28.0	27.2	26.4
9	31.5	30.6	29.7

	32	31	30
2	6.4	6.2	6.0
3	9.6	9.3	9.0
4	12.8	12.4	12.0
5	16.0	15.5	15.0
6	19.2	18.6	18.0
7	22.4	21.7	21.0
8	25.6	24.8	24.0
9	28.8	27.9	27.0

* The asterisk * indicates that the second digit of the mantissa is increased one.

N.	0	1	2	3	4	5	6	7	8	9
150	17 609	638	667	696	725	754	782	811	840	869
151	898	926	955	984	*013	*041	*070	*099	*127	*156
152	18 184	213	241	270	298	327	355	384	412	441
153	469	498	526	554	583	611	639	667	696	724
154	752	780	808	837	865	893	921	949	977	*005
155	19 033	061	089	117	145	173	201	229	257	285
156	312	340	368	396	424	451	479	507	535	562
157	590	618	645	673	700	728	756	783	811	838
158	866	893	921	948	976	*003	*030	*058	*085	*112
159	20 140	167	194	222	249	276	303	330	358	385
160	412	439	466	493	520	548	575	602	629	656
161	683	710	737	763	790	817	844	871	898	925
162	952	978	*005	*032	*059	*085	*112	*139	*165	*192
163	21 219	245	272	299	325	352	378	405	431	458
164	484	511	537	564	590	617	643	669	696	722
165	748	775	801	827	854	880	906	932	958	985
166	22 011	037	063	089	115	141	167	194	220	246
167	272	298	324	350	376	401	427	453	479	505
168	531	557	583	608	634	660	686	712	737	763
169	789	814	840	866	891	917	943	968	994	*019
170	23 045	070	096	121	147	172	198	223	249	274
171	300	325	350	376	401	426	452	477	502	528
172	553	578	603	629	654	679	704	729	754	779
173	805	830	855	880	905	930	955	980	*005	*030
174	24 055	080	105	130	155	180	204	229	254	279
175	304	329	353	378	403	428	452	477	502	527
176	551	576	601	625	650	674	699	724	748	773
177	797	822	846	871	895	920	944	969	993	*018
178	25 042	066	091	115	139	164	188	212	237	261
179	285	310	334	358	382	406	431	455	479	503
180	527	551	575	600	624	648	672	696	720	744
181	768	792	816	840	864	888	912	935	959	983
182	26 007	031	055	079	102	126	150	174	198	221
183	245	269	293	316	340	364	387	411	435	458
184	482	505	529	553	576	600	623	647	670	694
185	717	741	764	788	811	834	858	881	905	928
186	951	975	998	*021	*045	*068	*091	*114	*138	*161
187	27 184	207	231	254	277	300	323	346	370	393
188	416	439	462	485	508	531	554	577	600	623
189	646	669	692	715	738	761	784	807	830	852
190	875	898	921	944	967	989	*012	*035	*058	*081
191	28 103	126	149	171	194	217	240	262	285	307
192	330	353	375	398	421	443	466	488	511	533
193	556	578	601	623	646	668	691	713	735	758
194	780	803	825	847	870	892	914	937	959	981
195	29 003	026	048	070	092	115	137	159	181	203
196	226	248	270	292	314	336	358	380	403	425
197	447	469	491	513	535	557	579	601	623	645
198	667	688	710	732	754	776	798	820	842	863
199	885	907	929	951	973	994	*016	*038	*060	*081
200	30 103	125	146	168	190	211	233	255	276	298
N.	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**

PROP. PARTS

	29	28
2	5.8	5.6
3	8.7	8.4
4	11.6	11.2
5	14.5	14.0
6	17.4	16.8
7	20.3	19.6
8	23.2	22.4
9	26.1	25.2

	27	26
2	5.4	5.2
3	8.1	7.8
4	10.8	10.4
5	13.5	13.0
6	16.2	15.6
7	18.9	18.2
8	21.6	20.8
9	24.3	23.4

	25
2	5.0
3	7.5
4	10.0
5	12.5
6	15.0
7	17.5
8	20.0
9	22.5

	24	23
2	4.8	4.6
3	7.2	6.9
4	9.6	9.2
5	12.0	11.5
6	14.4	13.8
7	16.8	16.1
8	19.2	18.4
9	21.6	20.7

	22	21
2	4.4	4.2
3	6.6	6.3
4	8.8	8.4
5	11.0	10.5
6	13.2	12.6
7	15.4	14.7
8	17.6	16.8
9	19.8	18.9

N.	0	1	2	3	4	5	6	7	8	9
200	30 103	125	146	168	190	211	233	255	276	298
201	320	341	363	384	406	428	449	471	492	514
202	535	557	578	600	621	643	664	685	707	728
203	750	771	792	814	835	856	878	899	920	942
204	963	984	*006	*027	*048	*069	*091	*112	*133	*154
205	31 175	197	218	239	260	281	302	323	345	366
206	387	408	429	450	471	492	513	534	555	576
207	597	618	639	660	681	702	723	744	765	785
208	806	827	848	869	890	911	931	952	973	994
209	32 015	035	056	077	098	118	139	160	181	201
210	222	243	263	284	305	325	346	366	387	408
211	428	449	469	490	510	531	552	572	593	613
212	634	654	675	695	715	736	756	777	797	818
213	838	858	879	899	919	940	960	980	*001	*021
214	33 041	062	082	102	122	143	163	183	203	224
215	244	264	284	304	325	345	365	385	405	425
216	445	465	486	506	526	546	566	586	606	626
217	646	666	686	706	726	746	766	786	806	826
218	846	866	885	905	925	945	965	985	*005	*025
219	34 044	064	084	104	124	143	163	183	203	223
220	242	262	282	301	321	341	361	380	400	420
221	439	459	479	498	518	537	557	577	596	616
222	635	655	674	694	713	733	753	772	792	811
223	830	850	869	889	908	928	947	967	986	*005
224	35 025	044	064	083	102	122	141	160	180	199
225	218	238	257	276	295	315	334	353	372	392
226	411	430	449	468	488	507	526	545	564	583
227	603	622	641	660	679	698	717	736	755	774
228	793	813	832	851	870	889	908	927	946	965
229	984	*003	*021	*040	*059	*078	*097	*116	*135	*154
230	36 173	192	211	229	248	267	286	305	324	342
231	361	380	399	418	436	455	474	493	511	530
232	549	568	586	605	624	642	661	680	698	717
233	736	754	773	791	810	829	847	866	884	903
234	922	940	959	977	996	*014	*033	*051	*070	*088
235	37 107	125	144	162	181	199	218	236	254	273
236	291	310	328	346	365	383	401	420	438	457
237	475	493	511	530	548	566	585	603	621	639
238	658	676	694	712	731	749	767	785	803	822
239	840	858	876	894	912	931	949	967	985	*003
240	38 021	039	057	075	093	112	130	148	166	184
241	202	220	238	256	274	292	310	328	346	364
242	382	399	417	435	453	471	489	507	525	543
243	561	578	596	614	632	650	668	686	703	721
244	739	757	775	792	810	828	846	863	881	899
245	917	934	952	970	987	*005	*023	*041	*058	*076
246	39 094	111	129	146	164	182	199	217	235	252
247	270	287	305	322	340	358	375	393	410	428
248	445	463	480	498	515	533	550	568	585	602
249	620	637	655	672	690	707	724	742	759	777
250	794	811	829	846	863	881	898	915	933	950
N.	0	1	2	3	4	5	6	7	8	9

PROP. PARTS

	22	21
2	4.4	4.2
3	6.6	6.3
4	8.8	8.4
5	11.0	10.5
6	13.2	12.6
7	15.4	14.7
8	17.6	16.8
9	19.8	18.9

20	
2	4.0
3	6.0
4	8.0
5	10.0
6	12.0
7	14.0
8	16.0
9	18.0

19	
2	3.8
3	5.7
4	7.6
5	9.5
6	11.4
7	13.3
8	15.2
9	17.1

18	
2	3.6
3	5.4
4	7.2
5	9.0
6	10.8
7	12.6
8	14.4
9	16.2

17	
2	3.4
3	5.1
4	6.8
5	8.5
6	10.2
7	11.9
8	13.6
9	15.3

N.	0	1	2	3	4	5	6	7	8	9	PROP. PARTS
250	39 794	811	829	846	863	881	898	915	933	950	
251	967	985	*002	*019	*037	*054	*071	*088	*106	*123	
252	40 140	157	175	192	209	226	243	261	278	295	
253	312	329	346	364	381	398	415	432	449	466	**18**
254	483	500	518	535	552	569	586	603	620	637	2 | 3.6
255	654	671	688	705	722	739	756	773	790	807	3 | 5.4
256	824	841	858	875	892	909	926	943	960	976	4 | 7.2
257	993	*010	*027	*044	*061	*078	*095	*111	*128	*145	5 | 9.0
258	41 162	179	196	212	229	246	263	280	296	313	6 | 10.8
259	330	347	363	380	397	414	430	447	464	481	7 | 12.6
260	497	514	531	547	564	581	597	614	631	647	8 | 14.4
261	664	681	697	714	731	747	764	780	797	814	9 | 16.2
262	830	847	863	880	896	913	929	946	963	979	
263	996	*012	*029	*045	*062	*078	*095	*111	*127	*144	**17**
264	42 160	177	193	210	226	243	259	275	292	308	2 | 3.4
265	325	341	357	374	390	406	423	439	455	472	3 | 5.1
266	488	504	521	537	553	570	586	602	619	635	4 | 6.8
267	651	667	684	700	716	732	749	765	781	797	5 | 8.5
268	813	830	846	862	878	894	911	927	943	959	6 | 10.2
269	975	991	*008	*024	*040	*056	*072	*088	*104	*120	7 | 11.9
270	43 136	152	169	185	201	217	233	249	265	281	8 | 13.6
271	297	313	329	345	361	377	393	409	425	441	9 | 15.3
272	457	473	489	505	521	537	553	569	584	600	
273	616	632	648	664	680	696	712	727	743	759	**16**
274	775	791	807	823	838	854	870	886	902	917	2 | 3.2
275	933	949	965	981	996	*012	*028	*044	*059	*075	3 | 4.8
276	44 091	107	122	138	154	170	185	201	217	232	4 | 6.4
277	248	264	279	295	311	326	342	358	373	389	5 | 8.0
278	404	420	436	451	467	483	498	514	529	545	6 | 9.6
279	560	576	592	607	623	638	654	669	685	700	7 | 11.2
280	716	731	747	762	778	793	809	824	840	855	8 | 12.8
281	871	886	902	917	932	948	963	979	994	*010	9 | 14.4
282	45 025	040	056	071	086	102	117	133	148	163	
283	179	194	209	225	240	255	271	286	301	317	**15**
284	332	347	362	378	393	408	423	439	454	469	2 | 3.0
285	484	500	515	530	545	561	576	591	606	621	3 | 4.5
286	637	652	667	682	697	712	728	743	758	773	4 | 6.0
287	788	803	818	834	849	864	879	894	909	924	5 | 7.5
288	939	954	969	984	*000	*015	*030	*045	*060	*075	6 | 9.0
289	46 090	105	120	135	150	165	180	195	210	225	7 | 10.5
290	240	255	270	285	300	315	330	345	359	374	8 | 12.0
291	389	404	419	434	449	464	479	494	509	523	9 | 13.5
292	538	553	568	583	598	613	627	642	657	672	
293	687	702	716	731	746	761	776	790	805	820	**14**
294	835	850	864	879	894	909	923	938	953	967	2 | 2.8
295	982	997	*012	*026	*041	*056	*070	*085	*100	*114	3 | 4.2
296	47 129	144	159	173	188	202	217	232	246	261	4 | 5.6
297	276	290	305	319	334	349	363	378	392	407	5 | 7.0
298	422	436	451	465	480	494	509	524	538	553	6 | 8.4
299	567	582	596	611	625	640	654	669	683	698	7 | 9.8
											8 | 11.2
300	712	727	741	756	770	784	799	813	828	842	9 | 12.6
N.	0	1	2	3	4	5	6	7	8	9	

N.	0	1	2	3	4	5	6	7	8	9	PROP. PARTS
300	47 712	727	741	756	770	784	799	813	828	842	
301	857	871	885	900	914	929	943	958	972	986	
302	48 001	015	029	044	058	073	087	101	116	130	
303	144	159	173	187	202	216	230	244	259	273	
304	287	302	316	330	344	359	373	387	401	416	
305	430	444	458	473	487	501	515	530	544	558	**15**
306	572	586	601	615	629	643	657	671	686	700	2 | 3.0
307	714	728	742	756	770	785	799	813	827	841	3 | 4.5
308	855	869	883	897	911	926	940	954	968	982	4 | 6.0
309	996	*010	*024	*038	*052	*066	*080	*094	*108	*122	5 | 7.5
310	49 136	150	164	178	192	206	220	234	248	262	6 | 9.0
311	276	290	304	318	332	346	360	374	388	402	7 | 10.5
312	415	429	443	457	471	485	499	513	527	541	8 | 12.0
313	554	568	582	596	610	624	638	651	665	679	9 | 13.5
314	693	707	721	734	748	762	776	790	803	817	
315	831	845	859	872	886	900	914	927	941	955	
316	969	982	996	*010	*024	*037	*051	*065	*079	*092	
317	50 106	120	133	147	161	174	188	202	215	229	**14**
318	243	256	270	284	297	311	325	338	352	365	2 | 2.8
319	379	393	406	420	433	447	461	474	488	501	3 | 4.2
320	515	529	542	556	569	583	596	610	623	637	4 | 5.6
321	651	664	678	691	705	718	732	745	759	772	5 | 7.0
322	786	799	813	826	840	853	866	880	893	907	6 | 8.4
323	920	934	947	961	974	987	*001	*014	*028	*041	7 | 9.8
324	51 055	068	081	095	108	121	135	148	162	175	8 | 11.2
325	188	202	215	228	242	255	268	282	295	308	9 | 12.6
326	322	335	348	362	375	388	402	415	428	441	
327	455	468	481	495	508	521	534	548	561	574	
328	587	601	614	627	640	654	667	680	693	706	
329	720	733	746	759	772	786	799	812	825	838	**13**
330	851	865	878	891	904	917	930	943	957	970	2 | 2.6
331	983	996	*009	*022	*035	*048	*061	*075	*088	*101	3 | 3.9
332	52 114	127	140	153	166	179	192	205	218	231	4 | 5.2
333	244	257	270	284	297	310	323	336	349	362	5 | 6.5
334	376	388	401	414	427	440	453	466	479	492	6 | 7.8
335	504	517	530	543	556	569	582	595	608	621	7 | 9.1
336	634	647	660	673	686	699	711	724	737	750	8 | 10.4
337	763	776	789	802	815	827	840	853	866	879	9 | 11.7
338	892	905	917	930	943	956	969	982	994	*007	
339	53 020	033	046	058	071	084	097	110	122	135	
340	148	161	173	186	199	212	224	237	250	263	**12**
341	275	288	301	314	326	339	352	364	377	390	2 | 2.4
342	403	415	428	441	453	466	479	491	504	517	3 | 3.6
343	529	542	555	567	580	593	605	618	631	643	4 | 4.8
344	656	668	681	694	706	719	732	744	757	769	5 | 6.0
345	782	794	807	820	832	845	857	870	882	895	6 | 7.2
346	908	920	933	945	958	970	983	995	*008	*020	7 | 8.4
347	54 033	045	058	070	083	095	108	120	133	145	8 | 9.6
348	158	170	183	195	208	220	233	245	258	270	9 | 10.8
349	283	295	307	320	332	345	357	370	382	394	
350	407	419	432	444	456	469	481	494	506	518	
N.	0	1	2	3	4	5	6	7	8	9	

N.	0	1	2	3	4	5	6	7	8	9	PROP. PARTS
350	54 407	419	432	444	456	469	481	494	506	518	
351	531	543	555	568	580	593	605	617	630	642	
352	654	667	679	691	704	716	728	741	753	765	
353	777	790	802	814	827	839	851	864	876	888	
354	900	913	925	937	949	962	974	986	998	*011	
355	55 023	035	047	060	072	084	096	108	121	133	**13**
356	145	157	169	182	194	206	218	230	242	255	2 | 2.6
357	267	279	291	303	315	328	340	352	364	376	3 | 3.9
358	388	400	413	425	437	449	461	473	485	497	4 | 5.2
359	509	522	534	546	558	570	582	594	606	618	5 | 6.5
											6 | 7.8
											7 | 9.1
360	630	642	654	666	678	691	703	715	727	739	8 | 10.4
361	751	763	775	787	799	811	823	835	847	859	9 | 11.7
362	871	883	895	907	919	931	943	955	967	979	
363	991	*003	*015	*027	*038	*050	*062	*074	*086	*098	
364	56 110	122	134	146	158	170	182	194	205	217	
365	229	241	253	265	277	289	301	312	324	336	
366	348	360	372	384	396	407	419	431	443	455	
367	467	478	490	502	514	526	538	549	561	573	**12**
368	585	597	608	620	632	644	656	667	679	691	2 | 2.4
369	703	714	726	738	750	761	773	785	797	808	3 | 3.6
											4 | 4.8
											5 | 6.0
370	820	832	844	855	867	879	891	902	914	926	6 | 7.2
371	937	949	961	972	984	996	*008	*019	*031	*043	7 | 8.4
372	57 054	066	078	089	101	113	124	136	148	159	8 | 9.6
373	171	183	194	206	217	229	241	252	264	276	9 | 10.8
374	287	299	310	322	334	345	357	368	380	392	
375	403	415	426	438	449	461	473	484	496	507	
376	519	530	542	553	565	576	588	600	611	623	
377	634	646	657	669	680	692	703	715	726	738	
378	749	761	772	784	795	807	818	830	841	852	
379	864	875	887	898	910	921	933	944	955	967	**11**
											2 | 2.2
380	978	990	*001	*013	*024	*035	*047	*058	*070	*081	3 | 3.3
381	58 092	104	115	127	138	149	161	172	184	195	4 | 4.4
382	206	218	229	240	252	263	274	286	297	309	5 | 5.5
383	320	331	343	354	365	377	388	399	410	422	6 | 6.6
384	433	444	456	467	478	490	501	512	524	535	7 | 7.7
											8 | 8.8
											9 | 9.9
385	546	557	569	580	591	602	614	625	636	647	
386	659	670	681	692	704	715	726	737	749	760	
387	771	782	794	805	816	827	838	850	861	872	
388	883	894	906	917	928	939	950	961	973	984	
389	995	*006	*017	*028	*040	*051	*062	*073	*084	*095	
390	59 106	118	129	140	151	162	173	184	195	207	**10**
391	218	229	240	251	262	273	284	295	306	318	2 | 2.0
392	329	340	351	362	373	384	395	406	417	428	3 | 3.0
393	439	450	461	472	483	494	506	517	528	539	4 | 4.0
394	550	561	572	583	594	605	616	627	638	649	5 | 5.0
											6 | 6.0
											7 | 7.0
395	660	671	682	693	704	715	726	737	748	759	8 | 8.0
396	770	780	791	802	813	824	835	846	857	868	9 | 9.0
397	879	890	901	912	923	934	945	956	966	977	
398	988	999	*010	*021	*032	*043	*054	*065	*076	*086	
399	60 097	108	119	130	141	152	163	173	184	195	
400	206	217	228	239	249	260	271	282	293	304	
N.	0	1	2	3	4	5	6	7	8	9	

N.	0	1	2	3	4	5	6	7	8	9	PROP. PARTS
400	60 206	217	228	239	249	260	271	282	293	304	
401	314	325	336	347	358	369	379	390	401	412	
402	423	433	444	455	466	477	487	498	509	520	
403	531	541	552	563	574	584	595	606	617	627	
404	638	649	660	670	681	692	703	713	724	735	
405	746	756	767	778	788	799	810	821	831	842	
406	853	863	874	885	895	906	917	927	938	949	
407	959	970	981	991	*002	*013	*023	*034	*045	*055	
408	61 066	077	087	098	109	119	130	140	151	162	
409	172	183	194	204	215	225	236	247	257	268	**11**
410	278	289	300	310	321	331	342	352	363	374	2 | 2.2 3 | 3.3 4 | 4.4 5 | 5.5 6 | 6.6 7 | 7.7 8 | 8.8 9 | 9.9
411	384	395	405	416	426	437	448	458	469	479	
412	490	500	511	521	532	542	553	563	574	584	
413	595	606	616	627	637	648	658	669	679	690	
414	700	711	721	731	742	752	763	773	784	794	
415	805	815	826	836	847	857	868	878	888	899	
416	909	920	930	941	951	962	972	982	993	*003	
417	62 014	024	034	045	055	066	076	086	097	107	
418	118	128	138	149	159	170	180	190	201	211	
419	221	232	242	252	263	273	284	294	304	315	
420	325	335	346	356	366	377	387	397	408	418	
421	428	439	449	459	469	480	490	500	511	521	
422	531	542	552	562	572	583	593	603	613	624	
423	634	644	655	665	675	685	696	706	716	726	**10**
424	737	747	757	767	778	788	798	808	818	829	2 | 2.0 3 | 3.0 4 | 4.0 5 | 5.0 6 | 6.0 7 | 7.0 8 | 8.0 9 | 9.0
425	839	849	859	870	880	890	900	910	921	931	
426	941	951	961	972	982	992	*002	*012	*022	*033	
427	63 043	053	063	073	083	094	104	114	124	134	
428	144	155	165	175	185	195	205	215	225	236	
429	246	256	266	276	286	296	306	317	327	337	
430	347	357	367	377	387	397	407	417	428	438	
431	448	458	468	478	488	498	508	518	528	538	
432	548	558	568	579	589	599	609	619	629	639	
433	649	659	669	679	689	699	709	719	729	739	
434	749	759	769	779	789	799	809	819	829	839	
435	849	859	869	879	889	899	909	919	929	939	
436	949	959	969	979	988	998	*008	*018	*028	*038	**9**
437	64 048	058	068	078	088	098	108	118	128	137	
438	147	157	167	177	187	197	207	217	227	237	2 | 1.8 3 | 2.7 4 | 3.6 5 | 4.5 6 | 5.4 7 | 6.3 8 | 7.2 9 | 8.1
439	246	256	266	276	286	296	306	316	326	335	
440	345	355	365	375	385	395	404	414	424	434	
441	444	454	464	473	483	493	503	513	523	532	
442	542	552	562	572	582	591	601	611	621	631	
443	640	650	660	670	680	689	699	709	719	729	
444	738	748	758	768	777	787	797	807	816	826	
445	836	846	856	865	875	885	895	904	914	924	
446	933	943	953	963	972	982	992	*002	*011	*021	
447	65 031	040	050	060	070	079	089	099	108	118	
448	128	137	147	157	167	176	186	196	205	215	
449	225	234	244	254	263	273	283	292	302	312	
450	321	331	341	350	360	369	379	389	398	408	
N.	0	1	2	3	4	5	6	7	8	9	

N.	0	1	2	3	4	5	6	7	8	9	PROP. PARTS
450	65 321	331	341	350	360	369	379	389	398	408	
451	418	427	437	447	456	466	475	485	495	504	
452	514	523	533	543	552	562	571	581	591	600	
453	610	619	629	639	648	658	667	677	686	696	
454	706	715	725	734	744	753	763	772	782	792	
455	801	811	820	830	839	849	858	868	877	887	
456	896	906	916	925	935	944	954	963	973	982	
457	992	*001	*011	*020	*030	*039	*049	*058	*068	*077	
458	66 087	096	106	115	124	134	143	153	162	172	
459	181	191	200	210	219	229	238	247	257	266	**10**
460	276	285	295	304	314	323	332	342	351	361	2 | 2.0 3 | 3.0 4 | 4.0 5 | 5.0 6 | 6.0 7 | 7.0 8 | 8.0 9 | 9.0
461	370	380	389	398	408	417	427	436	445	455	
462	464	474	483	492	502	511	521	530	539	549	
463	558	567	577	586	596	605	614	624	633	642	
464	652	661	671	680	689	699	708	717	727	736	
465	745	755	764	773	783	792	801	811	820	829	
466	839	848	857	867	876	885	894	904	913	922	
467	932	941	950	960	969	978	987	997	*006	*015	
468	67 025	034	043	052	062	071	080	089	099	108	
469	117	127	136	145	154	164	173	182	191	201	
470	210	219	228	237	247	256	265	274	284	293	
471	302	311	321	330	339	348	357	367	376	385	
472	394	403	413	422	431	440	449	459	468	477	
473	486	495	504	514	523	532	541	550	560	569	**9**
474	578	587	596	605	614	624	633	642	651	660	2 | 1.8 3 | 2.7 4 | 3.6 5 | 4.5 6 | 5.4 7 | 6.3 8 | 7.2 9 | 8.1
475	669	679	688	697	706	715	724	733	742	752	
476	761	770	779	788	797	806	815	825	834	843	
477	852	861	870	879	888	897	906	916	925	934	
478	943	952	961	970	979	988	997	*006	*015	*024	
479	68 034	043	052	061	070	079	088	097	106	115	
480	124	133	142	151	160	169	178	187	196	205	
481	215	224	233	242	251	260	269	278	287	296	
482	305	314	323	332	341	350	359	368	377	386	
483	395	404	413	422	431	440	449	458	467	476	
484	485	494	502	511	520	529	538	547	556	565	
485	574	583	592	601	610	619	628	637	646	655	
486	664	673	681	690	699	708	717	726	735	744	
487	753	762	771	780	789	797	806	815	824	833	**8**
488	842	851	860	869	878	886	895	904	913	922	2 | 1.6
489	931	940	949	958	966	975	984	993	*002	*011	3 | 2.4 4 | 3.2 5 | 4.0 6 | 4.8 7 | 5.6 8 | 6.4 9 | 7.2
490	69 020	028	037	046	055	064	073	082	090	099	
491	108	117	126	135	144	152	161	170	179	188	
492	197	205	214	223	232	241	249	258	267	276	
493	285	294	302	311	320	329	338	346	355	364	
494	373	381	390	399	408	417	425	434	443	452	
495	461	469	478	487	496	504	513	522	531	539	
496	548	557	566	574	583	592	601	609	618	627	
497	636	644	653	662	671	679	688	697	705	714	
498	723	732	740	749	758	767	775	784	793	801	
499	810	819	827	836	845	854	862	871	880	888	
500	897	906	914	923	932	940	949	958	966	975	
N.	0	1	2	3	4	5	6	7	8	9	

N.	0	1	2	3	4	5	6	7	8	9	PROP. PARTS
500	69 897	906	914	923	932	940	949	958	966	975	
501	984	992	*001	*010	*018	*027	*036	*044	*053	*062	
502	70 070	079	088	096	105	114	122	131	140	148	
503	157	165	174	183	191	200	209	217	226	234	
504	243	252	260	269	278	286	295	303	312	321	
505	329	338	346	355	364	372	381	389	398	406	
506	415	424	432	441	449	458	467	475	484	492	
507	501	509	518	526	535	544	552	561	569	578	
508	586	595	603	612	621	629	638	646	655	663	
509	672	680	689	697	706	714	723	731	740	749	**9**
510	757	766	774	783	791	800	808	817	825	834	2 | 1.8
511	842	851	859	868	876	885	893	902	910	919	3 | 2.7
512	927	935	944	952	961	969	978	986	995	*003	4 | 3.6
513	71 012	020	029	037	046	054	063	071	079	088	5 | 4.5
514	096	105	113	122	130	139	147	155	164	172	6 | 5.4
515	181	189	198	206	214	223	231	240	248	257	7 | 6.3
516	265	273	282	290	299	307	315	324	332	341	8 | 7.2
517	349	357	366	374	383	391	399	408	416	425	9 | 8.1
518	433	441	450	458	466	475	483	492	500	508	
519	517	525	533	542	550	559	567	575	584	592	
520	600	609	617	625	634	642	650	659	667	675	
521	684	692	700	709	717	725	734	742	750	759	
522	767	775	784	792	800	809	817	825	834	842	
523	850	858	867	875	883	892	900	908	917	925	
524	933	941	950	958	966	975	983	991	999	*008	**8**
525	72 016	024	032	041	049	057	066	074	082	090	2 | 1.6
526	099	107	115	123	132	140	148	156	165	173	3 | 2.4
527	181	189	198	206	214	222	230	239	247	255	4 | 3.2
528	263	272	280	288	296	304	313	321	329	337	5 | 4.0
529	346	354	362	370	378	387	395	403	411	419	6 | 4.8
530	428	436	444	452	460	469	477	485	493	501	7 | 5.6
531	509	518	526	534	542	550	558	567	575	583	8 | 6.4
532	591	599	607	616	624	632	640	648	656	665	9 | 7.2
533	673	681	689	697	705	713	722	730	738	746	
534	754	762	770	779	787	795	803	811	819	827	
535	835	843	852	860	868	876	884	892	900	908	
536	916	925	933	941	949	957	965	973	981	989	
537	997	*006	*014	*022	*030	*038	*046	*054	*062	*070	**7**
538	73 078	086	094	102	111	119	127	135	143	151	2 | 1.4
539	159	167	175	183	191	199	207	215	223	231	3 | 2.1
540	239	247	255	263	272	280	288	296	304	312	4 | 2.8
541	320	328	336	344	352	360	368	376	384	392	5 | 3.5
542	400	408	416	424	432	440	448	456	464	472	6 | 4.2
543	480	488	496	504	512	520	528	536	544	552	7 | 4.9
544	560	568	576	584	592	600	608	616	624	632	8 | 5.6
545	640	648	656	664	672	679	687	695	703	711	9 | 6.3
546	719	727	735	743	751	759	767	775	783	791	
547	799	807	815	823	830	838	846	854	862	870	
548	878	886	894	902	910	918	926	933	941	949	
549	957	965	973	981	989	997	*005	*013	*020	*028	
550	74 036	044	052	060	068	076	084	092	099	107	
N.	0	1	2	3	4	5	6	7	8	9	

FORESTRY LIBRARY
260 FORESTRY BUILDING
UNIVERSITY OF CALIFORNIA
BERKELEY

N.	0	1	2	3	4	5	6	7	8	9	PROP. PARTS
550	74 036	044	052	060	068	076	084	092	099	107	
551	115	123	131	139	147	155	162	170	178	186	
552	194	202	210	218	225	233	241	249	257	265	
553	273	280	288	296	304	312	320	327	335	343	
554	351	359	367	374	382	390	398	406	414	421	
555	429	437	445	453	461	468	476	484	492	500	
556	507	515	523	531	539	547	554	562	570	578	
557	586	593	601	609	617	624	632	640	648	656	
558	663	671	679	687	695	702	710	718	726	733	
559	741	749	757	764	772	780	788	796	803	811	
560	819	827	834	842	850	858	865	873	881	889	
561	896	904	912	920	927	935	943	950	958	966	
562	974	981	989	997	*005	*012	*020	*028	*035	*043	
563	75 051	059	066	074	082	089	097	105	113	120	**8**
564	128	136	143	151	159	166	174	182	189	197	2 | 1.6
565	205	213	220	228	236	243	251	259	266	274	3 | 2.4
566	282	289	297	305	312	320	328	335	343	351	4 | 3.2
567	358	366	374	381	389	397	404	412	420	427	5 | 4.0
568	435	442	450	458	465	473	481	488	496	504	6 | 4.8
569	511	519	526	534	542	549	557	565	572	580	7 | 5.6
											8 | 6.4
											9 | 7.2
570	587	595	603	610	618	626	633	641	648	656	
571	664	671	679	686	694	702	709	717	724	732	
572	740	747	755	762	770	778	785	793	800	808	
573	815	823	831	838	846	853	861	868	876	884	
574	891	899	906	914	921	929	937	944	952	959	
575	967	974	982	989	997	*005	*012	*020	*027	*035	
576	76 042	050	057	065	072	080	087	095	103	110	
577	118	125	133	140	148	155	163	170	178	185	
578	193	200	208	215	223	230	238	245	253	260	
579	268	275	283	290	298	305	313	320	328	335	
580	343	350	358	365	373	380	388	395	403	410	
581	418	425	433	440	448	455	462	470	477	485	
582	492	500	507	515	522	530	537	545	552	559	**7**
583	567	574	582	589	597	604	612	619	626	634	2 | 1.4
584	641	649	656	664	671	678	686	693	701	708	3 | 2.1
585	716	723	730	738	745	753	760	768	775	782	4 | 2.8
586	790	797	805	812	819	827	834	842	849	856	5 | 3.5
587	864	871	879	886	893	901	908	916	923	930	6 | 4.2
588	938	945	953	960	967	975	982	989	997	*004	7 | 4.9
589	77 012	019	026	034	041	048	056	063	070	078	8 | 5.6
											9 | 6.3
590	085	093	100	107	115	122	129	137	144	151	
591	159	166	173	181	188	195	203	210	217	225	
592	232	240	247	254	262	269	276	283	291	298	
593	305	313	320	327	335	342	349	357	364	371	
594	379	386	393	401	408	415	422	430	437	444	
595	452	459	466	474	481	488	495	503	510	517	
596	525	532	539	546	554	561	568	576	583	590	
597	597	605	612	619	627	634	641	648	656	663	
598	670	677	685	692	699	706	714	721	728	735	
599	743	750	757	764	772	779	786	793	801	808	
600	815	822	830	837	844	851	859	866	873	880	
N.	0	1	2	3	4	5	6	7	8	9	

N.	0	1	2	3	4	5	6	7	8	9	PROP. PARTS
600	77 815	822	830	837	844	851	859	866	873	880	
601	887	895	902	909	916	924	931	938	945	952	
602	960	967	974	981	988	996	*003	*010	*017	*025	
603	78 032	039	046	053	061	068	075	082	089	097	
604	104	111	118	125	132	140	147	154	161	168	
605	176	183	190	197	204	211	219	226	233	240	
606	247	254	262	269	276	283	290	297	305	312	
607	319	326	333	340	347	355	362	369	376	383	
608	390	398	405	412	419	426	433	440	447	455	**8**
609	462	469	476	483	490	497	504	512	519	526	2 | 1.6
610	533	540	547	554	561	569	576	583	590	597	3 | 2.4
611	604	611	618	625	633	640	647	654	661	668	4 | 3.2
612	675	682	689	696	704	711	718	725	732	739	5 | 4.0
613	746	753	760	767	774	781	789	796	803	810	6 | 4.8
614	817	824	831	838	845	852	859	866	873	880	7 | 5.6
615	888	895	902	909	916	923	930	937	944	951	8 | 6.4
616	958	965	972	979	986	993	*000	*007	*014	*021	9 | 7.2
617	79 029	036	043	050	057	064	071	078	085	092	
618	099	106	113	120	127	134	141	148	155	162	
619	169	176	183	190	197	204	211	218	225	232	
620	239	246	253	260	267	274	281	288	295	302	
621	309	316	323	330	337	344	351	358	365	372	
622	379	386	393	400	407	414	421	428	435	442	
623	449	456	463	470	477	484	491	498	505	511	**7**
624	518	525	532	539	546	553	560	567	574	581	2 | 1.4
625	588	595	602	609	616	623	630	637	644	650	3 | 2.1
626	657	664	671	678	685	692	699	706	713	720	4 | 2.8
627	727	734	741	748	754	761	768	775	782	789	5 | 3.5
628	796	803	810	817	824	831	837	844	851	858	6 | 4.2
629	865	872	879	886	893	900	906	913	920	927	7 | 4.9
630	934	941	948	955	962	969	975	982	989	996	8 | 5.6
631	80 003	010	017	024	030	037	044	051	058	065	9 | 6.3
632	072	079	085	092	099	106	113	120	127	134	
633	140	147	154	161	168	175	182	188	195	202	
634	209	216	223	229	236	243	250	257	264	271	
635	277	284	291	298	305	312	318	325	332	339	
636	346	353	359	366	373	380	387	393	400	407	
637	414	421	428	434	441	448	455	462	468	475	
638	482	489	496	502	509	516	523	530	536	543	**6**
639	550	557	564	570	577	584	591	598	604	611	2 | 1.2
640	618	625	632	638	645	652	659	665	672	679	3 | 1.8
641	686	693	699	706	713	720	726	733	740	747	4 | 2.4
642	754	760	767	774	781	787	794	801	808	814	5 | 3.0
643	821	828	835	841	848	855	862	868	875	882	6 | 3.6
644	889	895	902	909	916	922	929	936	943	949	7 | 4.2
645	956	963	969	976	983	990	996	*003	*010	*017	8 | 4.8
646	81 023	030	037	043	050	057	064	070	077	084	9 | 5.4
647	090	097	104	111	117	124	131	137	144	151	
648	158	164	171	178	184	191	198	204	211	218	
649	224	231	238	245	251	258	265	271	278	285	
650	291	298	305	311	318	325	331	338	345	351	
N.	0	1	2	3	4	5	6	7	8	9	

N.	0	1	2	3	4	5	6	7	8	9	PROP. PARTS
650	81 291	298	305	311	318	325	331	338	345	351	
651	358	365	371	378	385	391	398	405	411	418	
652	425	431	438	445	451	458	465	471	478	485	
653	491	498	505	511	518	525	531	538	544	551	
654	558	564	571	578	584	591	598	604	611	617	
655	624	631	637	644	651	657	664	671	677	684	
656	690	697	704	710	717	723	730	737	743	750	
657	757	763	770	776	783	790	796	803	809	816	
658	823	829	836	842	849	856	862	869	875	882	
659	889	895	902	908	915	921	928	935	941	948	
660	954	961	968	974	981	987	994	*000	*007	*014	
661	82 020	027	033	040	046	053	060	066	073	079	
662	086	092	099	105	112	119	125	132	138	145	7
663	151	158	164	171	178	184	191	197	204	210	2 | 1.4
664	217	223	230	236	243	249	256	263	269	276	3 | 2.1
665	282	289	295	302	308	315	321	328	334	341	4 | 2.8
666	347	354	360	367	373	380	387	393	400	406	5 | 3.5
667	413	419	426	432	439	445	452	458	465	471	6 | 4.2
668	478	484	491	497	504	510	517	523	530	536	7 | 4.9
669	543	549	556	562	569	575	582	588	595	601	8 | 5.6
											9 | 6.3
670	607	614	620	627	633	640	646	653	659	666	
671	672	679	685	692	698	705	711	718	724	730	
672	737	743	750	756	763	769	776	782	789	795	
673	802	808	814	821	827	834	840	847	853	860	
674	866	872	879	885	892	898	905	911	918	924	
675	930	937	943	950	956	963	969	975	982	988	
676	995	*001	*008	*014	*020	*027	*033	*040	*046	*052	
677	83 059	065	072	078	085	091	097	104	110	117	
678	123	129	136	142	149	155	161	168	174	181	
679	187	193	200	206	213	219	225	232	238	245	
680	251	257	264	270	276	283	289	296	302	308	
681	315	321	327	334	340	347	353	359	366	372	
682	378	385	391	398	404	410	417	423	429	436	6
683	442	448	455	461	467	474	480	487	493	499	2 | 1.2
684	506	512	518	525	531	537	544	550	556	563	3 | 1.8
685	569	575	582	588	594	601	607	613	620	626	4 | 2.4
686	632	639	645	651	658	664	670	677	683	689	5 | 3.0
687	696	702	708	715	721	727	734	740	746	753	6 | 3.6
688	759	765	771	778	784	790	797	803	809	816	7 | 4.2
689	822	828	835	841	847	853	860	866	872	879	8 | 4.8
											9 | 5.4
690	885	891	897	904	910	916	923	929	935	942	
691	948	954	960	967	973	979	985	992	998	*004	
692	84 011	017	023	029	036	042	048	055	061	067	
693	073	080	086	092	098	105	111	117	123	130	
694	136	142	148	155	161	167	173	180	186	192	
695	198	205	211	217	223	230	236	242	248	255	
696	261	267	273	280	286	292	298	305	311	317	
697	323	330	336	342	348	354	361	367	373	379	
698	386	392	398	404	410	417	423	429	435	442	
699	448	454	460	466	473	479	485	491	497	504	
700	510	516	522	528	535	541	547	553	559	566	
N.	0	1	2	3	4	5	6	7	8	9	

N.	0	1	2	3	4	5	6	7	8	9	PROP. PARTS
700	84 510	516	522	528	535	541	547	553	559	566	
701	572	578	584	590	597	603	609	615	621	628	
702	634	640	646	652	658	665	671	677	683	689	
703	696	702	708	714	720	726	733	739	745	751	
704	757	763	770	776	782	788	794	800	807	813	
705	819	825	831	837	844	850	856	862	868	874	
706	880	887	893	899	905	911	917	924	930	936	
707	942	948	954	960	967	973	979	985	991	997	
708	85 003	009	016	022	028	034	040	046	052	058	**7**
709	065	071	077	083	089	095	101	107	114	120	2 | 1.4
710	126	132	138	144	150	156	163	169	175	181	3 | 2.1
711	187	193	199	205	211	217	224	230	236	242	4 | 2.8
712	248	254	260	266	272	278	285	291	297	303	5 | 3.5
713	309	315	321	327	333	339	345	352	358	364	6 | 4.2
714	370	376	382	388	394	400	406	412	418	425	7 | 4.9
715	431	437	443	449	455	461	467	473	479	485	8 | 5.6
716	491	497	503	509	516	522	528	534	540	546	9 | 6.3
717	552	558	564	570	576	582	588	594	600	606	
718	612	618	625	631	637	643	649	655	661	667	
719	673	679	685	691	697	703	709	715	721	727	
720	733	739	745	751	757	763	769	775	781	788	
721	794	800	806	812	818	824	830	836	842	848	
722	854	860	866	872	878	884	890	896	902	908	
723	914	920	926	932	938	944	950	956	962	968	**6**
724	974	980	986	992	998	*004	*010	*016	*022	*028	2 | 1.2
725	86 034	040	046	052	058	064	070	076	082	088	3 | 1.8
726	094	100	106	112	118	124	130	136	141	147	4 | 2.4
727	153	159	165	171	177	183	189	195	201	207	5 | 3.0
728	213	219	225	231	237	243	249	255	261	267	6 | 3.6
729	273	279	285	291	297	303	308	314	320	326	7 | 4.2
730	332	338	344	350	356	362	368	374	380	386	8 | 4.8
731	392	398	404	410	415	421	427	433	439	445	9 | 5.4
732	451	457	463	469	475	481	487	493	499	504	
733	510	516	522	528	534	540	546	552	558	564	
734	570	576	581	587	593	599	605	611	617	623	
735	629	635	641	646	652	658	664	670	676	682	
736	688	694	700	705	711	717	723	729	735	741	
737	747	753	759	764	770	776	782	788	794	800	
738	806	812	817	823	829	835	841	847	853	859	**5**
739	864	870	876	882	888	894	900	906	911	917	2 | 1.0
740	923	929	935	941	947	953	958	964	970	976	3 | 1.5
741	982	988	994	999	*005	*011	*017	*023	*029	*035	4 | 2.0
742	87 040	046	052	058	064	070	075	081	087	093	5 | 2.5
743	099	105	111	116	122	128	134	140	146	151	6 | 3.0
744	157	163	169	175	181	186	192	198	204	210	7 | 3.5
745	216	221	227	233	239	245	251	256	262	268	8 | 4.0
746	274	280	286	291	297	303	309	315	320	326	9 | 4.5
747	332	338	344	349	355	361	367	373	379	384	
748	390	396	402	408	413	419	425	431	437	442	
749	448	454	460	466	471	477	483	489	495	500	
750	506	512	518	523	529	535	541	547	552	558	
N.	0	1	2	3	4	5	6	7	8	9	

N.	0	1	2	3	4	5	6	7	8	9	PROP. PARTS
750	87 506	512	518	523	529	535	541	547	552	558	
751	564	570	576	581	587	593	599	604	610	616	
752	622	628	633	639	645	651	656	662	668	674	
753	679	685	691	697	703	708	714	720	726	731	
754	737	743	749	754	760	766	772	777	783	789	
755	795	800	806	812	818	823	829	835	841	846	
756	852	858	864	869	875	881	887	892	898	904	
757	910	915	921	927	933	938	944	950	955	961	
758	967	973	978	984	990	996	*001	*007	*013	*018	
759	88 024	030	036	041	047	053	058	064	070	076	
760	081	087	093	098	104	110	116	121	127	133	
761	138	144	150	156	161	167	173	178	184	190	
762	195	201	207	213	218	224	230	235	241	247	
763	252	258	264	270	275	281	287	292	298	304	
764	309	315	321	326	332	338	343	349	355	360	**6**
765	366	372	377	383	389	395	400	406	412	417	2 | 1.2
766	423	429	434	440	446	451	457	463	468	474	3 | 1.8
767	480	485	491	497	502	508	513	519	525	530	4 | 2.4
768	536	542	547	553	559	564	570	576	581	587	5 | 3.0
769	593	598	604	610	615	621	627	632	638	643	6 | 3.6
770	649	655	660	666	672	677	683	689	694	700	7 | 4.2
771	705	711	717	722	728	734	739	745	750	756	8 | 4.8
772	762	767	773	779	784	790	795	801	807	812	9 | 5.4
773	818	824	829	835	840	846	852	857	863	868	
774	874	880	885	891	897	902	908	913	919	925	
775	930	936	941	947	953	958	964	969	975	981	
776	986	992	997	*003	*009	*014	*020	*025	*031	*037	
777	89 042	048	053	059	064	070	076	081	087	092	
778	098	104	109	115	120	126	131	137	143	148	
779	154	159	165	170	176	182	187	193	198	204	
780	209	215	221	226	232	237	243	248	254	260	
781	265	271	276	282	287	293	298	304	310	315	
782	321	326	332	337	343	348	354	360	365	371	
783	376	382	387	393	398	404	409	415	421	426	**5**
784	432	437	443	448	454	459	465	470	476	481	2 | 1.0
785	487	492	498	504	509	515	520	526	531	537	3 | 1.5
786	542	548	553	559	564	570	575	581	586	592	4 | 2.0
787	597	603	609	614	620	625	631	636	642	647	5 | 2.5
788	653	658	664	669	675	680	686	691	697	702	6 | 3.0
789	708	713	719	724	730	735	741	746	752	757	7 | 3.5
790	763	768	774	779	785	790	796	801	807	812	8 | 4.0
791	818	823	829	834	840	845	851	856	862	867	9 | 4.5
792	873	878	883	889	894	900	905	911	916	922	
793	927	933	938	944	949	955	960	966	971	977	
794	982	988	993	998	*004	*009	*015	*020	*026	*031	
795	90 037	042	048	053	059	064	069	075	080	086	
796	091	097	102	108	113	119	124	129	135	140	
797	146	151	157	162	168	173	179	184	189	195	
798	200	206	211	217	222	227	233	238	244	249	
799	255	260	266	271	276	282	287	293	298	304	
800	309	314	320	325	331	336	342	347	352	358	
N.	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	

N.	0	1	2	3	4	5	6	7	8	9	PROP. PARTS
800	90 309	314	320	325	331	336	342	347	352	358	
801	363	369	374	380	385	390	396	401	407	412	
802	417	423	428	434	439	445	450	455	461	466	
803	472	477	482	488	493	499	504	509	515	520	
804	526	531	536	542	547	553	558	563	569	574	
805	580	585	590	596	601	607	612	617	623	628	
806	634	639	644	650	655	660	666	671	677	682	
807	687	693	698	703	709	714	720	725	730	736	
808	741	747	752	757	763	768	773	779	784	789	
809	795	800	806	811	816	822	827	832	838	843	
810	849	854	859	865	870	875	881	886	891	897	
811	902	907	913	918	924	929	934	940	945	950	
812	956	961	966	972	977	982	988	993	998	*004	
813	91 009	014	020	025	030	036	041	046	052	057	**6**
814	062	068	073	078	084	089	094	100	105	110	2 | 1.2
815	116	121	126	132	137	142	148	153	158	164	3 | 1.8
816	169	174	180	185	190	196	201	206	212	217	4 | 2.4
817	222	228	233	238	243	249	254	259	265	270	5 | 3.0
818	275	281	286	291	297	302	307	312	318	323	6 | 3.6
819	328	334	339	344	350	355	360	365	371	376	7 | 4.2
820	381	387	392	397	403	408	413	418	424	429	8 | 4.8
821	434	440	445	450	455	461	466	471	477	482	9 | 5.4
822	487	492	498	503	508	514	519	524	529	535	
823	540	545	551	556	561	566	572	577	582	587	
824	593	598	603	609	614	619	624	630	635	640	
825	645	651	656	661	666	672	677	682	687	693	
826	698	703	709	714	719	724	730	735	740	745	
827	751	756	761	766	772	777	782	787	793	798	
828	803	808	814	819	824	829	834	840	845	850	
829	855	861	866	871	876	882	887	892	897	903	
830	908	913	918	924	929	934	939	944	950	955	
831	960	965	971	976	981	986	991	997	*002	*007	
832	92 012	018	023	028	033	038	044	049	054	059	**5**
833	065	070	075	080	085	091	096	101	106	111	2 | 1.0
834	117	122	127	132	137	143	148	153	158	163	3 | 1.5
835	169	174	179	184	189	195	200	205	210	215	4 | 2.0
836	221	226	231	236	241	247	252	257	262	267	5 | 2.5
837	273	278	283	288	293	298	304	309	314	319	6 | 3.0
838	324	330	335	340	345	350	355	361	366	371	7 | 3.5
839	376	381	387	392	397	402	407	412	418	423	8 | 4.0
840	428	433	438	443	449	454	459	464	469	474	9 | 4.5
841	480	485	490	495	500	505	511	516	521	526	
842	531	536	542	547	552	557	562	567	572	578	
843	583	588	593	598	603	609	614	619	624	629	
844	634	639	645	650	655	660	665	670	675	681	
845	686	691	696	701	706	711	716	722	727	732	
846	737	742	747	752	758	763	768	773	778	783	
847	788	793	799	804	809	814	819	824	829	834	
848	840	845	850	855	860	865	870	875	881	886	
849	891	896	901	906	911	916	921	927	932	937	
850	942	947	952	957	962	967	973	978	983	988	
N.	0	1	2	3	4	5	6	7	8	9	

N.	0	1	2	3	4	5	6	7	8	9	PROP. PARTS
850	92 942	947	952	957	962	967	973	978	983	988	
851	993	998	*003	*008	*013	*018	*024	*029	*034	*039	
852	93 044	049	054	059	064	069	075	080	085	090	
853	095	100	105	110	115	120	125	131	136	141	
854	146	151	156	161	166	171	176	181	186	192	
855	197	202	207	212	217	222	227	232	237	242	
856	247	252	258	263	268	273	278	283	288	293	
857	298	303	308	313	318	323	328	334	339	344	
858	349	354	359	364	369	374	379	384	389	394	
859	399	404	409	414	420	425	430	435	440	445	**6**
860	450	455	460	465	470	475	480	485	490	495	2 | 1.2
861	500	505	510	515	520	526	531	536	541	546	3 | 1.8
862	551	556	561	566	571	576	581	586	591	596	4 | 2.4
863	601	606	611	616	621	626	631	636	641	646	5 | 3.0
864	651	656	661	666	671	676	682	687	692	697	6 | 3.6
											7 | 4.2
865	702	707	712	717	722	727	732	737	742	747	8 | 4.8
866	752	757	762	767	772	777	782	787	792	797	9 | 5.4
867	802	807	812	817	822	827	832	837	842	847	
868	852	857	862	867	872	877	882	887	892	897	
869	902	907	912	917	922	927	932	937	942	947	
870	952	957	962	967	972	977	982	987	992	997	
871	94 002	007	012	017	022	027	032	037	042	047	
872	052	057	062	067	072	077	082	086	091	096	
873	101	106	111	116	121	126	131	136	141	146	**5**
874	151	156	161	166	171	176	181	186	191	196	2 | 1.0
875	201	206	211	216	221	226	231	236	240	245	3 | 1.5
876	250	255	260	265	270	275	280	285	290	295	4 | 2.0
877	300	305	310	315	320	325	330	335	340	345	5 | 2.5
878	349	354	359	364	369	374	379	384	389	394	6 | 3.0
879	399	404	409	414	419	424	429	433	438	443	7 | 3.5
											8 | 4.0
880	448	453	458	463	468	473	478	483	488	493	9 | 4.5
881	498	503	507	512	517	522	527	532	537	542	
882	547	552	557	562	567	571	576	581	586	591	
883	596	601	606	611	616	621	626	630	635	640	
884	645	650	655	660	665	670	675	680	685	689	
885	694	699	704	709	714	719	724	729	734	738	
886	743	748	753	758	763	768	773	778	783	787	
887	792	797	802	807	812	817	822	827	832	836	**4**
888	841	846	851	856	861	866	871	876	880	885	2 | 0.8
889	890	895	900	905	910	915	919	924	929	934	3 | 1.2
											4 | 1.6
890	939	944	949	954	959	963	968	973	978	983	5 | 2.0
891	988	993	998	*002	*007	*012	*017	*022	*027	*032	6 | 2.4
892	95 036	041	046	051	056	061	066	071	075	080	7 | 2.8
893	085	090	095	100	105	109	114	119	124	129	8 | 3.2
894	134	139	143	148	153	158	163	168	173	177	9 | 3.6
895	182	187	192	197	202	207	211	216	221	226	
896	231	236	240	245	250	255	260	265	270	274	
897	279	284	289	294	299	303	308	313	318	323	
898	328	332	337	342	347	352	357	361	366	371	
899	376	381	386	390	395	400	405	410	415	419	
900	424	429	434	439	444	448	453	458	463	468	
N.	0	1	2	3	4	5	6	7	8	9	

N.	0	1	2	3	4	5	6	7	8	9	PROP. PARTS
900	95 424	429	434	439	444	448	453	458	463	468	
901	472	477	482	487	492	497	501	506	511	516	
902	521	525	530	535	540	545	550	554	559	564	
903	569	574	578	583	588	593	598	602	607	612	
904	617	622	626	631	636	641	646	650	655	660	
905	665	670	674	679	684	689	694	698	703	708	
906	713	718	722	727	732	737	742	746	751	756	
907	761	766	770	775	780	785	789	794	799	804	
908	809	813	818	823	828	832	837	842	847	852	
909	856	861	866	871	875	880	885	890	895	899	
910	904	909	914	918	923	928	933	938	942	947	
911	952	957	961	966	971	976	980	985	990	995	
912	999	*004	*009	*014	*019	*023	*028	*033	*038	*042	
913	96 047	052	057	061	066	071	076	080	085	090	**5**
914	095	099	104	109	114	118	123	128	133	137	2 | 1.0
915	142	147	152	156	161	166	171	175	180	185	3 | 1.5
916	190	194	199	204	209	213	218	223	227	232	4 | 2.0
917	237	242	246	251	256	261	265	270	275	280	5 | 2.5
918	284	289	294	298	303	308	313	317	322	327	6 | 3.0
919	332	336	341	346	350	355	360	365	369	374	7 | 3.5
920	379	384	388	393	398	402	407	412	417	421	8 | 4.0
921	426	431	435	440	445	450	454	459	464	468	9 | 4.5
922	473	478	483	487	492	497	501	506	511	515	
923	520	525	530	534	539	544	548	553	558	562	
924	567	572	577	581	586	591	595	600	605	609	
925	614	619	624	628	633	638	642	647	652	656	
926	661	666	670	675	680	685	689	694	699	703	
927	708	713	717	722	727	731	736	741	745	750	
928	755	759	764	769	774	778	783	788	792	797	
929	802	806	811	816	820	825	830	834	839	844	
930	848	853	858	862	867	872	876	881	886	890	
931	895	900	904	909	914	918	923	928	932	937	
932	942	946	951	956	960	965	970	974	979	984	**4**
933	988	993	997	*002	*007	*011	*016	*021	*025	*030	2 | 0.8
934	97 035	039	044	049	053	058	063	067	072	077	3 | 1.2
935	081	086	090	095	100	104	109	114	118	123	4 | 1.6
936	128	132	137	142	146	151	155	160	165	169	5 | 2.0
937	174	179	183	188	192	197	202	206	211	216	6 | 2.4
938	220	225	230	234	239	243	248	253	257	262	7 | 2.8
939	267	271	276	280	285	290	294	299	304	308	8 | 3.2
940	313	317	322	327	331	336	340	345	350	354	9 | 3.6
941	359	364	368	373	377	382	387	391	396	400	
942	405	410	414	419	424	428	433	437	442	447	
943	451	456	460	465	470	474	479	483	488	493	
944	497	502	506	511	516	520	525	529	534	539	
945	543	548	552	557	562	566	571	575	580	585	
946	589	594	598	603	607	612	617	621	626	630	
947	635	640	644	649	653	658	663	667	672	676	
948	681	685	690	695	699	704	708	713	717	722	
949	727	731	736	740	745	749	754	759	763	768	
950	772	777	782	786	791	795	800	804	809	813	
N.	0	1	2	3	4	5	6	7	8	9	

N.	0	1	2	3	4	5	6	7	8	9	PROP. PARTS
950	97 772	777	782	786	791	795	800	804	809	813	
951	818	823	827	832	836	841	845	850	855	859	
952	864	868	873	877	882	886	891	896	900	905	
953	909	914	918	923	928	932	937	941	946	950	
954	955	959	964	968	973	978	982	987	991	996	
955	98 000	005	009	014	019	023	028	032	037	041	
956	046	050	055	059	064	068	073	078	082	087	
957	091	096	100	105	109	114	118	123	127	132	
958	137	141	146	150	155	159	164	168	173	177	
959	182	186	191	195	200	204	209	214	218	223	
960	227	232	236	241	245	250	254	259	263	268	
961	272	277	281	286	290	295	299	304	308	313	
962	318	322	327	331	336	340	345	349	354	358	
963	363	367	372	376	381	385	390	394	399	403	
964	408	412	417	421	426	430	435	439	444	448	**5**
965	453	457	462	466	471	475	480	484	489	493	2 | 1.0
966	498	502	507	511	516	520	525	529	534	538	3 | 1.5
967	543	547	552	556	561	565	570	574	579	583	4 | 2.0
968	588	592	597	601	605	610	614	619	623	628	5 | 2.5
969	632	637	641	646	650	655	659	664	668	673	6 | 3.0
											7 | 3.5
											8 | 4.0
											9 | 4.5
970	677	682	686	691	695	700	704	709	713	717	
971	722	726	731	735	740	744	749	753	758	762	
972	767	771	776	780	784	789	793	798	802	807	
973	811	816	820	825	829	834	838	843	847	851	
974	856	860	865	869	874	878	883	887	892	896	
975	900	905	909	914	918	923	927	932	936	941	
976	945	949	954	958	963	967	972	976	981	985	
977	989	994	998	*003	*007	*012	*016	*021	*025	*029	
978	99 034	038	043	047	052	056	061	065	069	074	
979	078	083	087	092	096	100	105	109	114	118	
980	123	127	131	136	140	145	149	154	158	162	
981	167	171	176	180	185	189	193	198	202	207	
982	211	216	220	224	229	233	238	242	247	251	
983	255	260	264	269	273	277	282	286	291	295	**4**
984	300	304	308	313	317	322	326	330	335	339	2 | 0.8
985	344	348	352	357	361	366	370	374	379	383	3 | 1.2
986	388	392	396	401	405	410	414	419	423	427	4 | 1.6
987	432	436	441	445	449	454	458	463	467	471	5 | 2.0
988	476	480	484	489	493	498	502	506	511	515	6 | 2.4
989	520	524	528	533	537	542	546	550	555	559	7 | 2.8
											8 | 3.2
											9 | 3.6
990	564	568	572	577	581	585	590	594	599	603	
991	607	612	616	621	625	629	634	638	642	647	
992	651	656	660	664	669	673	677	682	686	691	
993	695	699	704	708	712	717	721	726	730	734	
994	739	743	747	752	756	760	765	769	774	778	
995	782	787	791	795	800	804	808	813	817	822	
996	826	830	835	839	843	848	852	856	861	865	
997	870	874	878	883	887	891	896	900	904	909	
998	913	917	922	926	930	935	939	944	948	952	
999	957	961	965	970	974	978	983	987	991	996	
1000	00 000	004	009	013	017	022	026	030	035	039	
N.	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	

Explanation of Table II

This table contains the numerical values of the sine, cosine, tangent, and cotangent for every minute from 0° to 90° correct to five places of decimals. The values of the secant and cosecant which are not so frequently used may be found by taking the reciprocal values of the cosine and the sine, respectively.

It should be borne in mind that the sine and tangent increase as the angle increases from 0° to 90°, while the other two functions decrease. Hence when interpolating for angles not found in the tables, the correction must be added to the functions of the smaller angle nearest it in the case of the sine and the tangent, and subtracted from it in the case of the cosine and cotangent.

Since the function of any angle can be expressed as the conamed function of the complementary angle, each column is doubly named, the names at the top of the page being taken with angles at the top of the page, and similarly for those at the bottom of the page. Thus the sine of 30° 18′ is found in the same place in the table as the cosine of 59° 42′. In the first instance the minute column at the left is used, and in the second, the minute column at the right.

Interpolation will not yield accurate results for the cotangents of angles near 0° nor for the tangents of angles near 90°, since these functions are changing very rapidly in these vicinities.

The following examples will make clear the use of the table:

Ex. 1. Find sin 34° 26′ 25″. This angle lies between 34° 26′ and 34° 27′. The tabular difference between the sines of these two angles is .00024 and the correction for 25″ is $\frac{25}{60}$ of this difference or .00010. Since the sine is an increasing function, we add this correction to the sine of the smaller angle. Hence sin 34° 26′ 25″ = .56545 + .00010 = .56555.

Ex. 2. Find cot 57° 19′.4. This angle lies between 57° 19′ and 57° 20′. The tabular difference for the cotangents of these angles is .00041. The correction is .4 of .00041 or .000164. But since the figure in the sixth place is less than 5, it is discarded, making the correction to five places .00016, which must be subtracted from the cotangent of the smaller angle, since the cotangent is a decreasing function. Hence cot 57° 19′.4 = .64158 − .00016 = .64142. (When the correction is for tenths of a minute, it may be read from the table of proportional parts given in Table I. Thus in this example $\frac{4}{10}$ of 41 may be found from the proportional parts of 41.)

Ex. 3. Find A if cos A = .46398. This number lies between .46381 and .46407. Therefore A lies between 62° 21′ and 62° 22′. The tabular difference is .00026 and cos 62° 21′ − cos A = .00009. Therefore A is $\frac{9}{26}$ of the way from 62° 21′ and 62° 22′, and the correction is $\frac{9}{26} \times 60''$ or 21″ to the nearest second. Hence A = 62° 21′ 21″.

TABLE II

NUMERICAL VALUES

OF THE

TRIGONOMETRIC FUNCTIONS

TO

FIVE DECIMAL PLACES

FOR EVERY MINUTE FROM 0° TO 90°

′	Sin	Tan	Cot	Cos	
0	.00000	.00000	———	1.0000	**60**
1	029	029	3437.7	000	59
2	058	058	1718.9	000	58
3	087	087	1145.9	000	57
4	116	116	859.44	000	56
5	.00145	.00145	687.55	1.0000	**55**
6	175	175	572.96	000	54
7	204	204	491.11	000	53
8	233	233	429.72	000	52
9	262	262	381.97	000	51
10	.00291	.00291	343.77	1.0000	**50**
11	320	320	312.52	.99999	49
12	349	349	286.48	999	48
13	378	378	264.44	999	47
14	407	407	245.55	999	46
15	.00436	.00436	229.18	.99999	**45**
16	465	465	214.86	999	44
17	495	495	202.22	999	43
18	524	524	190.98	999	42
19	553	553	180.93	998	41
20	.00582	.00582	171.89	.99998	**40**
21	611	611	163.70	998	39
22	640	640	156.26	998	38
23	669	669	149.47	998	37
24	698	698	143.24	998	36
25	.00727	.00727	137.51	.99997	**35**
26	756	756	132.22	997	34
27	785	785	127.32	997	33
28	814	815	122.77	997	32
29	844	844	118.54	996	31
30	.00873	.00873	114.59	.99996	**30**
31	902	902	110.89	996	29
32	931	931	107.43	996	28
33	960	960	104.17	995	27
34	.00989	.00989	101.11	995	26
35	.01018	.01018	98.218	.99995	**25**
36	047	047	95.489	995	24
37	076	076	92.908	994	23
38	105	105	90 463	994	22
39	134	135	88.144	994	21
40	.01164	.01164	85.940	.99993	**20**
41	193	193	83.844	993	19
42	222	222	81.847	993	18
43	251	251	79.943	992	17
44	280	280	78.126	992	16
45	.01309	.01309	76.390	.99991	**15**
46	338	338	74.729	991	14
47	367	367	73.139	991	13
48	396	396	71.615	990	12
49	425	425	70.153	990	11
50	.01454	.01455	68.750	.99989	**10**
51	483	484	67.402	989	9
52	513	513	66.105	989	8
53	542	542	64.858	988	7
54	571	571	63.657	988	6
55	.01600	.01600	62.499	.99987	**5**
56	629	629	61.383	987	4
57	658	658	60.306	986	3
58	687	687	59.266	986	2
59	716	716	58.261	985	1
60	.01745	.01746	57.290	.99985	**0**
	Cos	Cot	Tan	Sin	′

′	Sin	Tan	Cot	Cos	
0	.01745	.01746	57.290	.99985	**60**
1	774	775	56.351	984	59
2	803	804	55.442	984	58
3	832	833	54.561	983	57
4	862	862	53.709	983	56
5	.01891	.01891	52.882	.99982	**55**
6	920	920	52.081	982	54
7	949	949	51.303	981	53
8	.01978	.01978	50.549	980	52
9	.02007	.02007	49.816	980	51
10	.02036	.02036	49.104	.99979	**50**
11	065	066	48.412	979	49
12	094	095	47.740	978	48
13	123	124	47.085	977	47
14	152	153	46.449	977	46
15	.02181	.02182	45.829	.99976	**45**
16	211	211	45.226	976	44
17	240	240	44.639	975	43
18	269	269	44.066	974	42
19	298	298	43.508	974	41
20	.02327	.02328	42.964	.99973	**40**
21	356	357	42.433	972	39
22	385	386	41.916	972	38
23	414	415	41.411	971	37
24	443	444	40.917	970	36
25	.02472	.02473	40.436	.99969	**35**
26	501	502	39.965	969	34
27	530	531	39.506	968	33
28	560	560	39.057	967	32
29	589	589	38.618	966	31
30	.02618	.02619	38.188	.99966	**30**
31	647	648	37.769	965	29
32	676	677	37.358	964	28
33	705	706	36.956	963	27
34	734	735	36.563	963	26
35	.02763	.02764	36.178	.99962	**25**
36	792	793	35.801	961	24
37	821	822	35.431	960	23
38	850	851	35.070	959	22
39	879	881	34.715	959	21
40	.02908	.02910	34.368	.99958	**20**
41	938	939	34.027	957	19
42	967	968	33.694	956	18
43	.02996	.02997	33.366	955	17
44	.03025	.03026	33.045	954	16
45	.03054	.03055	32.730	.99953	**15**
46	083	084	32.421	952	14
47	112	114	32.118	952	13
48	141	143	31.821	951	12
49	170	172	31.528	950	11
50	.03199	.03201	31.242	.99949	**10**
51	228	230	30.960	948	9
52	257	259	30.683	947	8
53	286	288	30.412	946	7
54	316	317	30.145	945	6
55	.03345	.03346	29.882	.99944	**5**
56	374	376	29.624	943	4
57	403	405	29.371	942	3
58	432	434	29.122	941	2
59	461	463	28.877	940	1
60	.03490	.03492	28.636	.99939	**0**
	Cos	Cot	Tan	Sin	′

′	Sin	Tan	Cot	Cos	
0	.03490	.03492	28.636	.99939	**60**
1	519	521	.399	938	59
2	548	550	28.166	937	58
3	577	579	27.937	936	57
4	606	609	.712	935	56
5	.03635	.03638	27.490	.99934	**55**
6	664	667	.271	933	54
7	693	696	27.057	932	53
8	723	725	26.845	931	52
9	752	754	.637	930	51
10	.03781	.03783	26.432	.99929	**50**
11	810	812	.230	927	49
12	839	842	26.031	926	48
13	868	871	25.835	925	47
14	897	900	.642	924	46
15	.03926	.03929	25.452	.99923	**45**
16	955	958	.264	922	44
17	.03984	.03987	25.080	921	43
18	.04013	.04016	24.898	919	42
19	042	046	.719	918	41
20	.04071	.04075	24.542	.99917	**40**
21	100	104	.368	916	39
22	129	133	.196	915	38
23	159	162	24.026	913	37
24	188	191	23.859	912	36
25	.04217	.04220	23.695	.99911	**35**
26	246	250	.532	910	34
27	275	279	.372	909	33
28	304	308	.214	907	32
29	333	337	23.058	906	31
30	.04362	.04366	22.904	.99905	**30**
31	391	395	.752	904	29
32	420	424	.602	902	28
33	449	454	.454	901	27
34	478	483	.308	900	26
35	.04507	.04512	22.164	.99898	**25**
36	536	541	22.022	897	24
37	565	570	21.881	896	23
38	594	599	.743	894	22
39	623	628	.606	893	21
40	.04653	.04658	21.470	.99892	**20**
41	682	687	.337	890	19
42	711	716	.205	889	18
43	740	745	21.075	888	17
44	769	774	20.946	886	16
45	.04798	.04803	20.819	.99885	**15**
46	827	833	.693	883	14
47	856	862	.569	882	13
48	885	891	.446	881	12
49	914	920	.325	879	11
50	.04943	.04949	20.206	.99878	**10**
51	.04972	.04978	20.087	876	9
52	.05001	.05007	19.970	875	8
53	030	037	.855	873	7
54	059	066	.740	872	6
55	.05088	.05095	19.627	.99870	**5**
56	117	124	.516	869	4
57	146	153	.405	867	3
58	175	182	.296	866	2
59	205	212	.188	864	1
60	.05234	.05241	19.081	.99863	**0**
	Cos	Cot	Tan	Sin	′

′	Sin	Tan	Cot	Cos	
0	.05234	.05241	19.081	.99863	**60**
1	263	270	18.976	861	59
2	292	299	.871	860	58
3	321	328	.768	858	57
4	350	357	.666	857	56
5	.05379	.05387	18.564	.99855	**55**
6	408	416	.464	854	54
7	437	445	.366	852	53
8	466	474	.268	851	52
9	495	503	.171	849	51
10	.05524	.05533	18.075	.99847	**50**
11	553	562	17.980	846	49
12	582	591	.886	844	48
13	611	620	.793	842	47
14	640	649	.702	841	46
15	.05669	.05678	17.611	.99839	**45**
16	698	708	.521	838	44
17	727	737	.431	836	43
18	756	766	.343	834	42
19	785	795	.256	833	41
20	.05814	.05824	17.169	.99831	**40**
21	844	854	17.084	829	39
22	873	883	16.999	827	38
23	902	912	.915	826	37
24	931	941	.832	824	36
25	.05960	.05970	16.750	.99822	**35**
26	.05989	.05999	.668	821	34
27	.06018	.06029	.587	819	33
28	047	058	.507	817	32
29	076	087	.428	815	31
30	.06105	.06116	16.350	.99813	**30**
31	134	145	.272	812	29
32	163	175	.195	810	28
33	192	204	.119	808	27
34	221	233	16.043	806	26
35	.06250	.06262	15.969	.99804	**25**
36	279	291	.895	803	24
37	308	321	.821	801	23
38	337	350	.748	799	22
39	366	379	.676	797	21
40	.06395	.06408	15.605	.99795	**20**
41	424	438	.534	793	19
42	453	467	.464	792	18
43	482	496	.394	790	17
44	511	525	.325	788	16
45	.06540	.06554	15.257	.99786	**15**
46	569	584	.189	784	14
47	598	613	.122	782	13
48	627	642	15.056	780	12
49	656	671	14.990	778	11
50	.06685	.06700	14.924	.99776	**10**
51	714	730	.860	774	9
52	743	759	.795	772	8
53	773	788	.732	770	7
54	802	817	.669	768	6
55	.06831	.06847	14.606	.99766	**5**
56	860	876	.544	764	4
57	889	905	.482	762	3
58	918	934	.421	760	2
59	947	963	.361	758	1
60	.06976	.06993	14.301	.99756	**0**
	Cos	Cot	Tan	Sin	′

′	Sin	Tan	Cot	Cos	
0	.06976	.06993	14.301	.99756	**60**
1	.07005	.07022	.241	754	59
2	034	051	.182	752	58
3	063	080	.124	750	57
4	092	110	.065	748	56
5	.07121	.07139	14.008	.99746	**55**
6	150	168	13.951	744	54
7	179	197	.894	742	53
8	208	227	.838	740	52
9	237	256	.782	738	51
10	.07266	.07285	13.727	.99736	**50**
11	295	314	.672	734	49
12	324	344	.617	731	48
13	353	373	.563	729	47
14	382	402	.510	727	46
15	.07411	.07431	13.457	.99725	**45**
16	440	461	.404	723	44
17	469	490	.352	721	43
18	498	519	.300	719	42
19	527	548	.248	716	41
20	.07556	.07578	13.197	.99714	**40**
21	585	607	.146	712	39
22	614	636	.096	710	38
23	643	665	13.046	708	37
24	672	695	12.996	705	36
25	.07701	.07724	12.947	.99703	**35**
26	730	753	.898	701	34
27	759	782	.850	699	33
28	788	812	.801	696	32
29	817	841	.754	694	31
30	.07846	.07870	12.706	.99692	**30**
31	875	899	.659	689	29
32	904	929	.612	687	28
33	933	958	.566	685	27
34	962	.07987	.520	683	26
35	.07991	.08017	12.474	.99680	**25**
36	.08020	046	429	678	24
37	049	075	.384	676	23
38	078	104	.339	673	22
39	107	134	.295	671	21
40	.08136	.08163	12.251	.99668	**20**
41	165	192	.207	666	19
42	194	221	.163	664	18
43	223	251	.120	661	17
44	252	280	.077	659	16
45	.08281	.08309	12.035	.99657	**15**
46	310	339	11.992	654	14
47	339	368	.950	652	13
48	368	397	.909	649	12
49	397	427	.867	647	11
50	.08426	.08456	11.826	.99644	**10**
51	455	485	.785	642	9
52	484	514	.745	639	8
53	513	544	.705	637	7
54	542	573	.664	635	6
55	.08571	.08602	11.625	.99632	**5**
56	600	632	.585	630	4
57	629	661	.546	627	3
58	658	690	.507	625	2
59	687	720	.468	622	1
60	.08716	.08749	11.430	.99619	**0**
	Cos	Cot	Tan	Sin	′

′	Sin	Tan	Cot	Cos	
0	.08716	.08749	11.430	.99619	**60**
1	745	778	.392	617	59
2	774	807	.354	614	58
3	803	837	.316	612	57
4	831	866	.279	609	56
5	.08860	.08895	11.242	.99607	**55**
6	889	925	.205	604	54
7	918	954	.168	602	53
8	947	.08983	.132	599	52
9	.08976	.09013	.095	596	51
10	.09005	.09042	11.059	.99594	**50**
11	034	071	11.024	591	49
12	063	101	10.988	588	48
13	092	130	.953	586	47
14	121	159	.918	583	46
15	.09150	.09189	10.883	.99580	**45**
16	179	218	.848	578	44
17	208	247	.814	575	43
18	237	277	.780	572	42
19	266	306	.746	570	41
20	.09295	.09335	10.712	.99567	**40**
21	324	365	.678	564	39
22	353	394	.645	562	38
23	382	423	.612	559	37
24	411	453	.579	556	36
25	.09440	.09482	10.546	.99553	**35**
26	469	511	.514	551	34
27	498	541	.481	548	33
28	527	570	.449	545	32
29	556	600	.417	542	31
30	.09585	.09629	10.385	.99540	**30**
31	614	658	.354	537	29
32	642	688	.322	534	28
33	671	717	.291	531	27
34	700	746	.260	528	26
35	.09729	.09776	10.229	.99526	**25**
36	758	805	.199	523	24
37	787	834	.168	520	23
38	816	864	.138	517	22
39	845	893	.108	514	21
40	.09874	.09923	10.078	.99511	**20**
41	903	952	.048	508	19
42	932	.09981	10.019	506	18
43	961	.10011	9.9893	503	17
44	.09990	040	.9601	500	16
45	.10019	.10069	9.9310	.99497	**15**
46	048	099	.9021	494	14
47	077	128	.8734	491	13
48	106	158	.8448	488	12
49	135	187	.8164	485	11
50	.10164	.10216	9.7882	.99482	**10**
51	192	246	.7601	479	9
52	221	275	.7322	476	8
53	250	305	.7044	473	7
54	279	334	.6768	470	6
55	.10308	.10363	9.6493	.99467	**5**
56	337	393	6220	464	4
57	366	422	.5949	461	3
58	395	452	.5679	458	2
59	424	481	.5411	455	1
60	.10453	.10510	9.5144	.99452	**0**
	Cos	Cot	Tan	Sin	′

′	Sin	Tan	Cot	Cos	
0	.10453	.10510	9.5144	.99452	60
1	482	540	.4878	449	59
2	511	569	.4614	446	58
3	540	599	.4352	443	57
4	569	628	.4090	440	56
5	.10597	.10657	9.3831	.99437	55
6	626	687	.3572	434	54
7	655	716	.3315	431	53
8	684	746	.3060	428	52
9	713	775	.2806	424	51
10	.10742	.10805	9.2553	.99421	50
11	771	834	.2302	418	49
12	800	863	.2052	415	48
13	829	893	.1803	412	47
14	858	922	.1555	409	46
15	.10887	.10952	9.1309	.99406	45
16	916	.10981	.1065	402	44
17	945	.11011	.0821	399	43
18	.10973	040	.0579	396	42
19	.11002	070	.0338	393	41
20	.11031	.11099	9.0098	.99390	40
21	060	128	8.9860	386	39
22	089	158	.9623	383	38
23	118	187	.9387	380	37
24	147	217	.9152	377	36
25	.11176	.11246	8.8919	.99374	35
26	205	276	.8686	370	34
27	234	305	.8455	367	33
28	263	335	.8225	364	32
29	291	364	.7996	360	31
30	.11320	.11394	8.7769	.99357	30
31	349	423	.7542	354	29
32	378	452	.7317	351	28
33	407	482	.7093	347	27
34	436	511	.6870	344	26
35	.11465	.11541	8.6648	.99341	25
36	494	570	.6427	337	24
37	523	600	.6208	334	23
38	552	629	.5989	331	22
39	580	659	.5772	327	21
40	.11609	.11688	8.5555	.99324	20
41	638	718	.5340	320	19
42	667	747	.5126	317	18
43	696	777	.4913	314	17
44	725	806	.4701	310	16
45	.11754	.11836	8.4490	.99307	15
46	783	865	.4280	303	14
47	812	895	.4071	300	13
48	840	924	.3863	297	12
49	869	954	.3656	293	11
50	.11898	.11983	8.3450	.99290	10
51	927	.12013	.3245	286	9
52	956	042	.3041	283	8
53	.11985	072	.2838	279	7
54	.12014	101	.2636	276	6
55	.12043	.12131	8.2434	.99272	5
56	071	160	.2234	269	4
57	100	190	.2035	265	3
58	129	219	.1837	262	2
59	158	249	.1640	258	1
60	.12187	.12278	8.1443	.99255	0
	Cos	Cot	Tan	Sin	′

′	Sin	Tan	Cot	Cos	
0	.12187	.12278	8.1443	.99255	60
1	216	308	.1248	251	59
2	245	338	.1054	248	58
3	274	367	.0860	244	57
4	302	397	.0667	240	56
5	.12331	.12426	8.0476	.99237	55
6	360	456	.0285	233	54
7	389	485	8.0095	230	53
8	418	515	7.9906	226	52
9	447	544	.9718	222	51
10	.12476	.12574	7.9530	.99219	50
11	504	603	.9344	215	49
12	533	633	.9158	211	48
13	562	662	.8973	208	47
14	591	692	.8789	204	46
15	.12620	.12722	7.8606	.99200	45
16	649	751	.8424	197	44
17	678	781	.8243	193	43
18	706	810	.8062	189	42
19	735	840	.7882	186	41
20	.12764	.12869	7.7704	.99182	40
21	793	899	.7525	178	39
22	822	929	.7348	175	38
23	851	958	.7171	171	37
24	880	.12988	.6996	167	36
25	.12908	.13017	7.6821	.99163	35
26	937	047	.6647	160	34
27	966	076	.6473	156	33
28	.12995	106	.6301	152	32
29	.13024	136	.6129	148	31
30	.13053	.13165	7.5958	.99144	30
31	081	195	.5787	141	29
32	110	224	.5618	137	28
33	139	254	.5449	133	27
34	168	284	.5281	129	26
35	.13197	.13313	7.5113	.99125	25
36	226	343	.4947	122	24
37	254	372	.4781	118	23
38	283	402	.4615	114	22
39	312	432	.4451	110	21
40	.13341	.13461	7.4287	.99106	20
41	370	491	.4124	102	19
42	399	521	.3962	098	18
43	427	550	.3800	094	17
44	456	580	.3639	091	16
45	.13485	.13609	7.3479	.99087	15
46	514	639	.3319	083	14
47	543	669	.3160	079	13
48	572	698	.3002	075	12
49	600	728	.2844	071	11
50	.13629	.13758	7.2687	.99067	10
51	658	787	.2531	063	9
52	687	817	.2375	059	8
53	716	846	.2220	055	7
54	744	876	.2066	051	6
55	.13773	.13906	7.1912	.99047	5
56	802	935	.1759	043	4
57	831	965	.1607	039	3
58	860	.13995	.1455	035	2
59	889	.14024	.1304	031	1
60	.13917	.14054	7.1154	.99027	0
	Cos	Cot	Tan	Sin	′

′	Sin	Tan	Cot	Cos	
0	.13917	.14054	7.1154	.99027	**60**
1	946	084	.1004	023	59
2	.13975	113	.0855	019	58
3	.14004	143	.0706	015	57
4	033	173	.0558	011	56
5	.14061	.14202	7.0410	.99006	**55**
6	090	232	.0264	.99002	54
7	119	262	7.0117	.98998	53
8	148	291	6.9972	994	52
9	177	321	.9827	990	51
10	.14205	.14351	6.9682	.98986	**50**
11	234	381	.9538	982	49
12	263	410	.9395	978	48
13	292	440	.9252	973	47
14	320	470	.9110	969	46
15	.14349	.14499	6.8969	.98965	**45**
16	378	529	.8828	961	44
17	407	559	.8687	957	43
18	436	588	.8548	953	42
19	464	618	.8408	948	41
20	.14493	.14648	6.8269	.98944	**40**
21	522	678	.8131	940	39
22	551	707	.7994	936	38
23	580	737	.7856	931	37
24	608	767	.7720	927	36
25	.14637	.14796	6.7584	.98923	**35**
26	666	826	.7448	919	34
27	695	856	.7313	914	33
28	723	886	.7179	910	32
29	752	915	.7045	906	31
30	.14781	.14945	6.6912	.98902	**30**
31	810	.14975	.6779	897	29
32	838	.15005	.6646	893	28
33	867	034	.6514	889	27
34	896	064	.6383	884	26
35	.14925	.15094	6.6252	.98880	**25**
36	954	124	.6122	876	24
37	.14982	153	.5992	871	23
38	.15011	183	.5863	867	22
39	040	213	.5734	863	21
40	.15069	.15243	6.5606	.98858	**20**
41	097	272	.5478	854	19
42	126	302	.5350	849	18
43	155	332	.5223	845	17
44	184	362	.5097	841	16
45	.15212	.15391	6.4971	.98836	**15**
46	241	421	.4846	832	14
47	270	451	.4721	827	13
48	299	481	.4596	823	12
49	327	511	.4472	818	11
50	.15356	.15540	6.4348	.98814	**10**
51	385	570	.4225	809	9
52	414	600	.4103	805	8
53	442	630	.3980	800	7
54	471	660	.3859	796	6
55	.15500	.15689	6.3737	.98791	**5**
56	529	719	.3617	787	4
57	557	749	.3496	782	3
58	586	779	.3376	778	2
59	615	809	.3257	773	1
60	.15643	.15838	6.3138	.98769	**0**
	Cos	Cot	Tan	Sin	′

′	Sin	Tan	Cot	Cos	
0	.15643	.15838	6.3138	.98769	**60**
1	672	868	.3019	764	59
2	701	898	.2901	760	58
3	730	928	.2783	755	57
4	758	958	.2666	751	56
5	.15787	.15988	6.2549	.98746	**55**
6	816	.16017	.2432	741	54
7	845	047	.2316	737	53
8	873	077	.2200	732	52
9	902	107	.2085	728	51
10	.15931	.16137	6.1970	.98723	**50**
11	959	167	.1856	718	49
12	.15988	196	.1742	714	48
13	.16017	226	.1628	709	47
14	046	256	.1515	704	46
15	.16074	.16286	6.1402	.98700	**45**
16	103	316	.1290	695	44
17	132	346	.1178	690	43
18	160	376	.1066	686	42
19	189	405	.0955	681	41
20	.16218	.16435	6.0844	.98676	**40**
21	246	465	.0734	671	39
22	275	495	.0624	667	38
23	304	525	.0514	662	37
24	333	555	.0405	657	36
25	.16361	.16585	6.0296	.98652	**35**
26	390	615	.0188	648	34
27	419	645	6.0080	643	33
28	447	674	5.9972	638	32
29	476	704	.9865	633	31
30	.16505	.16734	5.9758	.98629	**30**
31	533	764	.9651	624	29
32	562	794	.9545	619	28
33	591	824	.9439	614	27
34	620	854	.9333	609	26
35	.16648	.16884	5.9228	.98604	**25**
36	677	914	.9124	600	24
37	706	944	.9019	595	23
38	734	.16974	.8915	590	22
39	763	.17004	.8811	585	21
40	.16792	.17033	5.8708	.98580	**20**
41	820	063	.8605	575	19
42	849	093	.8502	570	18
43	878	123	.8400	565	17
44	906	153	.8298	561	16
45	.16935	.17183	5.8197	.98556	**15**
46	964	213	.8095	551	14
47	.16992	243	.7994	546	13
48	.17021	273	.7894	541	12
49	050	303	.7794	536	11
50	.17078	.17333	5.7694	.98531	**10**
51	107	363	.7594	526	9
52	136	393	.7495	521	8
53	164	423	.7396	516	7
54	193	453	.7297	511	6
55	.17222	.17483	5.7199	.98506	**5**
56	250	513	.7101	501	4
57	279	543	.7004	496	3
58	308	573	.6906	491	2
59	336	603	.6809	486	1
60	.17365	.17633	5.6713	.98481	**0**
	Cos	Cot	Tan	Sin	′

′	Sin	Tan	Cot	Cos	
0	.17365	.17633	5.6713	.98481	60
1	393	663	.6617	476	59
2	422	693	.6521	471	58
3	451	723	.6425	466	57
4	479	753	.6329	461	56
5	.17508	.17783	5.6234	.98455	55
6	537	813	.6140	450	54
7	565	843	.6045	445	53
8	594	873	.5951	440	52
9	623	903	.5857	435	51
10	.17651	.17933	5.5764	.98430	50
11	680	963	.5671	425	49
12	708	.17993	.5578	420	48
13	737	.18023	.5485	414	47
14	766	053	.5393	409	46
15	.17794	.18083	5.5301	.98404	45
16	823	113	.5209	399	44
17	852	143	.5118	394	43
18	880	173	.5026	389	42
19	909	203	.4936	383	41
20	.17937	.18233	5.4845	.98378	40
21	966	263	.4755	373	39
22	.17995	293	.4665	368	38
23	.18023	323	.4575	362	37
24	052	353	.4486	357	36
25	.18081	.18384	5.4397	.98352	35
26	109	414	.4308	347	34
27	138	444	.4219	341	33
28	166	474	.4131	336	32
29	195	504	.4043	331	31
30	.18224	.18534	5.3955	.98325	30
31	252	564	.3868	320	29
32	281	594	.3781	315	28
33	309	624	.3694	310	27
34	338	654	.3607	304	26
35	.18367	.18684	5.3521	.98299	25
36	395	714	.3435	294	24
37	424	745	.3349	288	23
38	452	775	.3263	283	22
39	481	805	.3178	277	21
40	.18509	.18835	5.3093	.98272	20
41	538	865	.3008	267	19
42	567	895	.2924	261	18
43	595	925	.2839	256	17
44	624	955	.2755	250	16
45	.18652	.18986	5.2672	.98245	15
46	681	.19016	.2588	240	14
47	710	046	.2505	234	13
48	738	076	.2422	229	12
49	767	106	.2339	223	11
50	.18795	.19136	5.2257	.98218	10
51	824	166	.2174	212	9
52	852	197	.2092	207	8
53	881	227	.2011	201	7
54	910	257	.1929	196	6
55	.18938	.19287	5.1848	.98190	5
56	967	317	.1767	185	4
57	.18995	347	.1686	179	3
58	.19024	378	.1606	174	2
59	052	408	.1526	168	1
60	.19081	.19438	5.1446	.98163	0
	Cos	Cot	Tan	Sin	′

′	Sin	Tan	Cot	Cos	
0	.19081	.19438	5.1446	.98163	60
1	109	468	.1366	157	59
2	138	498	.1286	152	58
3	167	529	.1207	146	57
4	195	559	.1128	140	56
5	.19224	.19589	5.1049	.98135	55
6	252	619	.0970	129	54
7	281	649	.0892	124	53
8	309	680	.0814	118	52
9	338	710	.0736	112	51
10	.19366	.19740	5.0658	.98107	50
11	395	770	.0581	101	49
12	423	801	.0504	096	48
13	452	831	.0427	090	47
14	481	861	.0350	084	46
15	.19509	.19891	5.0273	.98079	45
16	538	921	.0197	073	44
17	566	952	.0121	067	43
18	595	.19982	5.0045	061	42
19	623	.20012	4.9969	056	41
20	.19652	.20042	4.9894	.98050	40
21	680	073	.9819	044	39
22	709	103	.9744	039	38
23	737	133	.9669	033	37
24	766	164	.9594	027	36
25	.19794	.20194	4.9520	.98021	35
26	823	224	.9446	016	34
27	851	254	.9372	010	33
28	880	285	.9298	.98004	32
29	908	315	.9225	.97998	31
30	.19937	.20345	4.9152	.97992	30
31	965	376	.9078	987	29
32	.19994	406	.9006	981	28
33	.20022	436	.8933	975	27
34	051	466	.8860	969	26
35	.20079	.20497	4.8788	.97963	25
36	108	527	.8716	958	24
37	136	557	.8644	952	23
38	165	588	.8573	946	22
39	193	618	.8501	940	21
40	.20222	.20648	4.8430	.97934	20
41	250	679	.8359	928	19
42	279	709	.8288	922	18
43	307	739	.8218	916	17
44	336	770	.8147	910	16
45	.20364	.20800	4.8077	.97905	15
46	393	830	.8007	899	14
47	421	861	.7937	893	13
48	450	891	.7867	887	12
49	478	921	.7798	881	11
50	.20507	.20952	4.7729	.97875	10
51	535	.20982	.7659	869	9
52	563	.21013	.7591	863	8
53	592	043	.7522	857	7
54	620	073	.7453	851	6
55	.20649	.21104	4.7385	.97845	5
56	677	134	.7317	839	4
57	706	164	.7249	833	3
58	734	195	.7181	827	2
59	763	225	.7114	821	1
60	.20791	.21256	4.7046	.97815	0
	Cos	Cot	Tan	Sin	′

′	Sin	Tan	Cot	Cos	
0	.20791	.21256	4.7046	.97815	**60**
1	820	286	.6979	809	59
2	848	316	.6912	803	58
3	877	347	.6845	797	57
4	905	377	.6779	791	56
5	.20933	.21408	4.6712	.97784	**55**
6	962	438	.6646	778	54
7	.20990	469	.6580	772	53
8	.21019	499	.6514	766	52
9	047	529	.6448	760	51
10	.21076	.21560	4 6382	.97754	**50**
11	104	590	.6317	748	49
12	132	621	.6252	742	48
13	161	651	.6187	735	47
14	189	682	.6122	729	46
15	.21218	.21712	4.6057	.97723	**45**
16	246	743	.5993	717	44
17	275	773	.5928	711	43
18	303	804	.5864	705	42
19	331	834	.5800	698	41
20	.21360	.21864	4.5736	.97692	**40**
21	388	895	.5673	686	39
22	417	925	.5609	680	38
23	445	956	.5546	673	37
24	474	.21986	.5483	667	36
25	.21502	.22017	4.5420	.97661	**35**
26	530	047	.5357	655	34
27	559	078	.5294	648	33
28	587	108	.5232	642	32
29	616	139	.5169	636	31
30	.21644	.22169	4.5107	.97630	**30**
31	672	200	.5045	623	29
32	701	231	.4983	617	28
33	729	261	.4922	611	27
34	758	292	.4860	604	26
35	.21786	.22322	4.4799	.97598	**25**
36	814	353	.4737	592	24
37	843	383	.4676	585	23
38	871	414	.4615	579	22
39	899	444	.4555	573	21
40	.21928	.22475	4.4494	.97566	**20**
41	956	505	.4434	560	19
42	.21985	536	.4373	553	18
43	.22013	567	.4313	547	17
44	041	597	.4253	541	16
45	.22070	.22628	4.4194	.97534	**15**
46	098	658	.4134	528	14
47	126	689	.4075	521	13
48	155	719	.4015	515	12
49	183	750	.3956	508	11
50	.22212	.22781	4.3897	.97502	**10**
51	240	811	.3838	496	9
52	268	842	.3779	489	8
53	297	872	.3721	483	7
54	325	903	.3662	476	6
55	.22353	.22934	4.3601	.97470	**5**
56	382	964	.3546	463	4
57	410	.22995	.3488	457	3
58	438	.23026	.3430	450	2
59	467	056	.3372	444	1
60	.22495	.23087	4.3315	.97437	**0**
	Cos	Cot	Tan	Sin	′

′	Sin	Tan	Cot	Cos	
0	.22495	.23087	4.3315	.97437	**60**
1	523	117	.3257	430	59
2	552	148	.3200	424	58
3	580	179	.3143	417	57
4	608	209	.3086	411	56
5	.22637	.23240	4.3029	.97404	**55**
6	665	271	.2972	398	54
7	693	301	.2916	391	53
8	722	332	.2859	384	52
9	750	363	.2803	378	51
10	.22778	.23393	4.2747	.97371	**50**
11	807	424	.2691	365	49
12	835	455	.2635	358	48
13	863	485	.2580	351	47
14	892	516	.2524	345	46
15	.22920	.23547	4.2468	.97338	**45**
16	948	578	.2413	331	44
17	.22977	608	.2358	325	43
18	.23005	639	.2303	318	42
19	033	670	.2248	311	41
20	.23062	.23700	4.2193	.97304	**40**
21	090	731	.2139	298	39
22	118	762	.2084	291	38
23	146	793	.2030	284	37
24	175	823	.1976	278	36
25	.23203	.23854	4.1922	.97271	**35**
26	231	885	.1868	264	34
27	260	916	.1814	257	33
28	288	946	.1760	251	32
29	316	.23977	.1706	244	31
30	.23345	.24008	4.1653	.97237	**30**
31	373	039	.1600	230	29
32	401	069	.1547	223	28
33	429	100	.1493	217	27
34	458	131	.1441	210	26
35	.23486	.24162	4.1388	.97203	**25**
36	514	193	.1335	196	24
37	542	223	.1282	189	23
38	571	254	.1230	182	22
39	599	285	.1178	176	21
40	.23627	.24316	4.1126	.97169	**20**
41	656	347	.1074	162	19
42	684	377	.1022	155	18
43	712	408	.0970	148	17
44	740	439	.0918	141	16
45	.23769	.24470	4.0867	.97134	**15**
46	797	501	.0815	127	14
47	825	532	.0764	120	13
48	853	562	.0713	113	12
49	882	593	.0662	106	11
50	.23910	.24624	4.0611	.97100	**10**
51	938	655	.0560	093	9
52	966	686	.0509	086	8
53	.23995	717	.0459	079	7
54	.24023	747	.0408	072	6
55	.24051	.24778	4.0358	.97065	**5**
56	079	809	.0308	058	4
57	108	840	.0257	051	3
58	136	871	.0207	044	2
59	164	902	.0158	037	1
60	.24192	.24933	4.0108	.97030	**0**
	Cos	Cot	Tan	Sin	′

′	Sin	Tan	Cot	Cos	
0	.24192	.24933	4.0108	.97030	**60**
1	220	964	.0058	023	59
2	249	.24995	4.0009	015	58
3	277	.25026	3.9959	008	57
4	305	056	.9910	.97001	56
5	.24333	.25087	3.9861	.96994	**55**
6	362	118	.9812	987	54
7	390	149	.9763	980	53
8	418	180	.9714	973	52
9	446	211	.9665	966	51
10	.24474	.25242	3.9617	.96959	**50**
11	503	273	.9568	952	49
12	531	304	.9520	945	48
13	559	335	.9471	937	47
14	587	366	.9423	930	46
15	.24615	.25397	3.9375	.96923	**45**
16	644	428	.9327	916	44
17	672	459	.9279	909	43
18	700	490	.9232	902	42
19	728	521	.9184	894	41
20	.24756	.25552	3.9136	.96887	**40**
21	784	583	.9089	880	39
22	813	614	.9042	873	38
23	841	645	.8995	866	37
24	869	676	.8947	858	36
25	.24897	.25707	3.8900	.96851	**35**
26	925	738	.8854	844	34
27	954	769	.8807	837	33
28	.24982	800	.8760	829	32
29	.25010	831	.8714	822	31
30	.25038	.25862	3.8667	.96815	**30**
31	066	893	.8621	807	29
32	094	924	.8575	800	28
33	122	955	.8528	793	27
34	151	.25986	.8482	786	26
35	.25179	.26017	3.8436	.96778	**25**
36	207	048	.8391	771	24
37	235	079	.8345	764	23
38	263	110	.8299	756	22
39	291	141	.8254	749	21
40	.25320	.26172	3.8208	.96742	**20**
41	348	203	.8163	734	19
42	376	235	.8118	727	18
43	404	266	.8073	719	17
44	432	297	.8028	712	16
45	.25460	.26328	3.7983	.96705	**15**
46	488	359	.7938	697	14
47	516	390	.7893	690	13
48	545	421	.7848	682	12
49	573	452	.7804	675	11
50	.25601	.26483	3.7760	.96667	**10**
51	629	515	.7715	660	9
52	657	546	.7671	653	8
53	685	577	.7627	645	7
54	713	608	.7583	638	6
55	.25741	.26639	3.7539	.96630	**5**
56	769	670	.7495	623	4
57	798	701	.7451	615	3
58	826	733	.7408	608	2
59	854	764	.7364	600	1
60	.25882	.26795	3.7321	.96593	**0**
	Cos	Cot	Tan	Sin	′

′	Sin	Tan	Cot	Cos	
0	.25882	.26795	3.7321	.96593	**60**
1	910	826	.7277	585	59
2	938	857	.7234	578	58
3	966	888	.7191	570	57
4	.25994	920	.7148	562	56
5	.26022	.26951	3.7105	.96555	**55**
6	050	.26982	.7062	547	54
7	079	.27013	.7019	540	53
8	107	044	.6976	532	52
9	135	076	.6933	524	51
10	.26163	.27107	3.6891	.96517	**50**
11	191	138	.6848	509	49
12	219	169	.6806	502	48
13	247	201	.6764	494	47
14	275	232	.6722	486	46
15	.26303	.27263	3.6680	.96479	**45**
16	331	294	.6638	471	44
17	359	326	.6596	463	43
18	387	357	.6554	456	42
19	415	388	.6512	448	41
20	.26443	.27419	3.6470	.96440	**40**
21	471	451	.6429	433	39
22	500	482	.6387	425	38
23	528	513	.6346	417	37
24	556	545	.6305	410	36
25	.26584	.27576	3.6264	.96402	**35**
26	612	607	.6222	394	34
27	640	638	.6181	386	33
28	668	670	.6140	379	32
29	696	701	.6100	371	31
30	.26724	.27732	3.6059	.96363	**30**
31	752	764	.6018	355	29
32	780	795	.5978	347	28
33	808	826	.5937	340	27
34	836	858	.5897	332	26
35	.26864	.27889	3.5856	.96324	**25**
36	892	921	.5816	316	24
37	920	952	.5776	308	23
38	948	.27983	.5736	301	22
39	.26976	.28015	.5696	293	21
40	.27004	.28046	3.5656	.96285	**20**
41	032	077	.5616	277	19
42	060	109	.5576	269	18
43	088	140	.5536	261	17
44	116	172	.5497	253	16
45	.27144	.28203	3.5457	.96246	**15**
46	172	234	.5418	238	14
47	200	266	.5379	230	13
48	228	297	.5339	222	12
49	256	329	.5300	214	11
50	.27284	.28360	3.5261	.96206	**10**
51	312	391	.5222	198	9
52	340	423	.5183	190	8
53	368	454	.5144	182	7
54	396	486	.5105	174	6
55	.27424	.28517	3.5067	.96166	**5**
56	452	549	.5028	158	4
57	480	580	.4989	150	3
58	508	612	.4951	142	2
59	536	643	.4912	134	1
60	.27564	.28675	3.4874	.96126	**0**
	Cos	Cot	Tan	Sin	′

′	Sin	Tan	Cot	Cos	
0	.27564	.28675	3.4874	.96126	**60**
1	592	706	.4836	118	59
2	620	738	.4798	110	58
3	648	769	.4760	102	57
4	676	801	.4722	094	56
5	.27704	.28832	3.4684	.96086	**55**
6	731	864	.4646	078	54
7	759	895	.4608	070	53
8	787	927	.4570	062	52
9	815	958	.4533	054	51
10	.27843	.28990	3.4495	.96046	**50**
11	871	.29021	.4458	037	49
12	899	053	.4420	029	48
13	927	084	.4383	021	47
14	955	116	.4346	013	46
15	.27983	.29147	3.4308	.96005	**45**
16	.28011	179	.4271	.95997	44
17	039	210	.4234	989	43
18	067	242	.4197	981	42
19	095	274	.4160	972	41
20	.28123	.29305	3.4124	.95964	**40**
21	150	337	.4087	956	39
22	178	368	.4050	948	38
23	206	400	.4014	940	37
24	234	432	.3977	931	36
25	.28262	.29463	3.3941	.95923	**35**
26	290	495	.3904	915	34
27	318	526	.3868	907	33
28	346	558	.3832	898	32
29	374	590	.3796	890	31
30	.28402	.29621	3.3759	.95882	**30**
31	429	653	.3723	874	29
32	457	685	.3687	865	28
33	485	716	.3652	857	27
34	513	748	.3616	849	26
35	.28541	.29780	3.3580	.95841	**25**
36	569	811	.3544	832	24
37	597	843	.3509	824	23
38	625	875	.3473	816	22
39	652	906	.3438	807	21
40	.28680	.29938	3.3402	.95799	**20**
41	708	.29970	.3367	791	19
42	736	.30001	.3332	782	18
43	764	033	.3297	774	17
44	792	065	.3261	766	16
45	.28820	.30097	3.3226	.95757	**15**
46	847	128	.3191	749	14
47	875	160	.3156	740	13
48	903	192	.3122	732	12
49	931	224	.3087	724	11
50	.28959	.30255	3.3052	.95715	**10**
51	.28987	287	.3017	707	9
52	.29015	319	.2983	698	8
53	042	351	.2948	690	7
54	070	382	.2914	681	6
55	.29098	.30414	3.2879	.95673	**5**
56	126	446	.2845	664	4
57	154	478	.2811	656	3
58	182	509	.2777	647	2
59	209	541	.2743	639	1
60	.29237	.30573	3.2709	.95630	**0**
	Cos	Cot	Tan	Sin	′

′	Sin	Tan	Cot	Cos	
0	.29237	.30573	3.2709	.95630	**60**
1	265	605	.2675	622	59
2	293	637	.2641	613	58
3	321	669	.2607	605	57
4	348	700	.2573	596	56
5	.29376	.30732	3.2539	.95588	**55**
6	404	764	.2506	579	54
7	432	796	.2472	571	53
8	460	828	.2438	562	52
9	487	860	.2405	554	51
10	.29515	.30891	3.2371	.95545	**50**
11	543	923	.2338	536	49
12	571	955	.2305	528	48
13	599	.30987	.2272	519	47
14	626	.31019	.2238	511	46
15	.29654	.31051	3.2205	.95502	**45**
16	682	083	.2172	493	44
17	710	115	.2139	485	43
18	737	147	.2106	476	42
19	765	178	.2073	467	41
20	.29793	.31210	3.2041	.95459	**40**
21	821	242	.2008	450	39
22	849	274	.1975	441	38
23	876	306	.1943	433	37
24	904	338	.1910	424	36
25	.29932	.31370	3.1878	.95415	**35**
26	960	402	.1845	407	34
27	.29987	434	.1813	398	33
28	.30015	466	.1780	389	32
29	043	498	.1748	380	31
30	.30071	.31530	3.1716	.95372	**30**
31	098	562	.1684	363	29
32	126	594	.1652	354	28
33	154	626	.1620	345	27
34	182	658	.1588	337	26
35	.30209	.31690	3.1556	.95328	**25**
36	237	722	.1524	319	24
37	265	754	.1492	310	23
38	292	786	.1460	301	22
39	320	818	.1429	293	21
40	.30348	.31850	3.1397	.95284	**20**
41	376	882	.1366	275	19
42	403	914	.1334	266	18
43	431	946	.1303	257	17
44	459	.31978	.1271	248	16
45	.30486	.32010	3.1240	.95240	**15**
46	514	042	.1209	231	14
47	542	074	.1178	222	13
48	570	106	.1146	213	12
49	597	139	.1115	204	11
50	.30625	.32171	3.1084	.95195	**10**
51	653	203	.1053	186	9
52	680	235	.1022	177	8
53	708	267	.0991	168	7
54	736	299	.0961	159	6
55	.30763	.32331	3.0930	.95150	**5**
56	791	363	.0899	142	4
57	819	396	.0868	133	3
58	846	428	.0838	124	2
59	874	460	.0807	115	1
60	.30902	.32492	3.0777	.95106	**0**
	Cos	Cot	Tan	Sin	′

′	Sin	Tan	Cot	Cos	
0	.30902	.32492	3.0777	.95106	**60**
1	929	524	.0746	097	59
2	957	556	.0716	088	58
3	.30985	588	.0686	079	57
4	.31012	621	.0655	070	56
5	.31040	.32653	3.0625	.95061	**55**
6	068	685	.0595	052	54
7	095	717	.0565	043	53
8	123	749	.0535	033	52
9	151	782	.0505	024	51
10	.31178	.32814	3.0475	.95015	**50**
11	206	846	.0445	.95006	49
12	233	878	.0415	.94997	48
13	261	911	.0385	988	47
14	289	943	.0356	979	46
15	.31316	.32975	3.0326	.94970	**45**
16	344	.33007	.0296	961	44
17	372	040	.0267	952	43
18	399	072	.0237	943	42
19	427	104	.0208	933	41
20	.31454	.33136	3.0178	.94924	**40**
21	482	169	.0149	915	39
22	510	201	.0120	906	38
23	537	233	.0090	897	37
24	565	266	.0061	888	36
25	.31593	.33298	3.0032	.94878	**35**
26	620	330	3.0003	869	34
27	648	363	2.9974	860	33
28	675	395	.9945	851	32
29	703	427	.9916	842	31
30	.31730	.33460	2.9887	.94832	**30**
31	758	492	.9858	823	29
32	786	524	.9829	814	28
33	813	557	.9800	805	27
34	841	589	.9772	795	26
35	.31868	.33621	2.9743	.94786	**25**
36	896	654	.9714	777	24
37	923	686	.9686	768	23
38	951	718	.9657	758	22
39	.31979	751	.9629	749	21
40	.32006	.33783	2.9600	.94740	**20**
41	034	816	.9572	730	19
42	061	848	.9544	721	18
43	089	881	.9515	712	17
44	116	913	.9487	702	16
45	.32144	.33945	2.9459	.94693	**15**
46	171	.33978	.9431	684	14
47	199	.34010	.9403	674	13
48	227	043	.9375	665	12
49	254	075	.9347	656	11
50	.32282	.34108	2.9319	.94646	**10**
51	309	140	.9291	637	9
52	337	173	.9263	627	8
53	364	205	.9235	618	7
54	392	238	.9208	609	6
55	.32419	.34270	2.9180	.94599	**5**
56	447	303	.9152	590	4
57	474	335	.9125	580	3
58	502	368	.9097	571	2
59	529	400	.9070	561	1
60	.32557	.34433	2.9042	.94552	**0**
	Cos	Cot	Tan	Sin	′

′	Sin	Tan	Cot	Cos	
0	.32557	.34433	2.9042	.94552	**60**
1	584	465	.9015	542	59
2	612	498	.8987	533	58
3	639	530	.8960	523	57
4	667	563	.8933	514	56
5	.32694	.34596	2.8905	.94504	**55**
6	722	628	.8878	495	54
7	749	661	.8851	485	53
8	777	693	.8824	476	52
9	804	726	.8797	466	51
10	.32832	.34758	2.8770	.94457	**50**
11	859	791	.8743	447	49
12	887	824	.8716	438	48
13	914	856	.8689	428	47
14	942	889	.8662	418	46
15	.32969	.34922	2.8636	.94409	**45**
16	.32997	954	.8609	399	44
17	.33024	.34987	.8582	390	43
18	051	.35020	.8556	380	42
19	079	052	.8529	370	41
20	.33106	.35085	2.8502	.94361	**40**
21	134	118	.8476	351	39
22	161	150	.8449	342	38
23	189	183	.8423	332	37
24	216	216	.8397	322	36
25	.33244	.35248	2.8370	.94313	**35**
26	271	281	.8344	303	34
27	298	314	.8318	293	33
28	326	346	.8291	284	32
29	353	379	.8265	274	31
30	.33381	.35412	2.8239	.94264	**30**
31	408	445	.8213	254	29
32	436	477	.8187	245	28
33	463	510	.8161	235	27
34	490	543	.8135	225	26
35	.33518	.35576	2.8109	.94215	**25**
36	545	608	.8083	206	24
37	573	641	.8057	196	23
38	600	674	.8032	186	22
39	627	707	.8006	176	21
40	.33655	.35740	2.7980	.94167	**20**
41	682	.772	.7955	157	19
42	710	805	.7929	147	18
43	737	838	.7903	137	17
44	764	871	.7878	127	16
45	.33792	.35904	2.7852	.94118	**15**
46	819	937	.7827	108	14
47	846	.35969	.7801	098	13
48	874	.36002	.7776	088	12
49	901	035	.7751	078	11
50	.33929	.36068	2.7725	.94068	**10**
51	956	101	.7700	058	9
52	.33983	134	.7675	049	8
53	.34011	167	.7650	039	7
54	038	199	.7625	029	6
55	.34065	.36232	2.7600	.94019	**5**
56	093	265	.7575	.94009	4
57	120	298	.7550	.93999	3
58	147	331	.7525	989	2
59	175	364	.7500	979	1
60	.34202	.36397	2.7475	.93969	**0**
	Cos	Cot	Tan	Sin	′

′	Sin	Tan	Cot	Cos	
0	.34202	.36397	2.7475	.93969	**60**
1	229	430	.7450	959	59
2	257	463	.7425	949	58
3	284	496	.7400	939	57
4	311	529	.7376	929	56
5	.34339	.36562	2.7351	.93919	**55**
6	366	595	.7326	909	54
7	393	628	.7302	899	53
8	421	661	.7277	889	52
9	448	694	.7253	879	51
10	.34475	.36727	2.7228	.93869	**50**
11	503	760	.7204	859	49
12	530	793	.7179	849	48
13	557	826	.7155	839	47
14	584	859	.7130	829	46
15	.34612	.36892	2.7106	.93819	**45**
16	639	925	.7082	809	44
17	666	958	.7058	799	43
18	694	.36991	.7034	789	42
19	721	.37024	.7009	779	41
20	.34748	.37057	2.6985	.93769	**40**
21	775	090	.6961	759	39
22	803	123	.6937	748	38
23	830	157	.6913	738	37
24	857	190	.6889	728	36
25	.34884	.37223	2.6865	.93718	**35**
26	912	256	.6841	708	34
27	939	289	.6818	698	33
28	966	322	.6794	688	32
29	.34993	355	.6770	677	31
30	.35021	.37388	2.6746	.93667	**30**
31	048	422	.6723	657	29
32	075	455	.6699	647	28
33	102	488	.6675	637	27
34	130	521	.6652	626	26
35	.35157	.37554	2.6628	.93616	**25**
36	184	588	.6605	606	24
37	211	621	.6581	596	23
38	239	654	.6558	585	22
39	266	687	.6534	575	21
40	.35293	.37720	2.6511	.93565	**20**
41	320	754	.6488	555	19
42	347	787	.6464	544	18
43	375	820	.6441	534	17
44	402	853	.6418	524	16
45	.35429	.37887	2.6395	.93514	**15**
46	456	920	.6371	503	14
47	484	953	.6348	493	13
48	511	.37986	.6325	483	12
49	538	.38020	.6302	472	11
50	.35565	.38053	2.6279	.93462	**10**
51	592	086	.6256	452	9
52	619	120	.6233	441	8
53	647	153	.6210	431	7
54	674	186	.6187	420	6
55	.35701	.38220	2.6165	.93410	**5**
56	728	253	.6142	400	4
57	755	286	.6119	389	3
58	782	320	.6096	379	2
59	810	353	.6074	368	1
60	.35837	.38386	2.6051	.93358	**0**
	Cos	Cot	Tan	Sin	′

′	Sin	Tan	Cot	Cos	
0	.35837	.38386	2.6051	.93358	**60**
1	864	420	.6028	348	59
2	891	453	.6006	337	58
3	918	487	.5983	327	57
4	945	520	.5961	316	56
5	.35973	.38553	2.5938	.93306	**55**
6	.36000	587	.5916	295	54
7	027	620	.5893	285	53
8	054	654	.5871	274	52
9	081	687	.5848	264	51
10	.36108	.38721	2.5826	.93253	**50**
11	135	754	.5804	243	49
12	162	787	.5782	232	48
13	190	821	.5759	222	47
14	217	854	.5737	211	46
15	.36244	.38888	2.5715	.93201	**45**
16	271	921	.5693	190	44
17	298	955	.5671	180	43
18	325	.38988	.5649	169	42
19	352	.39022	.5627	159	41
20	.36379	.39055	2.5605	.93148	**40**
21	406	089	.5583	137	39
22	434	122	.5561	127	38
23	461	156	.5539	116	37
24	488	190	.5517	106	36
25	.36515	.39223	2.5495	.93095	**35**
26	542	257	.5473	084	34
27	569	290	.5452	074	33
28	596	324	.5430	063	32
29	623	357	.5408	052	31
30	.36650	.39391	2.5386	.93042	**30**
31	677	425	.5365	031	29
32	704	458	.5343	020	28
33	731	492	.5322	.93010	27
34	758	526	.5300	.92999	26
35	.36785	.39559	2.5279	.92988	**25**
36	812	593	.5257	978	24
37	839	626	.5236	967	23
38	867	660	.5214	956	22
39	894	694	.5193	945	21
40	.36921	.39727	2.5172	.92935	**20**
41	948	761	.5150	924	19
42	.36975	795	.5129	913	18
43	.37002	829	.5108	902	17
44	029	862	.5086	892	16
45	.37056	.39896	2.5065	.92881	**15**
46	083	930	.5044	870	14
47	110	963	.5023	859	13
48	137	.39997	.5002	849	12
49	164	.40031	.4981	838	11
50	.37191	.40065	2.4960	.92827	**10**
51	218	098	.4939	816	9
52	245	132	.4918	805	8
53	272	166	.4897	794	7
54	299	200	.4876	784	6
55	.37326	.40234	2.4855	.92773	**5**
56	353	267	.4834	762	4
57	380	301	.4813	751	3
58	407	335	.4792	740	2
59	434	369	.4772	729	1
60	.37461	.40403	2.4751	.92718	**0**
	Cos	Cot	Tan	Sin	′

′	Sin	Tan	Cot	Cos	
0	.37461	.40403	2.4751	.92718	**60**
1	488	436	.4730	707	59
2	515	470	.4709	697	58
3	542	504	.4689	686	57
4	569	538	.4668	675	56
5	.37595	.40572	2.4648	.92664	**55**
6	622	606	.4627	653	54
7	649	640	.4606	642	53
8	676	674	.4586	631	52
9	703	707	.4566	620	51
10	.37730	.40741	2.4545	.92609	**50**
11	757	775	.4525	598	49
12	784	809	.4504	587	48
13	811	843	.4484	576	47
14	838	877	.4464	565	46
15	.37865	.40911	2.4443	.92554	**45**
16	892	945	.4423	513	44
17	919	.40979	.4403	532	43
18	946	.41013	.4383	521	42
19	973	047	.4362	510	41
20	.37999	.41081	2.4342	.92499	**40**
21	.38026	115	.4322	488	39
22	053	149	.4302	477	38
23	080	183	.4282	466	37
24	107	217	.4262	455	36
25	.38134	.41251	2.4242	.92444	**35**
26	161	285	.4222	432	34
27	188	319	.4202	421	33
28	215	353	.4182	410	32
29	241	387	.4162	399	31
30	.38268	.41421	2.4142	.92388	**30**
31	295	455	.4122	377	29
32	322	490	.4102	366	28
33	349	524	.4083	355	27
34	376	558	.4063	343	26
35	.38403	.41592	2.4043	.92332	**25**
36	430	626	.4023	321	24
37	456	660	.4004	310	23
38	483	694	.3984	299	22
39	510	728	.3964	287	21
40	.38537	.41763	2.3945	.92276	**20**
41	564	797	.3925	265	19
42	591	831	.3906	254	18
43	617	865	.3886	243	17
44	644	899	.3867	231	16
45	.38671	.41933	2.3847	.92220	**15**
46	698	.41968	.3828	209	14
47	725	.42002	.3808	198	13
48	752	036	.3789	186	12
49	778	070	.3770	175	11
50	.38805	.42105	2.3750	.92164	**10**
51	832	139	.3731	152	9
52	859	173	.3712	141	8
53	886	207	.3693	130	7
54	912	242	.3673	119	6
55	.38939	.42276	2.3654	.92107	**5**
56	966	310	.3635	096	4
57	.38993	345	.3616	085	3
58	.39020	379	.3597	073	2
59	046	413	.3578	062	1
60	.39073	.42447	2.3559	.92050	**0**
	Cos	Cot	Tan	Sin	′

′	Sin	Tan	Cot	Cos	
0	.39073	.42447	2.3559	.92050	**60**
1	100	482	.3539	039	59
2	127	516	.3520	028	58
3	153	551	.3501	016	57
4	180	585	.3483	.92005	56
5	.39207	.42619	2.3464	.91994	**55**
6	234	654	.3445	982	54
7	260	688	.3426	971	53
8	287	722	.3407	959	52
9	314	757	.3388	948	51
10	.39341	.42791	2.3369	.91936	**50**
11	367	826	.3351	925	49
12	394	860	.3332	914	48
13	421	894	.3313	902	47
14	448	929	.3294	891	46
15	.39474	.42963	2.3276	.91879	**45**
16	501	.42998	.3257	868	44
17	528	.43032	.3238	856	43
18	555	067	.3220	845	42
19	581	101	.3201	833	41
20	.39608	.43136	2.3183	.91822	**40**
21	635	170	.3164	810	39
22	661	205	.3146	799	38
23	688	239	.3127	787	37
24	715	274	.3109	775	36
25	.39741	.43308	2.3090	.91764	**35**
26	768	343	.3072	752	34
27	795	378	.3053	741	33
28	822	412	.3035	729	32
29	848	447	.3017	718	31
30	.39875	.43481	2.2998	.91706	**30**
31	902	516	.2980	694	29
32	928	550	.2962	683	28
33	955	585	.2944	671	27
34	.39982	620	.2925	660	26
35	.40008	.43654	2.2907	.91648	**25**
36	035	689	.2889	636	24
37	062	724	.2871	625	23
38	088	758	.2853	613	22
39	115	793	.2835	601	21
40	.40141	.43828	2.2817	.91590	**20**
41	168	862	.2799	578	19
42	195	897	.2781	566	18
43	221	932	.2763	555	17
44	248	.43966	.2745	543	16
45	.40275	.44001	2.2727	.91531	**15**
46	301	036	.2709	519	14
47	328	071	.2691	508	13
48	355	105	.2673	496	12
49	381	140	.2655	484	11
50	.40408	.44175	2.2637	.91472	**10**
51	434	210	.2620	461	9
52	461	244	.2602	449	8
53	488	279	.2584	437	7
54	514	314	.2566	425	6
55	.40541	.44349	2.2549	.91414	**5**
56	567	384	.2531	402	4
57	594	418	.2513	390	3
58	621	453	.2496	378	2
59	647	488	.2478	366	1
60	.40674	.44523	2.2460	.91355	**0**
	Cos	Cot	Tan	Sin	′

′	Sin	Tan	Cot	Cos	
0	.40674	.44523	2.2460	.91355	**60**
1	700	558	.2443	343	59
2	727	593	.2425	331	58
3	753	627	.2408	319	57
4	780	662	.2390	307	56
5	.40806	.44697	2.2373	.91295	**55**
6	833	732	.2355	283	54
7	860	767	.2338	272	53
8	886	802	.2320	260	52
9	913	837	.2303	248	51
10	.40939	.44872	2.2286	.91236	**50**
11	966	907	.2268	224	49
12	.40992	942	.2251	212	48
13	.41019	.44977	.2234	200	47
14	045	.45012	.2216	188	46
15	.41072	.45047	2.2199	.91176	**45**
16	098	082	.2182	164	44
17	125	117	.2165	152	43
18	151	152	.2148	140	42
19	178	187	.2130	128	41
20	.41204	.45222	2.2113	.91116	**40**
21	231	257	.2096	104	39
22	257	292	.2079	092	38
23	284	327	.2062	080	37
24	310	362	.2045	068	36
25	.41337	.45397	2.2028	.91056	**35**
26	363	432	.2011	044	34
27	390	467	.1994	032	33
28	416	502	.1977	020	32
29	443	538	.1960	.91008	31
30	.41469	.45573	2.1943	.90996	**30**
31	496	608	.1926	984	29
32	522	643	.1909	972	28
33	549	678	.1892	960	27
34	575	713	.1876	948	26
35	.41602	.45748	2.1859	.90936	**25**
36	628	784	.1842	924	24
37	655	819	.1825	911	23
38	681	854	.1808	899	22
39	707	889	.1792	887	21
40	.41734	.45924	2.1775	.90875	**20**
41	760	960	.1758	863	19
42	787	.45995	.1742	851	18
43	813	.46030	.1725	839	17
44	840	065	.1708	826	16
45	.41866	.46101	2.1692	.90814	**15**
46	892	136	.1675	802	14
47	919	171	.1659	790	13
48	945	206	.1642	778	12
49	972	242	.1625	766	11
50	.41998	.46277	2.1609	.90753	**10**
51	.42024	312	.1592	741	9
52	051	348	.1576	729	8
53	077	383	.1560	717	7
54	104	418	.1543	704	6
55	.42130	.46454	2.1527	.90692	**5**
56	156	489	.1510	680	4
57	183	525	.1494	668	3
58	209	560	.1478	655	2
59	235	595	.1461	643	1
60	.42262	.46631	2.1445	.90631	**0**
	Cos	Cot	Tan	Sin	′

′	Sin	Tan	Cot	Cos	
0	.42262	.46631	2.1445	.90631	**60**
1	288	666	.1429	618	59
2	315	702	.1413	606	58
3	341	737	.1396	594	57
4	367	772	.1380	582	56
5	.42394	.46808	2.1364	.90569	**55**
6	420	843	.1348	557	54
7	446	879	.1332	545	53
8	473	914	.1315	532	52
9	499	950	.1299	520	51
10	.42525	.46985	2.1283	.90507	**50**
11	552	.47021	.1267	495	49
12	578	056	.1251	483	48
13	604	092	.1235	470	47
14	631	128	.1219	458	46
15	.42657	.47163	2.1203	.90446	**45**
16	683	199	.1187	433	44
17	709	234	.1171	421	43
18	736	270	.1155	408	42
19	762	305	.1139	396	41
20	.42788	.47341	2.1123	.90383	**40**
21	815	377	.1107	371	39
22	841	412	.1092	358	38
23	867	448	.1076	346	37
24	894	483	.1060	334	36
25	.42920	.47519	2.1044	.90321	**35**
26	946	555	.1028	309	34
27	972	590	.1013	296	33
28	.42999	626	.0997	284	32
29	.43025	662	.0981	271	31
30	.43051	.47698	2.0965	.90259	**30**
31	077	733	.0950	246	29
32	104	769	.0934	233	28
33	130	805	.0918	221	27
34	156	840	.0903	208	26
35	.43182	.47876	2.0887	.90196	**25**
36	209	912	.0872	183	24
37	235	948	.0856	171	23
38	261	.47984	.0840	158	22
39	287	.48019	.0825	146	21
40	.43313	.48055	2.0809	.90133	**20**
41	340	091	.0794	120	19
42	366	127	.0778	108	18
43	392	163	.0763	095	17
44	418	198	.0748	082	16
45	.43445	.48234	2.0732	.90070	**15**
46	471	270	.0717	057	14
47	497	306	.0701	045	13
48	523	342	.0686	032	12
49	549	378	.0671	019	11
50	.43575	.48414	2.0655	.90007	**10**
51	602	450	.0640	.89994	9
52	628	486	.0625	981	8
53	654	521	.0609	968	7
54	680	557	.0594	956	6
55	.43706	.48593	2.0579	.89943	**5**
56	733	629	.0564	930	4
57	759	665	.0549	918	3
58	785	701	.0533	905	2
59	811	737	.0518	892	1
60	.43837	.48773	2.0503	.89879	**0**
	Cos	Cot	Tan	Sin	′

′	Sin	Tan	Cot	Cos	
0	.43837	.48773	2.0503	.89879	**60**
1	863	809	.0488	867	59
2	889	845	.0473	854	58
3	916	881	.0458	841	57
4	942	917	.0443	828	56
5	.43968	.48953	2.0428	.89816	**55**
6	.43994	.48989	.0413	803	54
7	.44020	.49026	.0398	790	53
8	046	062	.0383	777	52
9	072	098	.0368	764	51
10	.44098	.49134	2.0353	.89752	**50**
11	124	170	.0338	739	49
12	151	206	.0323	726	48
13	177	242	.0308	713	47
14	203	278	.0293	700	46
15	.44229	.49315	2.0278	.89687	**45**
16	255	351	.0263	674	44
17	281	387	.0248	662	43
18	307	423	.0233	649	42
19	333	459	.0219	636	41
20	.44359	.49495	2.0204	.89623	**40**
21	385	532	.0189	610	39
22	411	568	.0174	597	38
23	437	604	.0160	584	37
24	464	640	.0145	571	36
25	.44490	.49677	2.0130	.89558	**35**
26	516	713	.0115	545	34
27	542	749	.0101	532	33
28	568	786	.0086	519	32
29	594	822	.0072	506	31
30	.44620	.49858	2.0057	.89493	**30**
31	646	894	.0042	480	29
32	672	931	.0028	467	28
33	698	.49967	2.0013	454	27
34	724	.50004	1.9999	441	26
35	.44750	.50040	1.9984	.89428	**25**
36	776	076	.9970	415	24
37	802	113	.9955	402	23
38	828	149	.9941	389	22
39	854	185	.9926	376	21
40	.44880	.50222	1.9912	.89363	**20**
41	906	258	.9897	350	19
42	932	295	.9883	337	18
43	958	331	.9868	324	17
44	.44984	368	.9854	311	16
45	.45010	.50404	1.9840	.89298	**15**
46	036	441	.9825	285	14
47	062	477	.9811	272	13
48	088	514	.9797	259	12
49	114	550	.9782	245	11
50	.45140	.50587	1.9768	.89232	**10**
51	166	623	.9754	219	9
52	192	660	.9740	206	8
53	218	696	.9725	193	7
54	243	733	.9711	180	6
55	.45269	.50769	1.9697	.89167	**5**
56	295	806	.9683	153	4
57	321	843	.9669	140	3
58	347	879	.9654	127	2
59	373	916	.9640	114	1
60	.45399	.50953	1.9626	.89101	**0**
	Cos	Cot	Tan	Sin	′

′	Sin	Tan	Cot	Cos	
0	.45399	.50953	1.9626	.89101	**60**
1	425	.50989	.9612	087	59
2	451	.51026	.9598	074	58
3	477	063	.9584	061	57
4	503	099	.9570	048	56
5	.45529	.51136	1.9556	.89035	**55**
6	554	173	.9542	021	54
7	580	209	.9528	.89008	53
8	606	246	.9514	.88995	52
9	632	283	.9500	981	51
10	.45658	.51319	1.9486	.88968	**50**
11	684	356	.9472	955	49
12	710	393	.9458	942	48
13	736	430	.9444	928	47
14	762	467	.9430	915	46
15	.45787	.51503	1.9416	.88902	**45**
16	813	540	.9402	888	44
17	839	577	.9388	875	43
18	865	614	.9375	862	42
19	891	651	.9361	848	41
20	.45917	.51688	1.9347	.88835	**40**
21	942	724	.9333	822	39
22	968	761	.9319	808	38
23	.45994	798	.9306	795	37
24	.46020	835	.9292	782	36
25	.46046	.51872	1.9278	.88768	**35**
26	072	909	.9265	755	34
27	097	946	.9251	741	33
28	123	.51983	.9237	728	32
29	149	.52020	.9223	715	31
30	.46175	.52057	1.9210	.88701	**30**
31	201	094	.9196	688	29
32	226	131	.9183	674	28
33	252	168	.9169	661	27
34	278	205	.9155	647	26
35	.46304	.52242	1.9142	.88634	**25**
36	330	279	.9128	620	24
37	355	316	.9115	607	23
38	381	353	.9101	593	22
39	407	390	.9088	580	21
40	.46433	.52427	1.9074	.88566	**20**
41	458	464	.9061	553	19
42	484	501	.9047	539	18
43	510	538	.9034	526	17
44	536	575	.9020	512	16
45	.46561	.52613	1.9007	.88499	**15**
46	587	650	.8993	485	14
47	613	687	.8980	472	13
48	639	724	.8967	458	12
49	664	761	.8953	445	11
50	.46690	.52798	1.8940	.88431	**10**
51	716	836	.8927	417	9
52	742	873	.8913	404	8
53	767	910	.8900	390	7
54	793	947	.8887	377	6
55	.46819	.52985	1.8873	.88363	**5**
56	844	.53022	.8860	349	4
57	870	059	.8847	336	3
58	896	096	.8834	322	2
59	921	134	.8820	308	1
60	.46947	.53171	1.8807	.88295	**0**
	Cos	Cot	Tan	Sin	′

′	Sin	Tan	Cot	Cos	
0	.46947	.53171	1.8807	.88295	60
1	973	208	.8794	281	59
2	.46999	246	.8781	267	58
3	.47024	283	.8768	254	57
4	050	320	.8755	240	56
5	.47076	.53358	1.8741	.88226	55
6	101	395	.8728	213	54
7	127	432	.8715	199	53
8	153	470	.8702	185	52
9	178	507	.8689	172	51
10	.47204	.53545	1.8676	.88158	50
11	229	582	.8663	144	49
12	255	620	.8650	130	48
13	281	657	.8637	117	47
14	306	694	.8624	103	46
15	.47332	.53732	1.8611	.88089	45
16	358	769	.8598	075	44
17	383	807	.8585	062	43
18	409	844	.8572	048	42
19	434	882	.8559	034	41
20	.47460	.53920	1.8546	.88020	40
21	486	957	.8533	.88006	39
22	511	.53995	.8520	.87993	38
23	537	.54032	.8507	979	37
24	562	070	.8495	965	36
25	.47588	.54107	1.8482	.87951	35
26	614	145	.8469	937	34
27	639	183	.8456	923	33
28	665	220	.8443	909	32
29	690	258	.8430	896	31
30	.47716	.54296	1.8418	.87882	30
31	741	333	.8405	868	29
32	767	371	.8392	854	28
33	793	409	.8379	840	27
34	818	446	.8367	826	26
35	.47844	.54484	1.8354	.87812	25
36	869	522	.8341	798	24
37	895	560	.8329	784	23
38	920	597	.8316	770	22
39	946	635	.8303	756	21
40	.47971	.54673	1.8291	.87743	20
41	.47997	711	.8278	729	19
42	.48022	748	.8265	715	18
43	048	786	.8253	701	17
44	073	824	.8240	687	16
45	.48099	.54862	1.8228	.87673	15
46	124	900	.8215	659	14
47	150	938	.8202	645	13
48	175	.54975	.8190	631	12
49	201	.55013	.8177	617	11
50	.48226	.55051	1.8165	.87603	10
51	252	089	.8152	589	9
52	277	127	.8140	575	8
53	303	165	.8127	561	7
54	328	203	.8115	546	6
55	.48354	.55241	1.8103	.87532	5
56	379	279	.8090	518	4
57	405	317	.8078	504	3
58	430	355	.8065	490	2
59	456	393	.8053	476	1
60	.48481	.55431	1.8040	.87462	0
	Cos	Cot	Tan	Sin	′

′	Sin	Tan	Cot	Cos	
0	.48481	.55431	1.8040	.87462	60
1	506	469	.8028	448	59
2	532	507	.8016	434	58
3	557	545	.8003	420	57
4	583	583	.7991	406	56
5	.48608	.55621	1.7979	.87391	55
6	634	659	.7966	377	54
7	659	697	.7954	363	53
8	684	736	.7942	349	52
9	710	774	.7930	335	51
10	.48735	.55812	1.7917	.87321	50
11	761	850	.7905	306	49
12	786	888	.7893	292	48
13	811	926	.7881	278	47
14	837	.55964	.7868	264	46
15	.48862	.56003	1.7856	.87250	45
16	888	041	.7844	235	44
17	913	079	.7832	221	43
18	938	117	.7820	207	42
19	964	156	.7808	193	41
20	.48989	.56194	1.7796	.87178	40
21	.49014	232	.7783	164	39
22	040	270	.7771	150	38
23	065	309	.7759	136	37
24	090	347	.7747	121	36
25	.49116	.56385	1.7735	.87107	35
26	141	424	.7723	093	34
27	166	462	.7711	079	33
28	192	501	.7699	064	32
29	217	539	.7687	050	31
30	.49242	.56577	1.7675	.87036	30
31	268	616	.7663	021	29
32	293	654	.7651	.87007	28
33	318	693	.7639	.86993	27
34	344	731	.7627	978	26
35	.49369	.56769	1.7615	.86964	25
36	394	808	.7603	949	24
37	419	846	.7591	935	23
38	445	885	.7579	921	22
39	470	923	.7567	906	21
40	.49495	.56962	1.7556	.86892	20
41	521	.57000	.7544	878	19
42	546	039	.7532	863	18
43	571	078	.7520	849	17
44	596	116	.7508	834	16
45	.49622	.57155	1.7496	.86820	15
46	647	193	.7485	805	14
47	672	232	.7473	791	13
48	697	271	.7461	777	12
49	723	309	.7449	762	11
50	.49748	.57348	1.7437	.86748	10
51	773	386	.7426	733	9
52	798	425	.7414	719	8
53	824	464	.7402	704	7
54	849	503	.7391	690	6
55	.49874	.57541	1.7379	.86675	5
56	899	580	.7367	661	4
57	924	619	.7355	646	3
58	950	657	.7344	632	2
59	.49975	696	.7332	617	1
60	.50000	.57735	1.7321	.86603	0
	Cos	Cot	Tan	Sin	′

′	Sin	Tan	Cot	Cos	
0	.50000	.57735	1.7321	.86603	60
1	025	774	.7309	588	59
2	050	813	.7297	573	58
3	076	851	.7286	559	57
4	101	890	.7274	544	56
5	.50126	.57929	1.7262	.86530	55
6	151	.57968	.7251	515	54
7	176	.58007	.7239	501	53
8	201	046	.7228	486	52
9	227	085	.7216	471	51
10	.50252	.58124	1.7205	.86457	50
11	277	162	.7193	442	49
12	302	201	.7182	427	48
13	327	240	.7170	413	47
14	352	279	.7159	398	46
15	.50377	.58318	1.7147	.86384	45
16	403	357	.7136	369	44
17	428	396	.7124	354	43
18	453	435	.7113	340	42
19	478	474	.7102	325	41
20	.50503	.58513	1.7090	.86310	40
21	528	552	.7079	295	39
22	553	591	.7067	281	38
23	578	631	.7056	266	37
24	603	670	.7045	251	36
25	.50628	.58709	1.7033	.86237	35
26	654	748	.7022	222	34
27	679	787	.7011	207	33
28	704	826	.6999	192	32
29	729	865	.6988	178	31
30	.50754	.58905	1.6977	.86163	30
31	779	944	.6965	148	29
32	804	.58983	.6954	133	28
33	829	.59022	.6943	119	27
34	854	061	.6932	104	26
35	.50879	.59101	1.6920	.86089	25
36	904	140	.6909	074	24
37	929	179	.6898	059	23
38	954	218	.6887	045	22
39	.50979	258	.6875	030	21
40	.51004	.59297	1.6864	.86015	20
41	029	336	.6853	.86000	19
42	054	376	.6842	.85985	18
43	079	415	.6831	970	17
44	104	454	.6820	956	16
45	.51129	.59494	1.6808	.85941	15
46	154	533	.6797	926	14
47	179	573	.6786	911	13
48	204	612	.6775	896	12
49	229	651	.6764	881	11
50	.51254	.59691	1.6753	.85866	10
51	279	730	.6742	851	9
52	304	770	.6731	836	8
53	329	809	.6720	821	7
54	354	849	.6709	806	6
55	.51379	.59888	1.6698	.85792	5
56	404	928	.6687	777	4
57	429	.59967	.6676	762	3
58	454	.60007	.6665	747	2
59	479	046	.6654	732	1
60	.51504	.60086	1.6643	.85717	0
	Cos	Cot	Tan	Sin	′

′	Sin	Tan	Cot	Cos	
0	.51504	.60086	1.6643	.85717	60
1	529	126	.6632	702	59
2	554	165	.6621	687	58
3	579	205	.6610	672	57
4	604	245	.6599	657	56
5	.51628	.60284	1.6588	.85642	55
6	653	324	.6577	627	54
7	678	364	.6566	612	53
8	703	403	.6555	597	52
9	728	443	.6545	582	51
10	.51753	.60483	1.6534	.85567	50
11	778	522	.6523	551	49
12	803	562	.6512	536	48
13	828	602	.6501	521	47
14	852	642	.6490	506	46
15	.51877	.60681	1.6479	.85491	45
16	902	721	.6469	476	44
17	927	761	.6458	461	43
18	952	801	.6447	446	42
19	.51977	841	.6436	431	41
20	.52002	.60881	1.6426	.85416	40
21	026	921	.6415	401	39
22	051	.60960	.6404	385	38
23	076	.61000	.6393	370	37
24	101	040	.6383	355	36
25	.52126	.61080	1.6372	.85340	35
26	151	120	.6361	325	34
27	175	160	.6351	310	33
28	200	200	.6340	294	32
29	225	240	.6329	279	31
30	.52250	.61280	1.6319	.85264	30
31	275	320	.6308	249	29
32	299	360	.6297	234	28
33	324	400	.6287	218	27
34	349	440	.6276	203	26
35	.52374	.61480	1.6265	.85188	25
36	399	520	.6255	173	24
37	423	561	.6244	157	23
38	448	601	.6234	142	22
39	473	641	.6223	127	21
40	.52498	.61681	1.6212	.85112	20
41	522	721	.6202	096	19
42	547	761	.6191	081	18
43	572	801	.6181	066	17
44	597	842	.6170	051	16
45	.52621	.61882	1.6160	.85035	15
46	646	922	.6149	020	14
47	671	.61962	.6139	.85005	13
48	696	.62003	.6128	.84989	12
49	720	043	.6118	974	11
50	.52745	.62083	1.6107	.84959	10
51	770	124	.6097	943	9
52	794	164	.6087	928	8
53	819	204	.6076	913	7
54	844	245	.6066	897	6
55	.52869	.62285	1.6055	.84882	5
56	893	325	.6045	866	4
57	918	366	.6034	851	3
58	943	406	.6024	836	2
59	967	446	.6014	820	1
60	.52992	.62487	1.6003	.84805	0
	Cos	Cot	Tan	Sin	′

′	Sin	Tan	Cot	Cos	
0	.52992	.62487	1.6003	.84805	**60**
1	.53017	527	.5993	789	59
2	041	568	.5983	774	58
3	066	608	.5972	759	57
4	091	649	.5962	743	56
5	.53115	.62689	1.5952	.84728	**55**
6	140	730	.5941	712	54
7	164	770	.5931	697	53
8	189	811	.5921	681	52
9	214	852	.5911	666	51
10	.53238	.62892	1.5900	.84650	**50**
11	263	933	.5890	635	49
12	288	.62973	.5880	619	48
13	312	.63014	.5869	604	47
14	337	055	.5859	588	46
15	.53361	.63095	1.5849	.84573	**45**
16	386	136	.5839	557	44
17	411	177	.5829	542	43
18	435	217	.5818	526	42
19	460	258	.5808	511	41
20	.53484	.63299	1.5798	.84495	**40**
21	509	340	.5788	480	39
22	534	380	.5778	464	38
23	558	421	.5768	448	37
24	583	462	.5757	433	36
25	.53607	.63503	1.5747	.84417	**35**
26	632	544	.5737	402	34
27	656	584	.5727	386	33
28	681	625	.5717	370	32
29	705	666	.5707	355	31
30	.53730	.63707	1.5697	.84339	**30**
31	754	748	.5687	324	29
32	779	789	.5677	308	28
33	804	830	.5667	292	27
34	828	871	.5657	277	26
35	.53853	.63912	1.5647	.84261	**25**
36	877	953	.5637	245	24
37	902	.63994	.5627	230	23
38	926	.64035	.5617	214	22
39	951	076	.5607	198	21
40	.53975	.64117	1.5597	.84182	**20**
41	.54000	158	.5587	167	19
42	024	199	.5577	151	18
43	049	240	.5567	135	17
44	073	281	.5557	120	16
45	.54097	.64322	1.5547	.84104	**15**
46	122	363	.5537	088	14
47	146	404	.5527	072	13
48	171	446	.5517	057	12
49	195	487	.5507	041	11
50	.54220	.64528	1.5497	.84025	**10**
51	244	569	.5487	.84009	9
52	269	610	.5477	.83994	8
53	293	652	.5468	978	7
54	317	693	.5458	962	6
55	.54342	.64734	1.5448	.83946	**5**
56	366	775	.5438	930	4
57	391	817	.5428	915	3
58	415	858	.5418	899	2
59	440	899	.5408	883	1
60	.54464	.64941	1.5399	.83867	**0**
	Cos	Cot	Tan	Sin	′

′	Sin	Tan	Cot	Cos	
0	.54464	.64941	1.5399	.83867	**60**
1	488	.64982	.5389	851	59
2	513	.65024	.5379	835	58
3	537	065	.5369	819	57
4	561	106	.5359	804	56
5	.54586	.65148	1.5350	.83788	**55**
6	610	189	.5340	772	54
7	635	231	.5330	756	53
8	659	272	.5320	740	52
9	683	314	.5311	724	51
10	.54708	.65355	1.5301	.83708	**50**
11	732	397	.5291	692	49
12	756	438	.5282	676	48
13	781	480	.5272	660	47
14	805	521	.5262	645	46
15	.54829	.65563	1.5253	.83629	**45**
16	854	604	.5243	613	44
17	878	646	.5233	597	43
18	902	688	.5224	581	42
19	927	729	.5214	565	41
20	.54951	.65771	1.5204	.83549	**40**
21	975	813	.5195	533	39
22	.54999	854	.5185	517	38
23	.55024	896	.5175	501	37
24	048	938	.5166	485	36
25	.55072	.65980	1.5156	.83469	**35**
26	097	.66021	.5147	453	34
27	121	063	.5137	437	33
28	145	105	.5127	421	32
29	169	147	.5118	405	31
30	.55194	.66189	1.5108	.83389	**30**
31	218	230	.5099	373	29
32	242	272	.5089	356	28
33	266	314	.5080	340	27
34	291	356	.5070	324	26
35	.55315	.66398	1.5061	.83308	**25**
36	339	440	.5051	292	24
37	363	482	.5042	276	23
38	388	524	.5032	260	22
39	412	566	.5023	244	21
40	.55436	.66608	1.5013	.83228	**20**
41	460	650	.5004	212	19
42	484	692	.4994	195	18
43	509	734	.4985	179	17
44	533	776	.4975	163	16
45	.55557	.66818	1.4966	.83147	**15**
46	581	860	.4957	131	14
47	605	902	.4947	115	13
48	630	944	.4938	098	12
49	654	.66986	.4928	082	11
50	.55678	.67028	1.4919	.83066	**10**
51	702	071	.4910	050	9
52	726	113	.4900	034	8
53	750	155	.4891	017	7
54	775	197	.4882	.83001	6
55	.55799	.67239	1.4872	.82985	**5**
56	823	282	.4863	969	4
57	847	324	.4854	953	3
58	871	366	.4844	936	2
59	895	409	.4835	920	1
60	.55919	.67451	1.4826	.82904	**0**
	Cos	Cot	Tan	Sin	′

′	Sin	Tan	Cot	Cos	
0	.55919	.67451	1.4826	.82904	60
1	943	493	.4816	887	59
2	968	536	.4807	871	58
3	.55992	578	.4798	855	57
4	.56016	620	.4788	839	56
5	.56040	.67663	1.4779	.82822	55
6	064	705	.4770	806	54
7	088	748	.4761	790	53
8	112	790	.4751	773	52
9	136	832	.4742	757	51
10	.56160	.67875	1.4733	.82741	50
11	184	917	.1724	724	49
12	208	.67960	.4715	708	48
13	232	.68002	.4705	692	47
14	256	045	.4696	675	46
15	.56280	.68088	1.4687	.82659	45
16	305	130	.4678	643	44
17	329	173	.4669	626	43
18	353	215	.4659	610	42
19	377	258	.4650	593	41
20	.56401	.68301	1.4641	.82577	40
21	425	343	.4632	561	39
22	449	386	.4623	544	38
23	473	429	.4614	528	37
24	497	471	.4605	511	36
25	.56521	.68514	1.4596	.82495	35
26	545	557	.4586	478	34
27	569	600	.4577	462	33
28	593	642	.4568	446	32
29	617	685	.4559	429	31
30	.56641	.68728	1.4550	.82413	30
31	665	771	.4541	396	29
32	689	814	.4532	380	28
33	713	857	.4523	363	27
34	736	900	.4514	347	26
35	.56760	.68942	1.4505	.82330	25
36	784	.68985	.4496	314	24
37	808	.69028	.4487	297	23
38	832	071	.4478	281	22
39	856	114	.4469	264	21
40	.56880	.69157	1.4460	.82248	20
41	904	200	.4451	231	19
42	928	243	.4442	214	18
43	952	286	.4433	198	17
44	.56976	329	.4424	181	16
45	.57000	.69372	1.4415	.82165	15
46	024	416	.4406	148	14
47	047	459	.4397	132	13
48	071	502	.4388	115	12
49	095	545	.4379	098	11
50	.57119	.69588	1.4370	.82082	10
51	143	631	.4361	065	9
52	167	675	.4352	048	8
53	191	718	.4344	032	7
54	215	761	.4335	.82015	6
55	.57238	.69804	1.4326	.81999	5
56	262	847	.4317	982	4
57	286	891	.4308	965	3
58	310	934	.4299	949	2
59	334	.69977	.4290	932	1
60	.57358	.70021	1.4281	.81915	0
	Cos	Cot	Tan	Sin	′

′	Sin	Tan	Cot	Cos	
0	.57358	.70021	1.4281	.81915	60
1	381	064	.4273	899	59
2	405	107	.4264	882	58
3	429	151	.4255	865	57
4	453	194	.4246	848	56
5	.57477	.70238	1.4237	.81832	55
6	501	281	.4229	815	54
7	524	325	.4220	798	53
8	548	368	.4211	782	52
9	572	412	.4202	765	51
10	.57596	.70455	1.4193	.81748	50
11	619	499	.4185	731	49
12	643	542	.4176	714	48
13	667	586	.4167	698	47
14	691	629	.4158	681	46
15	.57715	.70673	1.4150	.81664	45
16	738	717	.4141	647	44
17	762	760	.4132	631	43
18	786	804	.4124	614	42
19	810	848	.4115	597	41
20	.57833	.70891	1.4106	.81580	40
21	857	935	.4097	563	39
22	881	.70979	.4089	546	38
23	904	.71023	.4080	530	37
24	928	066	.4071	513	36
25	.57952	.71110	1.4063	.81496	35
26	976	154	.4054	479	34
27	.57999	198	.4045	462	33
28	.58023	242	.4037	445	32
29	047	285	.4028	428	31
30	.58070	.71329	1.4019	.81412	30
31	094	373	.4011	395	29
32	118	417	.4002	378	28
33	141	461	.3994	361	27
34	165	505	.3985	344	26
35	.58189	.71549	1.3976	.81327	25
36	212	593	.3968	310	24
37	236	637	.3959	293	23
38	260	681	.3951	276	22
39	283	725	.3942	259	21
40	.58307	.71769	1.3934	.81242	20
41	330	813	.3925	225	19
42	354	857	.3916	208	18
43	378	901	.3908	191	17
44	401	946	.3899	174	16
45	.58425	.71990	1.3891	.81157	15
46	449	.72034	.3882	140	14
47	472	078	.3874	123	13
48	496	122	.3865	106	12
49	519	167	.3857	089	11
50	.58543	.72211	1.3848	.81072	10
51	567	255	.3840	055	9
52	590	299	.3831	038	8
53	614	344	.3823	021	7
54	637	388	.3814	.81004	6
55	.58661	.72432	1.3806	.80987	5
56	684	477	.3798	970	4
57	708	521	.3789	953	3
58	731	565	.3781	936	2
59	755	610	.3772	919	1
60	.58779	.72654	1.3764	.80902	0
	Cos	Cot	Tan	Sin	′

′	Sin	Tan	Cot	Cos	
0	.58779	.72654	1.3764	.80902	**60**
1	802	699	.3755	885	59
2	826	743	.3747	867	58
3	849	788	.3739	850	57
4	873	832	.3730	833	56
5	.58896	.72877	1.3722	.80816	**55**
6	920	921	.3713	799	54
7	943	.72966	.3705	782	53
8	967	.73010	.3697	765	52
9	.58990	055	.3688	748	51
10	.59014	.73100	1.3680	.80730	**50**
11	037	144	.3672	713	49
12	061	189	.3663	696	48
13	084	234	.3655	679	47
14	108	278	.3647	662	46
15	.59131	.73323	1.3638	.80644	**45**
16	154	368	.3630	627	44
17	178	413	.3622	610	43
18	201	457	.3613	593	42
19	225	502	.3605	576	41
20	.59248	.73547	1.3597	.80558	**40**
21	272	592	.3588	541	39
22	295	637	.3580	524	38
23	318	681	.3572	507	37
24	342	726	.3564	489	36
25	.59365	.73771	1.3555	.80472	**35**
26	389	816	.3547	455	34
27	412	861	.3539	438	33
28	436	906	.3531	420	32
29	459	951	.3522	403	31
30	.59482	.73996	1.3514	.80386	**30**
31	506	.74041	.3506	368	29
32	529	086	.3498	351	28
33	552	131	.3490	334	27
34	576	176	.3481	316	26
35	.59599	.74221	1.3473	.80299	**25**
36	622	267	.3465	282	24
37	646	312	.3457	264	23
38	669	357	.3449	247	22
39	693	402	.3440	230	21
40	.59716	.74447	1.3432	.80212	**20**
41	739	492	.3424	195	19
42	763	538	.3416	178	18
43	786	583	.3408	160	17
44	809	628	.3400	143	16
45	.59832	.74674	1.3392	.80125	**15**
46	856	719	.3384	108	14
47	879	764	.3375	091	13
48	902	810	.3367	073	12
49	926	855	.3359	056	11
50	.59949	.74900	1.3351	.80038	**10**
51	972	946	.3343	021	9
52	.59995	.74991	.3335	.80003	8
53	.60019	.75037	.3327	.79986	7
54	042	082	.3319	968	6
55	.60065	.75128	1.3311	.79951	**5**
56	089	173	.3303	934	4
57	112	219	.3295	916	3
58	135	264	.3287	899	2
59	158	310	.3278	881	1
60	.60182	.75355	1.3270	.79864	**0**
	Cos	Cot	Tan	Sin	′

′	Sin	Tan	Cot	Cos	
0	.60182	.75355	1.3270	.79864	**60**
1	205	401	.3262	846	59
2	228	447	.3254	829	58
3	251	492	.3246	811	57
4	274	538	.3238	793	56
5	.60298	.75584	1.3230	.79776	**55**
6	321	629	.3222	758	54
7	344	675	.3214	741	53
8	367	721	.3206	723	52
9	390	767	.3198	706	51
10	.60414	.75812	1.3190	.79688	**50**
11	437	858	.3182	671	49
12	460	904	.3175	653	48
13	483	950	.3167	635	47
14	506	.75996	.3159	618	46
15	.60529	.76042	1.3151	.79600	**45**
16	553	088	.3143	583	44
17	576	134	.3135	565	43
18	599	180	.3127	547	42
19	622	226	.3119	530	41
20	.60645	.76272	1.3111	.79512	**40**
21	668	318	.3103	494	39
22	691	364	.3095	477	38
23	714	410	.3087	459	37
24	738	456	.3079	441	36
25	.60761	.76502	1.3072	.79424	**35**
26	784	548	.3064	406	34
27	807	594	.3056	388	33
28	830	640	.3048	371	32
29	853	686	.3040	353	31
30	.60876	.76733	1.3032	.79335	**30**
31	899	779	.3024	318	29
32	922	825	.3017	300	28
33	945	871	.3009	282	27
34	968	918	.3001	264	26
35	.60991	.76964	1.2993	.79247	**25**
36	.61015	.77010	.2985	229	24
37	038	057	.2977	211	23
38	061	103	.2970	193	22
39	084	149	.2962	176	21
40	.61107	.77196	1.2954	.79158	**20**
41	130	242	.2946	140	19
42	153	289	.2938	122	18
43	176	335	.2931	105	17
44	199	382	.2923	087	16
45	.61222	.77428	1.2915	.79069	**15**
46	245	475	.2907	051	14
47	268	521	.2900	033	13
48	291	568	.2892	.79016	12
49	314	615	.2884	.78998	11
50	.61337	.77661	1.2876	.78980	**10**
51	360	708	.2869	962	9
52	383	754	.2861	944	8
53	406	801	.2853	926	7
54	429	848	.2846	908	6
55	.61451	.77895	1.2838	.78891	**5**
56	474	941	.2830	873	4
57	497	.77988	.2822	855	3
58	520	.78035	.2815	837	2
59	543	082	.2807	819	1
60	.61566	.78129	1.2799	.78801	**0**
	Cos	Cot	Tan	Sin	′

′	Sin	Tan	Cot	Cos	
0	.61566	.78129	1.2799	.78801	60
1	589	175	.2792	783	59
2	612	222	.2784	765	58
3	635	269	.2776	747	57
4	658	316	.2769	729	56
5	.61681	.78363	1.2761	.78711	55
6	704	410	.2753	694	54
7	726	457	.2746	676	53
8	749	504	.2738	658	52
9	772	551	.2731	640	51
10	.61795	.78598	1.2723	.78622	50
11	818	645	.2715	604	49
12	841	692	.2708	586	48
13	864	739	.2700	568	47
14	887	786	.2693	550	46
15	.61909	.78834	1.2685	.78532	45
16	932	881	.2677	514	44
17	955	928	.2670	496	43
18	.61978	.78975	.2662	478	42
19	.62001	.79022	.2655	460	41
20	.62024	.79070	1.2647	.78442	40
21	046	117	.2640	424	39
22	069	164	.2632	405	38
23	092	212	.2624	387	37
24	115	259	.2617	369	36
25	.62138	.79306	1.2609	.78351	35
26	160	354	.2602	333	34
27	183	401	.2594	315	33
28	206	449	.2587	297	32
29	229	496	.2579	279	31
30	.62251	.79544	1.2572	.78261	30
31	274	591	.2564	243	29
32	297	639	.2557	225	28
33	320	686	.2549	206	27
34	342	734	.2542	188	26
35	.62365	.79781	1.2534	.78170	25
36	388	829	.2527	152	24
37	411	877	.2519	134	23
38	433	924	.2512	116	22
39	456	.79972	.2504	098	21
40	.62479	.80020	1.2497	.78079	20
41	502	067	.2489	061	19
42	524	115	.2482	043	18
43	547	163	.2475	025	17
44	570	211	.2467	.78007	16
45	.62592	.80258	1.2460	.77988	15
46	615	306	.2452	970	14
47	638	354	.2445	952	13
48	660	402	.2437	934	12
49	683	450	.2430	916	11
50	.62706	.80498	1.2423	.77897	10
51	728	546	.2415	879	9
52	751	594	.2408	861	8
53	774	642	.2401	843	7
54	796	690	.2393	824	6
55	.62819	.80738	1.2386	.77806	5
56	842	786	.2378	788	4
57	864	834	.2371	769	3
58	887	882	.2364	751	2
59	909	930	.2356	733	1
60	.62932	.80978	1.2349	.77715	0
	Cos	Cot	Tan	Sin	′

′	Sin	Tan	Cot	Cos	
0	.62932	.80978	1.2349	.77715	60
1	955	.81027	.2342	696	59
2	.62977	075	.2334	678	58
3	.63000	123	.2327	660	57
4	022	171	.2320	641	56
5	.63045	.81220	1.2312	.77623	55
6	068	268	.2305	605	54
7	090	316	.2298	586	53
8	113	364	.2290	568	52
9	135	413	.2283	550	51
10	.63158	.81461	1.2276	.77531	50
11	180	510	.2268	513	49
12	203	558	.2261	494	48
13	225	606	.2254	476	47
14	248	655	.2247	458	46
15	.63271	.81703	1.2239	.77439	45
16	293	752	.2232	421	44
17	316	800	.2225	402	43
18	338	849	.2218	384	42
19	361	898	.2210	366	41
20	.63383	.81946	1.2203	.77347	40
21	406	.81995	.2196	329	39
22	428	.82044	.2189	310	38
23	451	092	.2181	292	37
24	473	141	.2174	273	36
25	.63496	.82190	1.2167	.77255	35
26	518	238	.2160	236	34
27	540	287	.2153	218	33
28	563	336	.2145	199	32
29	585	385	.2138	181	31
30	.63608	.82434	1.2131	.77162	30
31	630	483	.2124	144	29
32	653	531	.2117	125	28
33	675	580	.2109	107	27
34	698	629	.2102	088	26
35	.63720	.82678	1.2095	.77070	25
36	742	727	.2088	051	24
37	765	776	.2081	033	23
38	787	825	.2074	.77014	22
39	810	874	.2066	.76996	21
40	.63832	.82923	1.2059	.76977	20
41	854	.82972	.2052	959	19
42	877	.83022	.2045	940	18
43	899	071	.2038	921	17
44	922	120	.2031	903	16
45	.63941	.83169	1.2024	.76884	15
46	966	218	.2017	866	14
47	.63989	268	.2009	847	13
48	.64011	317	.2002	828	12
49	033	366	.1995	810	11
50	.64056	.83415	1.1988	.76791	10
51	078	465	.1981	772	9
52	100	514	.1974	754	8
53	123	564	.1967	735	7
54	145	613	.1960	717	6
55	.64167	.83662	1.1953	.76698	5
56	190	712	.1946	679	4
57	212	761	.1939	661	3
58	234	811	.1932	642	2
59	256	860	.1925	623	1
60	.64279	.83910	1.1918	.76604	0
	Cos	Cot	Tan	Sin	′

′	Sin	Tan	Cot	Cos	
0	.64279	.83910	1.1918	.76604	60
1	301	.83960	.1910	586	59
2	323	.84009	.1903	567	58
3	346	059	.1896	548	57
4	368	108	.1889	530	56
5	.64390	.84158	1.1882	.76511	55
6	412	208	.1875	492	54
7	435	258	.1868	473	53
8	457	307	.1861	455	52
9	479	357	.1854	436	51
10	.64501	.84407	1.1847	.76417	50
11	524	457	.1840	398	49
12	546	507	.1833	380	48
13	568	556	.1826	361	47
14	590	606	.1819	342	46
15	.64612	.84656	1.1812	.76323	45
16	635	706	.1806	304	44
17	657	756	.1799	286	43
18	679	806	.1792	267	42
19	701	856	.1785	248	41
20	.64723	.84906	1.1778	.76229	40
21	746	.84956	.1771	210	39
22	768	.85006	.1764	192	38
23	790	057	.1757	173	37
24	812	107	.1750	154	36
25	.64834	.85157	1.1743	.76135	35
26	856	207	.1736	116	34
27	878	257	.1729	097	33
28	901	308	.1722	078	32
29	923	358	.1715	059	31
30	.64945	.85408	1.1708	.76041	30
31	967	458	.1702	022	29
32	.64989	509	.1695	.76003	28
33	.65011	559	.1688	.75984	27
34	033	609	.1681	965	26
35	.65055	.85660	1.1674	.75946	25
36	077	710	.1667	927	24
37	100	761	.1660	908	23
38	122	811	.1653	889	22
39	144	862	.1647	870	21
40	.65166	.85912	1.1640	.75851	20
41	188	.85963	.1633	832	19
42	210	.86014	.1626	813	18
43	232	064	.1619	794	17
44	254	115	.1612	775	16
45	.65276	.86166	1.1606	.75756	15
46	298	216	.1599	738	14
47	320	267	.1592	719	13
48	342	318	.1585	700	12
49	364	368	.1578	680	11
50	.65386	.86419	1.1571	.75661	10
51	408	470	.1565	642	9
52	430	521	.1558	623	8
53	452	572	.1551	604	7
54	474	623	.1544	585	6
55	.65496	.86674	1.1538	.75566	5
56	518	725	.1531	547	4
57	540	776	.1524	528	3
58	562	827	.1517	509	2
59	584	878	.1510	490	1
60	.65606	.86929	1.1504	.75471	0
	Cos	Cot	Tan	Sin	′

′	Sin	Tan	Cot	Cos	
0	.65606	.86929	1.1504	.75471	60
1	628	.86980	.1497	452	59
2	650	.87031	.1490	433	58
3	672	082	.1483	414	57
4	694	133	.1477	395	56
5	.65716	.87184	1.1470	.75375	55
6	738	236	.1463	356	54
7	759	287	.1456	337	53
8	781	338	.1450	318	52
9	803	389	.1443	299	51
10	.65825	.87441	1.1436	.75280	50
11	847	492	.1430	261	49
12	869	543	.1423	241	48
13	891	595	.1416	222	47
14	913	646	.1410	203	46
15	.65935	.87698	1.1403	.75184	45
16	956	749	.1396	165	44
17	65978	801	.1389	146	43
18	.66000	852	.1383	126	42
19	022	904	.1376	107	41
20	.66044	.87955	1.1369	.75088	40
21	066	.88007	.1363	069	39
22	088	059	.1356	050	38
23	109	110	.1349	030	37
24	131	162	.1343	.75011	36
25	.66153	.88214	1.1336	.74992	35
26	175	265	.1329	973	34
27	197	317	.1323	953	33
28	218	369	.1316	934	32
29	240	421	.1310	915	31
30	.66262	.88473	1.1303	.74896	30
31	284	524	.1296	876	29
32	306	576	.1290	857	28
33	327	628	.1283	838	27
34	349	680	.1276	818	26
35	.66371	.88732	1.1270	.74799	25
36	393	784	.1263	780	24
37	414	836	.1257	760	23
38	436	888	.1250	741	22
39	458	940	.1243	722	21
40	.66480	.88992	1.1237	.74703	20
41	501	.89045	.1230	683	19
42	523	097	.1224	664	18
43	545	149	.1217	644	17
44	566	201	.1211	625	16
45	.66588	.89253	1.1204	.74606	15
46	610	306	.1197	586	14
47	632	358	.1191	567	13
48	653	410	.1184	548	12
49	675	463	.1178	528	11
50	.66697	.89515	1.1171	.74509	10
51	718	567	.1165	489	9
52	740	620	.1158	470	8
53	762	672	.1152	451	7
54	783	725	.1145	431	6
55	.66805	.89777	1.1139	.74412	5
56	827	830	.1132	392	4
57	848	883	.1126	373	3
58	870	935	.1119	353	2
59	891	.89988	.1113	334	1
60	.66913	.90040	1.1106	.74314	0
	Cos	Cot	Tan	Sin	′

′	Sin	Tan	Cot	Cos	
0	.66913	.90040	1.1106	.74314	**60**
1	935	093	.1100	295	59
2	956	146	.1093	276	58
3	978	199	.1087	256	57
4	.66999	251	.1080	237	56
5	.67021	.90304	1.1074	.74217	**55**
6	043	357	.1067	198	54
7	064	410	.1061	178	53
8	086	463	.1054	159	52
9	107	516	.1048	139	51
10	.67129	.90569	1.1041	.74120	**50**
11	151	621	.1035	100	49
12	172	674	.1028	080	48
13	194	727	.1022	061	47
14	215	781	.1016	041	46
15	.67237	.90834	1.1009	.74022	**45**
16	258	887	.1003	.74002	44
17	280	940	.0996	.73983	43
18	301	.90993	.0990	963	42
19	323	.91046	.0983	944	41
20	.67344	.91099	1.0977	.73924	**40**
21	366	153	.0971	904	39
22	387	206	.0964	885	38
23	409	259	.0958	865	37
24	430	313	.0951	846	36
25	.67452	.91366	1.0945	.73826	**35**
26	473	419	.0939	806	34
27	495	473	.0932	787	33
28	516	526	.0926	767	32
29	538	580	.0919	747	31
30	.67559	.91633	1.0913	.73728	**30**
31	580	687	.0907	708	29
32	602	740	.0900	688	28
33	623	794	.0894	669	27
34	645	847	.0888	649	26
35	.67666	.91901	1.0881	.73629	**25**
36	688	.91955	.0875	610	24
37	709	.92008	.0869	590	23
38	730	062	.0862	570	22
39	752	116	.0856	551	21
40	.67773	.92170	1.0850	.73531	**20**
41	795	224	.0843	511	19
42	816	277	.0837	491	18
43	837	331	.0831	472	17
44	859	385	.0824	452	16
45	.67880	.92439	1.0818	.73432	**15**
46	901	493	.0812	413	14
47	923	547	.0805	393	13
48	944	601	.0799	373	12
49	965	655	.0793	353	11
50	.67987	.92709	1.0786	.73333	**10**
51	.68008	763	.0780	314	9
52	029	817	.0774	294	8
53	051	872	.0768	274	7
54	072	926	.0761	254	6
55	.68093	.92980	1.0755	.73234	**5**
56	115	.93034	.0749	215	4
57	136	088	.0742	195	3
58	157	143	.0736	175	2
59	179	197	.0730	155	1
60	.68200	.93252	1.0724	.73135	**0**
	Cos	Cot	Tan	Sin	′

′	Sin	Tan	Cot	Cos	
0	.68200	.93252	1.0724	.73135	**60**
1	221	306	.0717	116	59
2	242	360	.0711	096	58
3	264	415	.0705	076	57
4	285	469	.0699	056	56
5	.68306	.93524	1.0692	.73036	**55**
6	327	578	.0686	.73016	54
7	349	633	.0680	.72996	53
8	370	688	.0674	976	52
9	391	742	.0668	957	51
10	.68412	.93797	1.0661	.72937	**50**
11	434	852	.0655	917	49
12	455	906	.0649	897	48
13	476	.93961	.0643	877	47
14	497	.94016	.0637	857	46
15	.68518	.94071	1.0630	.72837	**45**
16	539	125	.0624	817	44
17	561	180	.0618	797	43
18	582	235	.0612	777	42
19	603	290	.0606	757	41
20	.68624	.94345	1.0599	.72737	**40**
21	645	400	.0593	717	39
22	666	455	.0587	697	38
23	688	510	.0581	677	37
24	709	565	.0575	657	36
25	.68730	.94620	1.0569	.72637	**35**
26	751	676	.0562	617	34
27	772	731	.0556	597	33
28	793	786	.0550	577	32
29	814	841	.0544	557	31
30	.68835	.94896	1.0538	.72537	**30**
31	857	.94952	.0532	517	29
32	878	.95007	.0526	497	28
33	899	062	.0519	477	27
34	920	118	.0513	457	26
35	.68941	.95173	1.0507	.72437	**25**
36	962	229	.0501	417	24
37	.68983	284	.0495	397	23
38	.69004	340	.0489	377	22
39	025	395	.0483	357	21
40	.69046	.95451	1.0477	.72337	**20**
41	067	506	.0470	317	19
42	088	562	.0464	297	18
43	109	618	.0458	277	17
44	130	673	.0452	257	16
45	.69151	.95729	1.0446	.72236	**15**
46	172	785	.0440	216	14
47	193	841	.0434	196	13
48	214	897	.0428	176	12
49	235	.95952	.0422	156	11
50	.69256	.96008	1.0416	.72136	**10**
51	277	064	.0410	116	9
52	298	120	.0404	095	8
53	319	176	.0398	075	7
54	340	232	.0392	055	6
55	.69361	.96288	1.0385	.72035	**5**
56	382	344	.0379	.72015	4
57	403	400	.0373	.71995	3
58	424	457	.0367	974	2
59	445	513	.0361	954	1
60	.69466	.96569	1.0355	.71934	**0**
	Cos	Cot	Tan	Sin	′

′	Sin	Tan	Cot	Cos	
0	.69466	.96569	1.0355	.71934	**60**
1	487	625	.0349	914	59
2	508	681	.0343	894	58
3	529	738	.0337	873	57
4	549	794	.0331	853	56
5	.69570	.96850	1.0325	.71833	**55**
6	591	907	.0319	813	54
7	612	.96963	.0313	792	53
8	633	.97020	.0307	772	52
9	654	076	.0301	752	51
10	.69675	.97133	1.0295	.71732	**50**
11	696	189	.0289	711	49
12	717	246	.0283	691	48
13	737	302	.0277	671	47
14	758	359	.0271	650	46
15	.69779	.97416	1.0265	.71630	**45**
16	800	472	.0259	610	44
17	821	529	.0253	590	43
18	842	586	.0247	569	42
19	862	643	.0241	549	41
20	.69883	.97700	1.0235	.71529	**40**
21	904	756	.0230	508	39
22	925	813	.0224	488	38
23	946	870	.0218	468	37
24	966	927	.0212	447	36
25	.69987	.97984	1.0206	.71427	**35**
26	.70008	.98041	.0200	407	34
27	029	098	.0194	386	33
28	049	155	.0188	366	32
29	070	213	.0182	345	31
30	.70091	.98270	1.0176	.71325	**30**
31	112	327	.0170	305	29
32	132	384	.0164	284	28
33	153	441	.0158	264	27
34	174	499	.0152	243	26
35	.70195	.98556	1.0147	.71223	**25**
36	215	613	.0141	203	24
37	236	671	.0135	182	23
38	257	728	.0129	162	22
39	277	786	.0123	141	21
40	.70298	.98843	1.0117	.71121	**20**
41	319	901	.0111	100	19
42	339	.98958	.0105	080	18
43	360	.99016	.0099	059	17
44	381	073	.0094	039	16
45	.70401	.99131	1.0088	.71019	**15**
46	422	189	.0082	.70998	14
47	443	247	.0076	978	13
48	463	304	.0070	957	12
49	484	362	.0064	937	11
50	.70505	.99420	1.0058	.70916	**10**
51	525	478	.0052	896	9
52	546	536	.0047	875	8
53	567	594	.0041	855	7
54	587	652	.0035	834	6
55	.70608	.99710	1.0029	.70813	**5**
56	628	768	.0023	793	4
57	649	826	.0017	772	3
58	670	884	.0012	752	2
59	690	.99942	.0006	731	1
60	.70711	1.0000	1.0000	.70711	**0**
	Cos	Cot	Tan	Sin	′

Explanation of Table III

This table contains the logarithms of the sine, cosine, tangent, and cotangent to five places of decimals for every minute of arc from 0° to 90°. These are the logarithms of the numbers in Table II, and the arrangement of the table is like that of Table II. Hence the explanation given for that table is applicable here. In this table, however, the tabular differences are given for the sine and cosine in the columns headed *d*, and for the tangent and cotangent in a single column headed *c d*, since one common difference serves for both functions. There are tables of proportional parts for all differences which occur frequently. Since the numerical values of sines and cosines of angles between 0° and 90° and of the tangents of angles between 0° and 45° are less than 1, the logarithms of these functions are negative. The characteristics of these logarithms are taken in the form 9–10, but the –10 is omitted to save space and hence must be supplied with all logarithms in the first, second, and fourth columns. Since the sine, tangent, and cotangent are changing rapidly near 0°, and the cosine, cotangent, and tangent near 90°, the ordinary methods of interpolation will not give very great accuracy. When greater accuracy is necessary for angles between 0° and 2° or between 88° and 90° use Table IV.

Angles greater than 90° are given at the upper right and lower left corners of each page. Bold-faced type indicates that the co-named function is to be used.

The use of this table is illustrated by the following examples:

Ex. 1. Find log cot 29° 30′ 42″. Log cot 29° 30′ = .24736 and the tabular difference is 30. Hence log cot 29° 30′ 42″ = .24736 $-\frac{42}{60}$ (.00030) = .24736 − .00021 = .24715. The correction has to be subtracted because the cotangent is a decreasing function. To find the correction by using the table of proportional parts for 30, change 42″ into decimal parts of a minute. Thus 42″ = .7′. The correction for .7 is found in the table to be 21 as above.

Ex. 2. Find log sin 134° 38′ 28″. 134° 38′28″ = 180° − 45° 21′ 32″. Hence log sin 134° 38′ 28″ = log sin 45° 21′ 32″. Log sin 45° 21′ = 9.85212–10. The tabular difference is 13. Hence log sin 45° 21′ 32″ = 9.85212 − 10 + $\frac{32}{60}$ (.00013) = 9.85212–10 + .00007 = 9.85219–10, which is the desired logarithm. Here the correction was added since the sine from 0° to 90° is an increasing function.

Ex. 3. Log sin 98° 47′. Since 98° is bold-faced we take the log cos 8° 47′. Hence log sin 98° 47′ = 9.99488 − 10.

TABLE III

COMMON LOGARITHMS
OF THE
TRIGONOMETRIC FUNCTIONS
TO
FIVE DECIMAL PLACES
FOR EVERY MINUTE OF ARC FROM 0° TO 90°

′	L Sin	d	L Tan	c d	L Cot	L Cos	
0						0.00 000	**60**
1	6.46 373	30103	6.46 373	30103	3.53 627	0.00 000	59
2	6.76 476	17609	6.76 476	17609	3.23 524	0.00 000	58
3	6.94 085	12494	6.94 085	12494	3.05 915	0.00 000	57
4	7.06 579	9691	7.06 579	9691	2.93 421	0.00 000	56
5	7.16 270	7918	7.16 270	7918	2.83 730	0.00 000	**55**
6	7.24 188	6694	7.24 188	6694	2.75 812	0.00 000	54
7	7.30 882	5800	7.30 882	5800	2.69 118	0.00 000	53
8	7.36 682	5115	7.36 682	5115	2.63 318	0.00 000	52
9	7.41 797	4576	7.41 797	4576	2.58 203	0.00 000	51
10	7.46 373	4139	7.46 373	4139	2.53 627	0.00 000	**50**
11	7.50 512	3779	7.50 512	3779	2.49 488	0.00 000	49
12	7.54 291	3476	7.54 291	3476	2.45 709	0.00 000	48
13	7.57 767	3218	7.57 767	3219	2.42 233	0.00 000	47
14	7.60 985	2997	7.60 986	2996	2.39 014	0.00 000	46
15	7.63 982	2802	7.63 982	2803	2.36 018	0.00 000	**45**
16	7.66 784	2633	7.66 785	2633	2.33 215	0.00 000	44
17	7.69 417	2483	7.69 418	2482	2.30 582	9.99 999	43
18	7.71 900	2348	7.71 900	2348	2.28 100	9.99 999	42
19	7.74 248	2227	7.74 248	2228	2.25 752	9.99 999	41
20	7.76 475	2119	7.76 476	2119	2.23 524	9.99 999	**40**
21	7.78 594	2021	7.78 595	2020	2.21 405	9.99 999	39
22	7.80 615	1930	7.80 615	1931	2.19 385	9.99 999	38
23	7.82 545	1848	7.82 546	1848	2.17 454	9.99 999	37
24	7.84 393	1773	7.84 394	1773	2.15 606	9.99 999	36
25	7.86 166	1704	7.86 167	1704	2.13 833	9.99 999	**35**
26	7.87 870	1639	7.87 871	1639	2.12 129	9.99 999	34
27	7.89 509	1579	7.89 510	1579	2.10 490	9.99 999	33
28	7.91 088	1524	7.91 089	1524	2.08 911	9.99 999	32
29	7.92 612	1472	7.92 613	1473	2.07 387	9.99 998	31
30	7.94 084	1424	7.94 086	1424	2.05 914	9.99 998	**30**
31	7.95 508	1379	7.95 510	1379	2.04 490	9.99 998	29
32	7.96 887	1336	7.96 889	1336	2.03 111	9.99 998	28
33	7.98 223	1297	7.98 225	1297	2.01 775	9.99 998	27
34	7.99 520	1259	7.99 522	1259	2.00 478	9.99 998	26
35	8.00 779	1223	8.00 781	1223	1.99 219	9.99 998	**25**
36	8.02 002	1190	8.02 004	1190	1.97 996	9.99 998	24
37	8.03 192	1158	8.03 194	1159	1.96 806	9.99 997	23
38	8.04 350	1128	8.04 353	1128	1.95 647	9.99 997	22
39	8.05 478	1100	8.05 481	1100	1.94 519	9.99 997	21
40	8.06 578	1072	8.06 581	1072	1.93 419	9.99 997	**20**
41	8.07 650	1046	8.07 653	1047	1.92 347	9.99 997	19
42	8.08 696	1022	8.08 700	1022	1.91 300	9.99 997	18
43	8.09 718	999	8.09 722	998	1.90 278	9.99 997	17
44	8.10 717	976	8.10 720	976	1.89 280	9.99 996	16
45	8.11 693	954	8.11 696	955	1.88 304	9.99 996	**15**
46	8.12 647	934	8.12 651	934	1.87 349	9.99 996	14
47	8.13 581	914	8.13 585	915	1.86 415	9.99 996	13
48	8.14 495	896	8.14 500	895	1.85 500	9.99 996	12
49	8.15 391	877	8.15 395	878	1.84 605	9.99 996	11
50	8.16 268	860	8.16 273	860	1.83 727	9.99 995	**10**
51	8.17 128	843	8.17 133	843	1.82 867	9.99 995	9
52	8.17 971	827	8.17 976	828	1.82 024	9.99 995	8
53	8.18 798	812	8.18 804	812	1.81 196	9.99 995	7
54	8.19 610	797	8.19 616	797	1.80 384	9.99 995	6
55	8.20 407	782	8.20 413	782	1.79 587	9.99 994	**5**
56	8.21 189	769	8.21 195	769	1.78 805	9.99 994	4
57	8.21 958	755	8.21 964	756	1.78 036	9.99 994	3
58	8.22 713	743	8.22 720	742	1.77 280	9.99 994	2
59	8.23 456	730	8.23 462	730	1.76 538	9.99 994	1
60	8.24 186		8.24 192		1.75 808	9.99 993	**0**
	L Cos	d	L Cot	c d	L Tan	L Sin	′

179° 269° 359° 89°

(If greater accuracy is necessary use Table IV.)

91° 181° 271°

′	L Sin	d	L Tan	c d	L Cot	L Cos		
0	8.24 186	717	8.24 192	718	1.75 808	9.99 993	**60**	
1	8.24 903	706	8.24 910	706	1.75 090	9.99 993	59	
2	8.25 609	695	8.25 616	696	1.74 384	9.99 993	58	
3	8.26 304	684	8.26 312	684	1.73 688	9.99 993	57	
4	8.26 988	673	8.26 996	673	1.73 004	9.99 992	56	
5	8.27 661	663	8.27 669	663	1.72 331	9.99 992	**55**	
6	8.28 324	653	8.28 332	654	1.71 668	9.99 992	54	
7	8.28 977	644	8.28 986	643	1.71 014	9.99 992	53	
8	8.29 621	634	8.29 629	634	1.70 371	9.99 992	52	
9	8.30 255	624	8.30 263	625	1.69 737	9.99 991	51	
10	8.30 879	616	8.30 888	617	1.69 112	9.99 991	**50**	
11	8.31 495	608	8.31 505	607	1.68 495	9.99 991	49	
12	8.32 103	599	8.32 112	599	1.67 888	9.99 990	48	
13	8.32 702	590	8.32 711	591	1.67 289	9.99 990	47	
14	8.33 292	583	8.33 302	584	1.66 698	9.99 990	46	
15	8.33 875	575	8.33 886	575	1.66 114	9.99 990	**45**	
16	8.34 450	568	8.34 461	568	1.65 539	9.99 989	44	
17	8.35 018	560	8.35 029	561	1.64 971	9.99 989	43	
18	8.35 578	553	8.35 590	553	1.64 410	9.99 989	42	
19	8.36 131	547	8.36 143	546	1.63 857	9.99 989	41	
20	8.36 678	539	8.36 689	540	1.63 311	9.99 988	**40**	
21	8.37 217	533	8.37 229	533	1.62 771	9.99 988	39	
22	8.37 750	526	8.37 762	527	1.62 238	9.99 988	38	
23	8.38 276	520	8.38 289	520	1.61 711	9.99 987	37	
24	8.38 796	514	8.38 809	514	1.61 191	9.99 987	36	
25	8.39 310	508	8.39 323	509	1.60 677	9.99 987	**35**	
26	8.39 818	502	8.39 832	502	1.60 168	9.99 986	34	
27	8.40 320	496	8.40 334	496	1.59 666	9.99 986	33	
28	8.40 816	491	8.40 830	491	1.59 170	9.99 986	32	
29	8.41 307	485	8.41 321	486	1.58 679	9.99 985	31	
30	8.41 792	480	8.41 807	480	1.58 193	9.99 985	**30**	
31	8.42 272	474	8.42 287	475	1.57 713	9.99 985	29	
32	8.42 746	470	8.42 762	470	1.57 238	9.99 984	28	
33	8.43 216	464	8.43 232	464	1.56 768	9.99 984	27	
34	8.43 680	459	8.43 696	460	1.56 304	9.99 984	26	
35	8.44 139	455	8.44 156	455	1.55 844	9.99 983	**25**	
36	8.44 594	450	8.44 611	450	1.55 389	9.99 983	24	
37	8.45 044	445	8.45 061	446	1.54 939	9.99 983	23	
38	8.45 489	441	8.45 507	441	1.54 493	9.99 982	22	
39	8.45 930	436	8.45 948	437	1.54 052	9.99 982	21	
40	8.46 366	433	8.46 385	432	1.53 615	9.99 982	**20**	
41	8.46 799	427	8.46 817	428	1.53 183	9.99 981	19	
42	8.47 226	424	8.47 245	424	1.52 755	9.99 981	18	
43	8.47 650	419	8.47 669	420	1.52 331	9.99 981	17	
44	8.48 069	416	8.48 089	416	1.51 911	9.99 980	16	
45	8.48 485	411	8.48 505	412	1.51 495	9.99 980	**15**	
46	8.48 896	408	8.48 917	408	1.51 083	9.99 979	14	
47	8.49 304	404	8.49 325	404	1.50 675	9.99 979	13	
48	8.49 708	400	8.49 729	401	1.50 271	9.99 979	12	
49	8.50 108	396	8.50 130	397	1.49 870	9.99 978	11	
50	8.50 504	393	8.50 527	393	1.49 473	9.99 978	**10**	
51	8.50 897	390	8.50 920	390	1.49 080	9.99 977	9	
52	8.51 287	386	8.51 310	386	1.48 690	9.99 977	8	
53	8.51 673	382	8.51 696	383	1.48 304	9.99 977	7	
54	8.52 055	379	8.52 079	380	1.47 921	9.99 976	6	
55	8.52 434	376	8.52 459	376	1.47 541	9.99 976	**5**	
56	8.52 810	373	8.52 835	373	1.47 165	9.99 975	4	
57	8.53 183	369	8.53 208	370	1.46 792	9.99 975	3	
58	8.53 552	367	8.53 578	367	1.46 422	9.99 974	2	
59	8.53 919	363	8.53 945	363	1.46 055	9.99 974	1	
60	8.54 282		8.54 308		1.45 692	9.99 974	**0**	
	L Cos	d	L Cot	c d	L Tan	L Sin	′	

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(If greater accuracy is necessary use Table IV.)

′	L Sin	d	L Tan	c d	L Cot	L Cos		Prop. Parts			
								360	355	350	345
0	8.54 282	360	8.54 308	361	1.45 692	9.99 974	60				
1	8.54 642	357	8.54 669	358	1.45 331	9.99 973	59				
2	8.54 999	355	8.55 027	355	1.44 973	9.99 973	58	2	72	71.0	70	69.0
3	8.55 354	351	8.55 382	352	1.44 618	9.99 972	57	3	108	106.5	105	103.5
4	8.55 705	349	8.55 734	349	1.44 266	9.99 972	56	4	144	142.0	140	138.0
5	8.56 054	346	8.56 083	346	1.43 917	9.99 971	55	5	180	177.5	175	172.5
6	8.56 400	343	8.56 429	344	1.43 571	9.99 971	54	6	216	213.0	210	207.0
7	8.56 743	341	8.56 773	341	1.43 227	9.99 970	53	7	252	248.5	245	241.5
8	8.57 084	337	8.57 114	338	1.42 886	9.99 970	52	8	288	284.0	280	276.0
9	8.57 421	336	8.57 452	336	1.42 548	9.99 969	51	9	324	319.5	315	310.5
								340	335	330	325
10	8.57 757	332	8.57 788	333	1.42 212	9.99 969	50				
11	8.58 089	330	8.58 121	330	1.41 879	9.99 968	49				
12	8.58 419	328	8.58 451	328	1.41 549	9.99 968	48	2	68	67.0	66	65.0
13	8.58 747	325	8.58 779	326	1.41 221	9.99 967	47	3	102	100.5	99	97.5
14	8.59 072	323	8.59 105	323	1.40 895	9.99 967	46	4	136	134.0	132	130.0
15	8.59 395	320	8.59 428	321	1.40 572	9.99 967	45	5	170	167.5	165	162.5
16	8.59 715	318	8.59 749	319	1.40 251	9.99 966	44	6	204	201.0	198	195.0
17	8.60 033	316	8.60 068	316	1.39 932	9.99 966	43	7	238	234.5	231	227.5
18	8.60 349	313	8.60 384	314	1.39 616	9.99 965	42	8	272	268.0	264	260.0
19	8.60 662	311	8.60 698	311	1.39 302	9.99 964	41	9	306	301.5	297	292.5
20	8.60 973	309	8.61 009	310	1.38 991	9.99 964	40					
21	8.61 282	307	8.61 319	307	1.38 681	9.99 963	39		320	315	310	305
22	8.61 589	305	8.61 626	305	1.38 374	9.99 963	38	2	64	63.0	62	61.0
23	8.61 894	302	8.61 931	303	1.38 069	9.99 962	37	3	96	94.5	93	91.5
24	8.62 196	301	8.62 234	301	1.37 766	9.99 962	36	4	128	126.0	124	122.0
25	8.62 497	298	8.62 535	299	1.37 465	9.99 961	35	5	160	157.5	155	152.5
26	8.62 795	296	8.62 834	297	1.37 166	9.99 961	34	6	192	189.0	186	183.0
27	8.63 091	294	8.63 131	295	1.36 869	9.99 960	33	7	224	220.5	217	213.5
28	8.63 385	293	8.63 426	292	1.36 574	9.99 960	32	8	256	252.0	248	244.0
29	8.63 678	290	8.63 718	291	1.36 282	9.99 959	31	9	288	283.5	279	274.5
30	8.63 968	288	8.64 009	289	1.35 991	9.99 959	30					
31	8.64 256	287	8.64 298	287	1.35 702	9.99 958	29		300	295	290	285
32	8.64 543	284	8.64 585	285	1.35 415	9.99 958	28					
33	8.64 827	283	8.64 870	284	1.35 130	9.99 957	27	2	60	59.0	58	57.0
34	8.65 110	281	8.65 154	281	1.34 846	9.99 956	26	3	90	88.5	87	85.5
35	8.65 391	279	8.65 435	280	1.34 565	9.99 956	25	4	120	118.0	116	114.0
36	8.65 670	277	8.65 715	278	1.34 285	9.99 955	24	5	150	147.5	145	142.5
37	8.65 947	276	8.65 993	276	1.34 007	9.99 955	23	6	180	177.0	174	171.0
38	8.66 223	274	8.66 269	274	1.33 731	9.99 954	22	7	210	206.5	203	199.5
39	8.66 497	272	8.66 543	273	1.33 457	9.99 954	21	8	240	236.0	232	228.0
40	8.66 769	270	8.66 816	271	1.33 184	9.99 953	20	9	270	265.5	261	256.5
41	8.67 039	269	8.67 087	269	1.32 913	9.99 952	19					
42	8.67 308	267	8.67 356	268	1.32 644	9.99 952	18		280	275	270	265
43	8.67 575	266	8.67 624	266	1.32 376	9.99 951	17	2	56	55.0	54	53.0
44	8.67 841	263	8.67 890	264	1.32 110	9.99 951	16	3	84	82.5	81	79.5
45	8.68 104	263	8.68 154	263	1.31 846	9.99 950	15	4	112	110.0	108	106.0
46	8.68 367	260	8.68 417	261	1.31 583	9.99 949	14	5	140	137.5	135	132.5
47	8.68 627	259	8.68 678	260	1.31 322	9.99 949	13	6	168	165.0	162	159.0
48	8.68 886	258	8.68 938	258	1.31 062	9.99 948	12	7	196	192.5	189	185.5
49	8.69 144	256	8.69 196	257	1.30 804	9.99 948	11	8	224	220.0	216	212.0
50	8.69 400	254	8.69 453	255	1.30 547	9.99 947	10	9	252	247.5	243	238.5
51	8.69 654	253	8.69 708	254	1.30 292	9.99 946	9					
52	8.69 907	252	8.69 962	252	1.30 038	9.99 946	8		260	255	250	245
53	8.70 159	250	8.70 214	251	1.29 786	9.99 945	7					
54	8.70 409	249	8.70 465	249	1.29 535	9.99 944	6	2	52	51.0	50	49.0
55	8.70 658	247	8.70 714	248	1.29 286	9.99 944	5	3	78	76.5	75	73.5
56	8.70 905	246	8.70 962	246	1.29 038	9.99 943	4	4	104	102.0	100	98.0
57	8.71 151	244	8.71 208	245	1.28 792	9.99 942	3	5	130	127.5	125	122.5
58	8.71 395	243	8.71 453	244	1.28 547	9.99 942	2	6	156	153.0	150	147.0
59	8.71 638	242	8.71 697	243	1.28 303	9.99 941	1	7	182	178.5	175	171.5
60	8.71 880		8.71 940		1.28 060	9.99 940	0	8	208	204.0	200	196.0
								9	234	229.5	225	220.5
	L Cos	d	L Cot	c d	L Tan	L Sin	′					

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′	L Sin	d	L Tan	c d	L Cot	L Cos		Prop. Parts
0	8.71 880	240	8.71 940	241	1.28 060	9.99 940	**60**	**241 239 237 235**
1	8.72 120	239	8.72 181	239	1.27 819	9.99 940	59	2 48.2 47.8 47.4 47.0
2	8.72 359	238	8.72 420	239	1.27 580	9.99 939	58	3 72.3 71.7 71.1 70.5
3	8.72 597	237	8.72 659	237	1.27 341	9.99 938	57	4 96.4 95.6 94.8 94.0
4	8.72 834	235	8.72 896	236	1.27 104	9.99 938	56	5 120.5 119.5 118.5 117.5
5	8.73 069	234	8.73 132	234	1.26 868	9.99 937	**55**	6 144.6 143.4 142.2 141.0
6	8.73 303	232	8.73 366	234	1.26 634	9.99 936	54	7 168.7 167.3 165.9 164.5
7	8.73 535	232	8.73 600	232	1.26 400	9.99 936	53	8 192.8 191.2 189.6 188.0
8	8.73 767	230	8.73 832	231	1.26 168	9.99 935	52	9 216.9 215.1 213.3 211.5
9	8.73 997	229	8.74 063	229	1.25 937	9.99 934	51	**234 232 229 227**
10	8.74 226	228	8.74 292	229	1.25 708	9.99 934	**50**	2 46.8 46.4 45.8 45.4
11	8.74 454	226	8.74 521	227	1.25 479	9.99 933	49	3 70.2 69.6 68.7 68.1
12	8.74 680	226	8.74 748	226	1.25 252	9.99 932	48	4 93.6 92.8 91.6 90.8
13	8.74 906	224	8.74 974	225	1.25 026	9.99 932	47	5 117.0 116.0 114.5 113.5
14	8.75 130	223	8.75 199	224	1.24 801	9.99 931	46	6 140.4 139.2 137.4 136.2
15	8.75 353	222	8.75 423	222	1.24 577	9.99 930	**45**	7 163.8 162.4 160.3 158.9
16	8.75 575	220	8.75 645	222	1.24 355	9.99 929	44	8 187.2 185.6 183.2 181.6
17	8.75 795	220	8.75 867	220	1.24 133	9.99 929	43	9 210.6 208.8 206.1 204.3
18	8.76 015	219	8.76 087	219	1.23 913	9.99 928	42	**226 224 222 220**
19	8.76 234	217	8.76 306	219	1.23 694	9.99 927	41	2 45.2 44.8 44.4 44.0
20	8.76 451	216	8.76 525	217	1.23 475	9.99 926	**40**	3 67.8 67.2 66.6 66.0
21	8.76 667	216	8.76 742	216	1.23 258	9.99 926	39	4 90.4 89.6 88.8 88.0
22	8.76 883	214	8.76 958	215	1.23 042	9.99 925	38	5 113.0 112.0 111.0 110.0
23	8.77 097	213	8.77 173	214	1.22 827	9.99 924	37	6 135.6 134.4 133.2 132.0
24	8.77 310	212	8.77 387	213	1.22 613	9.99 923	36	7 158.2 156.8 155.4 154.0
25	8.77 522	211	8.77 600	211	1.22 400	9.99 923	**35**	8 180.8 179.2 177.6 176.0
26	8.77 733	210	8.77 811	211	1.22 189	9.99 922	34	9 203.4 201.6 199.8 198.0
27	8.77 943	209	8.78 022	210	1.21 978	9.99 921	33	**219 217 215 213**
28	8.78 152	208	8.78 232	209	1.21 768	9.99 920	32	2 43.8 43.4 43.0 42.6
29	8.78 360	208	8.78 441	208	1.21 559	9.99 920	31	3 65.7 65.1 64.5 63.9
30	8.78 568	206	8.78 649	208	1.21 351	9.99 919	**30**	4 87.6 86.8 86.0 85.2
31	8.78 774	205	8.78 855	206	1.21 145	9.99 918	29	5 109.5 108.5 107.5 106.5
32	8.78 979	204	8.79 061	206	1.20 939	9.99 917	28	6 131.4 130.2 129.0 127.8
33	8.79 183	203	8.79 266	205	1.20 734	9.99 917	27	7 153.3 151.9 150.5 149.1
34	8.79 386	202	8.79 470	204	1.20 530	9.99 916	26	8 175.2 173.6 172.0 170.4
35	8.79 588	201	8.79 673	203	1.20 327	9.99 915	**25**	9 197.1 195.3 193.5 191.7
36	8.79 789	201	8.79 875	202	1.20 125	9.99 914	24	**211 208 206 203**
37	8.79 990	199	8.80 076	201	1.19 924	9.99 913	23	2 42.2 41.6 41.2 40.6
38	8.80 189	199	8.80 277	201	1.19 723	9.99 913	22	3 63.3 62.4 61.8 60.9
39	8.80 388	197	8.80 476	199	1.19 524	9.99 912	21	4 84.4 83.2 82.4 81.2
40	8.80 585	197	8.80 674	198	1.19 326	9.99 911	**20**	5 105.5 104.0 103.0 101.5
41	8.80 782	196	8.80 872	196	1.19 128	9.99 910	19	6 126.6 124.8 123.6 121.8
42	8.80 978	195	8.81 068	196	1.18 932	9.99 909	18	7 147.7 145.6 144.2 142.1
43	8.81 173	194	8.81 264	196	1.18 736	9.99 909	17	8 168.8 166.4 164.8 162.4
44	8.81 367	193	8.81 459	195	1.18 541	9.99 908	16	9 189.9 187.2 185.4 182.7
45	8.81 560	192	8.81 653	194	1.18 347	9.99 907	**15**	**201 199 197 195**
46	8.81 752	192	8.81 846	193	1.18 154	9.99 906	14	2 40.2 39.8 39.4 39.0
47	8.81 944	190	8.82 038	192	1.17 962	9.99 905	13	3 60.3 59.7 59.1 58.5
48	8.82 134	190	8.82 230	192	1.17 770	9.99 904	12	4 80.4 79.6 78.8 78.0
49	8.82 324	189	8.82 420	190	1.17 580	9.99 904	11	5 100.5 99.5 98.5 97.5
50	8.82 513	188	8.82 610	190	1.17 390	9.99 903	**10**	6 120.6 119.4 118.2 117.0
51	8.82 701	187	8.82 799	189	1.17 201	9.99 902	9	7 140.7 139.3 137.9 136.5
52	8.82 888	187	8.82 987	188	1.17 013	9.99 901	8	8 160.8 159.2 157.6 156.0
53	8.83 075	186	8.83 175	188	1.16 825	9.99 900	7	9 180.9 179.1 177.3 175.5
54	8.83 261	185	8.83 361	186	1.16 639	9.99 899	6	**193 192 190 188**
55	8.83 446	184	8.83 547	185	1.16 453	9.99 898	**5**	2 38.6 38.4 38.0 37.6
56	8.83 630	183	8.83 732	184	1.16 268	9.99 898	4	3 57.9 57.6 57.0 56.4
57	8.83 813	183	8.83 916	184	1.16 084	9.99 897	3	4 77.2 76.8 76.0 75.2
58	8.83 996	181	8.84 100	182	1.15 900	9.99 896	2	5 96.5 96.0 95.0 94.0
59	8.84 177	181	8.84 282	182	1.15 718	9.99 895	1	6 115.8 115.2 114.0 112.8
60	8.84 358		8.84 464		1.15 536	9.99 894	**0**	7 135.1 134.4 133.0 131.6
								8 154.4 153.6 152.0 150.4
	L Cos	d	L Cot	c d	L Tan	L Sin	′	9 173.7 172.8 171.0 169.2
								186 184 182 181
								2 37.2 36.8 36.4 36.2
								3 55.8 55.2 54.6 54.3
								4 74.4 73.6 72.8 72.4
								5 93.0 92.0 91.0 90.5
								6 111.6 110.4 109.2 108.6
								7 130.2 128.8 127.4 126.7
								8 148.8 147.2 145.6 144.8
								9 167.4 165.6 163.8 162.9

′	L Sin	d	L Tan	c d	L Cot	L Cos		Prop. Parts			
								182	181	180	179
0	8.84 358	181	8.84 464	182	1.15 536	9.99 894	60	2 36.4	36.2	36.0	35.8
1	8.84 539	179	8.84 646	180	1.15 354	9.99 893	59	3 54.6	54.3	54.0	53.7
2	8.84 718	179	8.84 826	180	1.15 174	9.99 892	58	4 72.8	72.4	72.0	71.6
3	8.84 897	178	8.85 006	179	1.14 994	9.99 891	57	5 91.0	90.5	90.0	89.5
4	8.85 075	177	8.85 185	178	1.14 815	9.99 891	56	6 109.2	108.6	108.0	107.4
5	8.85 252	177	8.85 363	177	1.14 637	9.99 890	55	7 127.4	126.7	126.0	125.3
6	8.85 429	176	8.85 540	177	1.14 460	9.99 889	54	8 145.6	144.8	144.0	143.2
7	8.85 605	175	8.85 717	176	1.14 283	9.99 888	53	9 163.8	162.9	162.0	161.1
8	8.85 780	175	8.85 893	176	1.14 107	9.99 887	52				
9	8.85 955	173	8.86 069	174	1.13 931	9.99 886	51	178	177	176	175
10	8.86 128	173	8.86 243	174	1.13 757	9.99 885	50	2 35.6	35.4	35.2	35.0
11	8.86 301	173	8.86 417	174	1.13 583	9.99 884	49	3 53.4	53.1	52.8	52.5
12	8.86 474	171	8.86 591	172	1.13 409	9.99 883	48	4 71.2	70.8	70.4	70.0
13	8.86 645	171	8.86 763	172	1.13 237	9.99 882	47	5 89.0	88.5	88.0	87.5
14	8.86 816	171	8.86 935	171	1.13 065	9.99 881	46	6 106.8	106.2	105.6	105.0
15	8.86 987	169	8.87 106	171	1.12 894	9.99 880	45	7 124.6	123.9	123.2	122.5
16	8.87 156	169	8.87 277	170	1.12 723	9.99 879	44	8 142.4	141.6	140.8	140.0
17	8.87 325	169	8.87 447	169	1.12 553	9.99 879	43	9 160.2	159.3	158.4	157.5
18	8.87 494	167	8.87 616	169	1.12 384	9.99 878	42				
19	8.87 661	168	8.87 785	169	1.12 215	9.99 877	41	174	173	172	171
20	8.87 829	166	8.87 953	168	1.12 047	9.99 876	40	2 34.8	34.6	34.4	34.2
21	8.87 995	166	8.88 120	167	1.11 880	9.99 875	39	3 52.2	51.9	51.6	51.3
22	8.88 161	165	8.88 287	167	1.11 713	9.99 874	38	4 69.6	69.2	68.8	68.4
23	8.88 326	164	8.88 453	166	1.11 547	9.99 873	37	5 87.0	86.5	86.0	85.5
24	8.88 490	164	8.88 618	165	1.11 382	9.99 872	36	6 104.4	103.8	103.2	102 6
25	8.88 654	163	8.88 783	165	1.11 217	9.99 871	35	7 121.8	121.1	120.4	119.7
26	8.88 817	163	8.88 948	163	1.11 052	9.99 870	34	8 139.2	138.4	137.6	136.8
27	8.88 980	162	8.89 111	163	1.10 889	9.99 869	33	9 156.6	155.7	154.8	153.9
28	8.89 142	162	8.89 274	163	1.10 726	9.99 868	32				
29	8.89 304	160	8.89 437	161	1.10 563	9.99 867	31	170	169	168	167
30	8.89 464	161	8.89 598	162	1.10 402	9.99 866	30	2 34.0	33.8	33.6	33.4
31	8.89 625	159	8.89 760	160	1.10 240	9.99 865	29	3 51.0	50.7	50.4	50.1
32	8.89 784	159	8.89 920	160	1.10 080	9.99 864	28	4 68.0	67.6	67.2	66.8
33	8.89 943	159	8.90 080	160	1.09 920	9.99 863	27	5 85.0	84.5	84.0	83.5
34	8.90 102	158	8.90 240	159	1.09 760	9.99 862	26	6 102.0	101.4	100.8	100.2
35	8.90 260	157	8.90 399	158	1.09 601	9.99 861	25	7 119.0	118.3	117.6	116.9
36	8.90 417	157	8.90 557	158	1.09 443	9.99 860	24	8 136.0	135.2	134.4	133.6
37	8.90 574	156	8.90 715	157	1.09 285	9.99 859	23	9 153.0	152.1	151.2	150.3
38	8.90 730	155	8.90 872	157	1.09 128	9.99 858	22				
39	8.90 885	155	8.91 029	156	1.08 971	9.99 857	21	166	165	164	163
40	8.91 040	155	8.91 185	155	1.08 815	9.99 856	20	2 33.2	33.0	32.8	32.6
41	8.91 195	154	8.91 340	155	1.08 660	9.99 855	19	3 49.8	49.5	49.2	48.9
42	8.91 349	153	8.91 495	155	1.08 505	9.99 854	18	4 66.4	66.0	65.6	65.2
43	8.91 502	153	8.91 650	153	1.08 350	9.99 853	17	5 83.0	82.5	82.0	81.5
44	8.91 655	152	8.91 803	154	1.08 197	9.99 852	16	6 99.6	99.0	98.4	97.8
45	8.91 807	152	8.91 957	153	1.08 043	9.99 851	15	7 116.2	115.5	114.8	114.1
46	8.91 959	151	8.92 110	152	1.07 890	9.99 850	14	8 132.8	132.0	131.2	130.4
47	8.92 110	151	8.92 262	152	1.07 738	9.99 848	13	9 149.4	148.5	147.6	146.7
48	8.92 261	150	8.92 414	151	1.07 586	9.99 847	12				
49	8.92 411	150	8.92 565	151	1.07 435	9.99 846	11	162	161	160	159
50	8.92 561	149	8.92 716	150	1.07 284	9.99 845	10	2 32.4	32.2	32.0	31.8
51	8.92 710	149	8.92 866	150	1.07 134	9.99 844	9	3 48.6	48.3	48.0	47.7
52	8.92 859	148	8.93 016	149	1.06 984	9.99 843	8	4 64.8	64.4	64.0	63.6
53	8.93 007	147	8.93 165	148	1.06 835	9.99 842	7	5 81.0	80.5	80.0	79.5
54	8.93 154	147	8.93 313	149	1.06 687	9.99 841	6	6 97.2	96.6	96.0	95.4
55	8.93 301	147	8.93 462	147	1.06 538	9.99 840	5	7 113.4	112.7	112.0	111.3
56	8.93 448	146	8.93 609	147	1.06 391	9.99 839	4	8 129.6	128.8	128.0	127.2
57	8.93 594	146	8.93 756	147	1.06 244	9.99 838	3	9 145.8	144.9	144.0	143.1
58	8.93 740	145	8.93 903	146	1.06 097	9.99 837	2				
59	8.93 885	145	8.94 049	146	1.05 951	9.99 836	1	158	157	156	155
60	8.94 030		8.94 195		1.05 805	9.99 834	0	2 31.6	31.4	31.2	31.0
								3 47.4	47.1	46.8	46.5
								4 63.2	62.8	62.4	62.0
								5 79.0	78.5	78.0	77.5
								6 94.8	94.2	93.6	93.0
								7 110.6	109.9	109.2	108.5
								8 126.4	125.6	124.8	124.0
								9 142.2	141.3	140.4	139.5
								154	153	152	151
								2 30.8	30.6	30.4	30.2
								3 46.2	45.9	45.6	45.3
								4 61.6	61.2	60.8	60.4
								5 77.0	76.5	76.0	75.5
								6 92.4	91.8	91.2	90.6
								7 107.8	107.1	106.4	105.7
								8 123.2	122.4	121.6	120.8
								9 138.6	137.7	136.8	135.9
	L Cos	d	L Cot	c d	L Tan	L Sin	′				

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′	L Sin	d	L Tan	c d	L Cot	L Cos		Prop. Parts
0	8.94 030	144	8.94 195	145	1.05 805	9.99 834	60	150 | 149 | 148 | 147
1	8.94 174	143	8.94 340	145	1.05 660	9.99 833	59	2 30.0 | 29.8 | 29.6 | 29.4
2	8.94 317	144	8.94 485	145	1.05 515	9.99 832	58	3 45.0 | 44.7 | 44.4 | 44.1
3	8.94 461	142	8.94 630	143	1.05 370	9.99 831	57	4 60.0 | 59.6 | 59.2 | 58.8
4	8.94 603	143	8.94 773	144	1.05 227	9.99 830	56	5 75.0 | 74.5 | 74.0 | 73.5
5	8.94 746	141	8.94 917	143	1.05 083	9.99 829	55	6 90 0 | 89.4 | 88.8 | 88.2
6	8.94 887	142	8.95 060	142	1.04 940	9.99 828	54	7 105 0 | 104.3 | 103.6 | 102.9
7	8.95 029	141	8.95 202	142	1.04 798	9.99 827	53	8 120.0 | 119.2 | 118.4 | 117.6
8	8.95 170	140	8.95 344	142	1.04 656	9.99 825	52	9 135.0 | 134.1 | 133.2 | 132.3
9	8.95 310	140	8.95 486	141	1.04 514	9.99 824	51	146 | 145 | 144 | 143
10	8.95 450	139	8.95 627	140	1.04 373	9.99 823	50	2 29.2 | 29.0 | 28.8 | 28.6
11	8.95 589	139	8.95 767	141	1.04 233	9.99 822	49	3 43.8 | 43.5 | 43.2 | 42.9
12	8.95 728	139	8.95 908	139	1.04 092	9.99 821	48	4 58.4 | 58.0 | 57.6 | 57.2
13	8.95 867	138	8.96 047	140	1.03 953	9.99 820	47	5 73.0 | 72.5 | 72.0 | 71.5
14	8.96 005	138	8.96 187	138	1.03 813	9.99 819	46	6 87.6 | 87.0 | 86.4 | 85.8
15	8.96 143	137	8.96 325	139	1.03 675	9.99 817	45	7 102.2 | 101.5 | 100.8 | 100.1
16	8.96 280	137	8.96 464	138	1.03 536	9.99 816	44	8 116.8 | 116.0 | 115.2 | 114.4
17	8.96 417	136	8.96 602	137	1.03 398	9.99 815	43	9 131.4 | 130.5 | 129 6 | 128.7
18	8.96 553	136	8.96 739	138	1.03 261	9.99 814	42	142 | 141 | 140 | 139
19	8.96 689	136	8.96 877	136	1.03 123	9.99 813	41	2 28.4 | 28.2 | 28.0 | 27.8
20	8.96 825	135	8.97 013	137	1.02 987	9.99 812	40	3 42.6 | 42.3 | 42.0 | 41.7
21	8.96 960	135	8.97 150	135	1.02 850	9.99 810	39	4 56.8 | 56.4 | 56.0 | 55.6
22	8.97 095	134	8.97 285	136	1.02 715	9.99 809	38	5 71.0 | 70.5 | 70.0 | 69.5
23	8.97 229	134	8.97 421	135	1.02 579	9.99 808	37	6 85.2 | 84.6 | 84.0 | 83.4
24	8.97 363	133	8.97 556	135	1.02 444	9.99 807	36	7 99.4 | 98.7 | 98.0 | 97.3
25	8.97 496	133	8.97 691	134	1.02 309	9.99 806	35	8 113.6 | 112.8 | 112.0 | 111.2
26	8.97 629	133	8.97 825	134	1.02 175	9.99 804	34	9 127.8 | 126.9 | 126 0 | 125.1
27	8.97 762	132	8.97 959	133	1.02 041	9.99 803	33	138 | 137 | 136 | 135
28	8.97 894	132	8.98 092	133	1.01 908	9.99 802	32	2 27.6 | 27.4 | 27.2 | 27.0
29	8.98 026	131	8.98 225	133	1.01 775	9.99 801	31	3 41.4 | 41.1 | 40.8 | 40.5
30	8.98 157	131	8.98 358	132	1.01 642	9.99 800	30	4 55.2 | 54.8 | 54.4 | 54.0
31	8.98 288	131	8.98 490	132	1.01 510	9.99 798	29	5 69.0 | 68.5 | 68.0 | 67.5
32	8.98 419	130	8.98 622	131	1.01 378	9.99 797	28	6 82.8 | 82.2 | 81.6 | 81.0
33	8.98 549	130	8.98 753	131	1.01 247	9.99 796	27	7 96.6 | 95.9 | 95.2 | 94.5
34	8.98 679	129	8.98 884	131	1.01 116	9.99 795	26	8 110.4 | 109.6 | 108.8 | 108 0
35	8.98 808	129	8.99 015	130	1.00 985	9.99 793	25	9 124.2 | 123.3 | 122.4 | 121 5
36	8.98 937	129	8.99 145	130	1.00 855	9.99 792	24	134 | 133 | 132 | 131
37	8.99 066	128	8.99 275	130	1.00 725	9.99 791	23	2 26 8 | 26.6 | 26.4 | 26.2
38	8.99 194	128	8.99 405	129	1.00 595	9.99 790	22	3 40.2 | 39.9 | 39.6 | 39.3
39	8.99 322	128	8.99 534	128	1.00 466	9.99 788	21	4 53 6 | 53.2 | 52.8 | 52.4
40	8.99 450	127	8.99 662	129	1.00 338	9.99 787	20	5 67.0 | 66.5 | 66.0 | 65.5
41	8.99 577	127	8.99 791	128	1.00 209	9.99 786	19	6 80.4 | 79.8 | 79.2 | 78.6
42	8.99 704	126	8.99 919	127	1.00 081	9.99 785	18	7 93.8 | 93.1 | 92.4 | 91.7
43	8.99 830	126	9.00 046	128	0.99 954	9.99 783	17	8 107 2 | 106.4 | 105 6 | 104.8
44	8.99 956	126	9.00 174	127	0.99 826	9.99 782	16	9 120.6 | 119.7 | 118.8 | 117.9
45	9.00 082	125	9.00 301	126	0.99 699	9.99 781	15	130 | 129 | 128 | 127
46	9.00 207	125	9.00 427	126	0.99 573	9.99 780	14	2 26 0 | 25.8 | 25 6 | 25.4
47	9.00 332	124	9.00 553	126	0.99 447	9.99 778	13	3 39.0 | 38 7 | 38.4 | 38.1
48	9.00 456	125	9.00 679	126	0.99 321	9.99 777	12	4 52.0 | 51.6 | 51.2 | 50.8
49	9.00 581	123	9.00 805	125	0.99 195	9.99 776	11	5 65.0 | 64.5 | 64 0 | 63.5
50	9.00 704	124	9.00 930	125	0.99 070	9.99 775	10	6 78.0 | 77 4 | 76.8 | 76.2
51	9.00 828	123	9.01 055	124	0.98 945	9.99 773	9	7 91.0 | 90.3 | 89.6 | 88.9
52	9.00 951	123	9.01 179	124	0.98 821	9.99 772	8	8 104 0 | 103.2 | 102.4 | 101.6
53	9.01 074	123	9.01 303	124	0.98 697	9.99 771	7	9 117 0 | 116.1 | 115.2 | 114.3
54	9.01 196	122	9.01 427	123	0.98 573	9.99 769	6	126 | 125 | 124 | 123
55	9.01 318	122	9.01 550	123	0.98 450	9.99 768	5	2 25.2 | 25 0 | 24.8 | 24 6
56	9.01 440	121	9.01 673	123	0.98 327	9.99 767	4	3 37 8 | 37.5 | 37.2 | 36 9
57	9.01 561	121	9.01 796	122	0.98 204	9.99 765	3	4 50.4 | 50 0 | 49.6 | 49 2
58	9.01 682	121	9.01 918	122	0.98 082	9.99 764	2	5 63 0 | 62 5 | 62.0 | 61.5
59	9.01 803	120	9.02 040	122	0.97 960	9.99 763	1	6 75 6 | 75 0 | 74 4 | 73.8
60	9.01 923		9.02 162		0.97 838	9.99 761	0	7 88.2 | 87.5 | 86.8 | 86.1
								8 100 8 | 100.0 | 99.2 | 98.4
								9 113.4 | 112.5 | 111.6 | 110.7
								122 | 121 | 120
								2 24 4 | 24.2 | 24.0
								3 36 6 | 36.3 | 36 0
								4 48.8 | 48.4 | 48.0
								5 61.0 | 60.5 | 60 0
								6 73 2 | 72.6 | 72 0
								7 85 4 | 84.7 | 84.0
								8 97.6 | 96.8 | 96.0
								9 109.8 | 108.9 | 108.0
	L Cos	d	L Cot	c d	L Tan	L Sin	′	

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′	L Sin	d	L Tan	c d	L Cot	L Cos		Prop. Parts
0	9.01 923	120	9.02 162	121	0.97 838	9.99 761	60	
1	9.02 043	120	9.02 283	121	0.97 717	9.99 760	59	
2	9.02 163	120	9.02 404	121	0.97 596	9.99 759	58	
3	9.02 283	119	9.02 525	120	0.97 475	9.99 757	57	
4	9.02 402	118	9.02 645	121	0.97 355	9.99 755	56	
5	9.02 520	119	9.02 766	119	0.97 234	9.99 755	55	
6	9.02 639	118	9.02 885	120	0.97 115	9.99 753	54	
7	9.02 757	117	9.03 005	119	0.96 995	9.99 752	53	121 | 120 | 119 | 118
8	9.02 874	118	9.03 124	118	0.96 876	9.99 751	52	2 | 24.2 | 24.0 | 23.8 | 23.6
9	9.02 992	117	9.03 242	119	0.96 758	9.99 749	51	3 | 36.3 | 36.0 | 35.7 | 35.4
10	9.03 109	117	9.03 361	118	0.96 639	9.99 748	50	4 | 48.4 | 48.0 | 47.6 | 47.2
11	9.03 226	116	9.03 479	118	0.96 521	9.99 747	49	5 | 60.5 | 60.0 | 59.5 | 59.0
12	9.03 342	116	9.03 597	117	0.96 403	9.99 745	48	6 | 72.6 | 72.0 | 71.4 | 70.8
13	9.03 458	116	9.03 714	118	0.96 286	9.99 744	47	7 | 84.7 | 84.0 | 83.3 | 82.6
14	9.03 574	116	9.03 832	116	0.96 168	9.99 742	46	8 | 96.8 | 96.0 | 95.2 | 94.4
15	9.03 690	115	9.03 948	117	0.96 052	9.99 741	45	9 | 108.9 | 108.0 | 107.1 | 106.2
16	9.03 805	115	9.04 065	116	0.95 935	9.99 740	44	
17	9.03 920	114	9.04 181	116	0.95 819	9.99 738	43	
18	9.04 034	115	9.04 297	116	0.95 703	9.99 737	42	
19	9.04 149	113	9.04 413	115	0.95 587	9.99 736	41	117 | 116 | 115 | 114
20	9.04 262	114	9.04 528	115	0.95 472	9.99 734	40	2 | 23.4 | 23.2 | 23.0 | 22.8
21	9.04 376	114	9.04 643	115	0.95 357	9.99 733	39	3 | 35.1 | 34.8 | 34.5 | 34.2
22	9.04 490	113	9.04 758	115	0.95 242	9.99 731	38	4 | 46.8 | 46.4 | 46.0 | 45.6
23	9.04 603	112	9.04 873	114	0.95 127	9.99 730	37	5 | 58.5 | 58.0 | 57.5 | 57.0
24	9.04 715	113	9.04 987	114	0.95 013	9.99 728	36	6 | 70.2 | 69.6 | 69.0 | 68.4
25	9.04 828	112	9.05 101	113	0.94 899	9.99 727	35	7 | 81.9 | 81.2 | 80.5 | 79.8
26	9.04 940	112	9.05 214	114	0.94 786	9.99 726	34	8 | 93.6 | 92.8 | 92.0 | 91.2
27	9.05 052	112	9.05 328	113	0.94 672	9.99 724	33	9 | 105.3 | 104.4 | 103.5 | 102.6
28	9.05 164	111	9.05 441	112	0.94 559	9.99 723	32	
29	9.05 275	111	9.05 553	113	0.94 447	9.99 721	31	
30	9.05 386	111	9.05 666	112	0.94 334	9.99 720	30	
31	9.05 497	110	9.05 778	112	0.94 222	9.99 718	29	
32	9.05 607	110	9.05 890	112	0.94 110	9.99 717	28	113 | 112 | 111 | 110
33	9.05 717	110	9.06 002	111	0.93 998	9.99 716	27	
34	9.05 827	110	9.06 113	111	0.93 887	9.99 714	26	2 | 22.6 | 22.4 | 22.2 | 22.0
35	9.05 937	109	9.06 224	111	0.93 776	9.99 713	25	3 | 33.9 | 33.6 | 33.3 | 33.0
36	9.06 046	109	9.06 335	110	0.93 665	9.99 711	24	4 | 45.2 | 44.8 | 44.4 | 44.0
37	9.06 155	109	9.06 445	111	0.93 555	9.99 710	23	5 | 56.5 | 56.0 | 55.5 | 55.0
38	9.06 264	108	9.06 556	110	0.93 444	9.99 708	22	6 | 67.8 | 67.2 | 66.6 | 66.0
39	9.06 372	109	9.06 666	109	0.93 334	9.99 707	21	7 | 79.1 | 78.4 | 77.7 | 77.0
40	9.06 481	108	9.06 775	110	0.93 225	9.99 705	20	8 | 90.4 | 89.6 | 88.8 | 88.0
41	9.06 589	107	9.06 885	109	0.93 115	9.99 704	19	9 | 101.7 | 100.8 | 99.9 | 99.0
42	9.06 696	108	9.06 994	109	0.93 006	9.99 702	18	
43	9.06 804	107	9.07 103	109	0.92 897	9.99 701	17	
44	9.06 911	107	9.07 211	108	0.92 789	9.99 699	16	
45	9.07 018	106	9.07 320	109	0.92 680	9.99 698	15	109 | 108 | 107 | 106
46	9.07 124	107	9.07 428	108	0.92 572	9.99 696	14	2 | 21.8 | 21.6 | 21.4 | 21.2
47	9.07 231	106	9.07 536	108	0.92 464	9.99 695	13	3 | 32.7 | 32.4 | 32.1 | 31.8
48	9.07 337	105	9.07 643	107	0.92 357	9.99 693	12	4 | 43.6 | 43.2 | 42.8 | 42.4
49	9.07 442	106	9.07 751	108	0.92 249	9.99 692	11	5 | 54.5 | 54.0 | 53.5 | 53.0
50	9.07 548	105	9.07 858	107	0.92 142	9.99 690	10	6 | 65.4 | 64.8 | 64.2 | 63.6
51	9.07 653	105	9.07 964	106	0.92 036	9.99 689	9	7 | 76.3 | 75.6 | 74.9 | 74.2
52	9.07 758	105	9.08 071	107	0.91 929	9.99 687	8	8 | 87.2 | 86.4 | 85.6 | 84.8
53	9.07 863	105	9.08 177	106	0.91 823	9.99 686	7	9 | 98.1 | 97.2 | 96.3 | 95.4
54	9.07 968	104	9.08 283	106	0.91 717	9.99 684	6	
55	9.08 072	104	9.08 389	106	0.91 611	9.99 683	5	
56	9.08 176	104	9.08 495	105	0.91 505	9.99 681	4	
57	9.08 280	103	9.08 600	105	0.91 400	9.99 680	3	
58	9.08 383	103	9.08 705	105	0.91 295	9.99 678	2	
59	9.08 486	103	9.08 810	104	0.91 190	9.99 677	1	
60	9.08 589		9.08 914		0.91 086	9.99 675	0	
	L Cos	d	L Cot	c d	L Tan	L Sin	′	

′	L Sin	d	L Tan	c d	L Cot	L Cos		Prop. Parts
0	9.08 589	103	9.08 914	105	0.91 086	9.99 675	60	
1	9.08 692	103	9.09 019	104	0.90 981	9.99 674	59	
2	9.08 795	102	9.09 123	104	0.90 877	9.99 672	58	
3	9.08 897	102	9.09 227	103	0.90 773	9.99 670	57	
4	9.08 999	102	9.09 330	104	0.90 670	9.99 669	56	
5	9.09 101	101	9.09 434	103	0.90 566	9.99 667	55	
6	9.09 202	102	9.09 537	103	0.90 463	9.99 666	54	
7	9.09 304	101	9.09 640	102	0.90 360	9.99 664	53	
8	9.09 405	101	9.09 742	103	0.90 258	9.99 663	52	**105 104 103 102**
9	9.09 506	100	9.09 845	102	0.90 155	9.99 661	51	2 21.0 20.8 20.6 20.4
10	9.09 606	101	9.09 947	102	0.90 053	9.99 659	50	3 31.5 31.2 30.9 30.6
11	9.09 707	100	9.10 049	101	0.89 951	9.99 658	49	4 42.0 41.6 41.2 40.8
12	9.09 807	100	9.10 150	102	0.89 850	9.99 656	48	5 52.5 52.0 51.5 51.0
13	9.09 907	99	9.10 252	101	0.89 743	9.99 655	47	6 63.0 62.4 61.8 61.2
14	9.10 006	100	9.10 353	101	0.89 647	9.99 653	46	7 73.5 72.8 72.1 71.4
15	9.10 106	99	9.10 454	101	0.89 546	9.99 651	45	8 84.0 83.2 82.4 81.6
16	9.10 205	99	9.10 555	101	0.89 445	9.99 650	44	9 94.5 93.6 92.7 91.8
17	9.10 304	98	9.10 656	100	0.89 344	9.99 648	43	
18	9.10 402	99	9.10 756	100	0.89 244	9.99 647	42	
19	9.10 501	98	9.10 856	100	0.89 144	9.99 645	41	
20	9.10 599	98	9.10 956	100	0.89 044	9.99 643	40	**101 99 98 97**
21	9.10 697	98	9.11 056	99	0.88 944	9.99 642	39	2 20.2 19.8 19.6 19.4
22	9.10 795	98	9.11 155	99	0.88 845	9.99 640	38	3 30.3 29.7 29.4 29.1
23	9.10 893	97	9.11 254	99	0.88 746	9.99 638	37	4 40.4 39.6 39.2 38.8
24	9.10 990	97	9.11 353	99	0.88 647	9.99 637	36	5 50.5 49.5 49.0 48.5
25	9.11 087	97	9.11 452	99	0.88 548	9.99 635	35	6 60.6 59.4 58.8 58.2
26	9.11 184	97	9.11 551	98	0.88 449	9.99 633	34	7 70.7 69.3 68.6 67.9
27	9.11 281	96	9.11 649	98	0.88 351	9.99 632	33	8 80.8 79.2 78.4 77.6
28	9.11 377	97	9.11 747	98	0.88 253	9.99 630	32	9 90.9 89.1 88.2 87.3
29	9.11 474	96	9.11 845	98	0.88 155	9.99 629	31	
30	9.11 570	96	9.11 943	97	0.88 057	9.99 627	30	
31	9.11 666	95	9.12 040	98	0.87 960	9.99 625	29	
32	9.11 761	96	9.12 138	97	0.87 862	9.99 624	28	**96 95 94 93**
33	9.11 857	95	9.12 235	97	0.87 765	9.99 622	27	2 19.2 19.0 18.8 18.6
34	9.11 952	95	9.12 332	97	0.87 668	9.99 620	26	3 28.8 28.5 28.2 27.9
35	9.12 047	95	9.12 428	96	0.87 572	9.99 618	25	4 38.4 38.0 37.6 37.2
36	9.12 142	94	9.12 525	97	0.87 475	9.99 617	24	5 48.0 47.5 47.0 46.5
37	9.12 236	95	9.12 621	96	0.87 379	9.99 615	23	6 57.6 57.0 56.4 55.8
38	9.12 331	94	9.12 717	96	0.87 283	9.99 613	22	7 67.2 66.5 65.8 65.1
39	9.12 425	94	9.12 813	96	0.87 187	9.99 612	21	8 76.8 76.0 75.2 74.4
40	9.12 519	93	9.12 909	95	0.87 091	9.99 610	20	9 86.4 85.5 84.6 83.7
41	9.12 612	94	9.13 004	95	0.86 996	9.99 608	19	
42	9.12 706	93	9.13 099	95	0.86 901	9.99 607	18	
43	9.12 799	93	9.13 194	95	0.86 806	9.99 605	17	
44	9.12 892	93	9.13 289	95	0.86 711	9.99 603	16	**92 91 90**
45	9.12 985	93	9.13 384	94	0.86 616	9.99 601	15	2 18.4 18.2 18.0
46	9.13 078	93	9.13 478	95	0.86 522	9.99 600	14	3 27.6 27.3 27.0
47	9.13 171	92	9.13 573	94	0.86 427	9.99 598	13	4 36.8 36.4 36.0
48	9.13 263	92	9.13 667	94	0.86 333	9.99 596	12	5 46.0 45.5 45.0
49	9.13 355	92	9.13 761	93	0.86 239	9.99 595	11	6 55.2 54.6 54.0
50	9.13 447	92	9.13 854	94	0.86 146	9.99 593	10	7 64.4 63.7 63.0
51	9.13 539	91	9.13 948	93	0.86 052	9.99 591	9	8 73.6 72.8 72.0
52	9.13 630	92	9.14 041	93	0.85 959	9.99 589	8	9 82.8 81.9 81.0
53	9.13 722	91	9.14 134	93	0.85 866	9.99 588	7	
54	9.13 813	91	9.14 227	93	0.85 773	9.99 586	6	
55	9.13 904	90	9.14 320	92	0.85 680	9.99 584	5	
56	9.13 994	91	9.14 412	92	0.85 588	9.99 582	4	
57	9.14 085	90	9.14 504	93	0.85 496	9.99 581	3	
58	9.14 175	91	9.14 597	91	0.85 403	9.99 579	2	
59	9.14 266	90	9.14 688	92	0.85 312	9.99 577	1	
60	9.14 356		9.14 780		0.85 220	9.99 575	0	
	L Cos	d	L Cot	c d	L Tan	L Sin	′	

′	L Sin	d	L Tan	c d	L Cot	L Cos		Prop. Parts
0	9.14 356	89	9.14 780	92	0.85 220	9.99 575	60	
1	9.14 445	90	9.14 872	91	0.85 128	9.99 574	59	
2	9.14 535	89	9.14 963	91	0.85 037	9.99 572	58	
3	9.14 624	90	9.15 054	91	0.84 946	9.99 570	57	
4	9.14 714	89	9.15 145	91	0.84 855	9.99 568	56	
5	9.14 803	88	9.15 236	91	0.84 764	9.99 566	55	
6	9.14 891	89	9.15 327	90	0.84 673	9.99 565	54	
7	9.14 980	89	9.15 417	91	0.84 583	9.99 563	53	
8	9.15 069	88	9.15 508	90	0.84 492	9.99 561	52	
9	9.15 157	88	9.15 598	90	0.84 402	9.99 559	51	
10	9.15 245	88	9.15 688	89	0.84 312	9.99 557	50	
11	9.15 333	88	9.15 777	90	0.84 223	9.99 556	49	
12	9.15 421	87	9.15 867	89	0.84 133	9.99 554	48	
13	9.15 508	88	9.15 956	90	0.84 044	9.99 552	47	
14	9.15 596	87	9.16 046	89	0.83 954	9.99 550	46	
15	9.15 683	87	9.16 135	89	0.83 865	9.99 548	45	
16	9.15 770	87	9.16 224	88	0.83 776	9.99 546	44	
17	9.15 857	87	9.16 312	89	0.83 688	9.99 545	43	
18	9.15 944	86	9.16 401	88	0.83 599	9.99 543	42	
19	9.16 030	86	9.16 489	88	0.83 511	9.99 541	41	
20	9.16 116	87	9.16 577	88	0.83 423	9.99 539	40	
21	9.16 203	86	9.16 665	88	0.83 335	9.99 537	39	
22	9.16 289	85	9.16 753	88	0.83 247	9.99 535	38	
23	9.16 374	86	9.16 841	88	0.83 159	9.99 533	37	
24	9.16 460	85	9.16 928	87	0.83 072	9.99 532	36	
25	9.16 545	86	9.17 016	88	0.82 984	9.99 530	35	
26	9.16 631	85	9.17 103	87	0.82 897	9.99 528	34	
27	9.16 716	85	9.17 190	87	0.82 810	9.99 526	33	
28	9.16 801	85	9.17 277	87	0.82 723	9.99 524	32	
29	9.16 886	84	9.17 363	86	0.82 637	9.99 522	31	
30	9.16 970	85	9.17 450	87	0.82 550	9.99 520	30	
31	9.17 055	84	9.17 536	86	0.82 464	9.99 518	29	
32	9.17 139	84	9.17 622	86	0.82 378	9.99 517	28	
33	9.17 223	84	9.17 708	86	0.82 292	9.99 515	27	
34	9.17 307	84	9.17 794	86	0.82 206	9.99 513	26	
35	9.17 391	83	9.17 880	85	0.82 120	9.99 511	25	
36	9.17 474	84	9.17 965	86	0.82 035	9.99 509	24	
37	9.17 558	83	9.18 051	85	0.81 949	9.99 507	23	
38	9.17 641	83	9.18 136	85	0.81 864	9.99 505	22	
39	9.17 724	83	9.18 221	85	0.81 779	9.99 503	21	
40	9.17 807	83	9.18 306	85	0.81 694	9.99 501	20	
41	9.17 890	83	9.18 391	84	0.81 609	9.99 499	19	
42	9.17 973	82	9.18 475	85	0.81 525	9.99 497	18	
43	9.18 055	82	9.18 560	84	0.81 440	9.99 495	17	
44	9.18 137	83	9.18 644	84	0.81 356	9.99 494	16	
45	9.18 220	82	9.18 728	84	0.81 272	9.99 492	15	
46	9.18 302	81	9.18 812	84	0.81 188	9.99 490	14	
47	9.18 383	82	9.18 896	84	0.81 104	9.99 488	13	
48	9.18 465	82	9.18 979	83	0.81 021	9.99 486	12	
49	9.18 547	81	9.19 063	84	0.80 937	9.99 484	11	
50	9.18 628	81	9.19 146	83	0.80 854	9.99 482	10	
51	9.18 709	81	9.19 229	83	0.80 771	9.99 480	9	
52	9.18 790	81	9.19 312	83	0.80 688	9.99 478	8	
53	9.18 871	81	9.19 395	83	0.80 605	9.99 476	7	
54	9.18 952	81	9.19 478	83	0.80 522	9.99 474	6	
55	9.19 033	80	9.19 561	83	0.80 439	9.99 472	5	
56	9.19 113	80	9.19 643	82	0.80 357	9.99 470	4	
57	9.19 193	80	9.19 725	82	0.80 275	9.99 468	3	
58	9.19 273	80	9.19 807	82	0.80 193	9.99 466	2	
59	9.19 353	80	9.19 889	82	0.80 111	9.99 464	1	
60	9.19 433		9.19 971		0.80 029	9.99 462	0	
	L Cos	d	L Cot	c d	L Tan	L Sin	′	

	92	91	90	89
2	18.4	18.2	18.0	17.8
3	27.6	27.3	27.0	26.7
4	36.8	36.4	36.0	35.6
5	46.0	45.5	45.0	44.5
6	55.2	54.6	54.0	53.4
7	64.4	63.7	63.0	62.3
8	73.6	72.8	72.0	71.2
9	82.8	81.9	81.0	80.1

	88	87	86
2	17.6	17.4	17.2
3	26.4	26.1	25.8
4	35.2	34.8	34.4
5	44.0	43.5	43.0
6	52.8	52.2	51.6
7	61.6	60.9	60.2
8	70.4	69.6	68.8
9	79.2	78.3	77.4

	85	84	83
2	17.0	16.8	16.6
3	25.5	25.2	24.9
4	34.0	33.6	33.2
5	42.5	42.0	41.5
6	51.0	50.4	49.8
7	59.5	58.8	58.1
8	68.0	67.2	66.4
9	76.5	75.6	74.7

	82	81	80
2	16.4	16.2	16.0
3	24.6	24.3	24.0
4	32.8	32.4	32.0
5	41.0	40.5	40.0
6	49.2	48.6	48.0
7	57.4	56.7	56.0
8	65.6	64.8	64.0
9	73.8	72.9	72.0

′	L Sin	d	L Tan	c d	L Cot	L Cos		Prop. Parts
0	9.19 433	80	9.19 971	82	0.80 029	9.99 462	60	
1	9.19 513	79	9.20 053	81	0.79 947	9.99 460	59	
2	9.19 592	80	9.20 134	82	0.79 866	9.99 458	58	
3	9.19 672	79	9.20 216	81	0.79 784	9.99 456	57	
4	9.19 751	79	9.20 297	81	0.79 703	9.99 454	56	
5	9.19 830	79	9.20 378	81	0.79 622	9.99 452	55	
6	9.19 909	79	9.20 459	81	0.79 541	9.99 450	54	
7	9.19 988	79	9.20 540	81	0.79 460	9.99 448	53	
8	9.20 067	78	9.20 621	80	0.79 379	9.99 446	52	
9	9.20 145	78	9.20 701	81	0.79 299	9.99 444	51	
10	9.20 223	79	9.20 782	80	0.79 218	9.99 442	50	
11	9.20 302	78	9.20 862	80	0.79 138	9.99 440	49	
12	9.20 380	78	9.20 942	80	0.79 058	9.99 438	48	
13	9.20 458	77	9.21 022	80	0.78 978	9.99 436	47	
14	9.20 535	78	9.21 102	80	0.78 898	9.99 434	46	
15	9.20 613	78	9.21 182	79	0.78 818	9.99 432	45	
16	9.20 691	77	9.21 261	80	0.78 739	9.99 429	44	
17	9.20 768	77	9.21 341	79	0.78 659	9.99 427	43	
18	9.20 845	77	9.21 420	79	0.78 580	9.99 425	42	
19	9.20 922	77	9.21 499	79	0.78 501	9.99 423	41	
20	9.20 999	77	9.21 578	79	0.78 422	9.99 421	40	
21	9.21 076	77	9.21 657	79	0.78 343	9.99 419	39	
22	9.21 153	76	9.21 736	78	0.78 264	9.99 417	38	
23	9.21 229	77	9.21 814	79	0.78 186	9.99 415	37	
24	9.21 306	76	9.21 893	78	0.78 107	9.99 413	36	
25	9.21 382	76	9.21 971	78	0.78 029	9.99 411	35	
26	9.21 458	76	9.22 049	78	0.77 951	9.99 409	34	
27	9.21 534	76	9.22 127	78	0.77 873	9.99 407	33	
28	9.21 610	75	9.22 205	78	0.77 795	9.99 404	32	
29	9.21 685	76	9.22 283	78	0.77 717	9.99 402	31	
30	9.21 761	75	9.22 361	77	0.77 639	9.99 400	30	
31	9.21 836	76	9.22 438	78	0.77 562	9.99 398	29	
32	9.21 912	75	9.22 516	77	0.77 484	9.99 396	28	
33	9.21 987	75	9.22 593	77	0.77 407	9.99 394	27	
34	9.22 062	75	9.22 670	77	0.77 330	9.99 392	26	
35	9.22 137	74	9.22 747	77	0.77 253	9.99 390	25	
36	9.22 211	75	9.22 824	77	0.77 176	9.99 388	24	
37	9.22 286	75	9.22 901	77	0.77 099	9.99 385	23	
38	9.22 361	74	9.22 977	76	0.77 023	9.99 383	22	
39	9.22 435	74	9.23 054	77	0.76 946	9.99 381	21	
40	9.22 509	74	9.23 130	76	0.76 870	9.99 379	20	
41	9.22 583	74	9.23 206	77	0.76 794	9.99 377	19	
42	9.22 657	74	9.23 283	76	0.76 717	9.99 375	18	
43	9.22 731	74	9.23 359	76	0.76 641	9.99 372	17	
44	9.22 805	73	9.23 435	75	0.76 565	9.99 370	16	
45	9.22 878	74	9.23 510	76	0.76 490	9.99 368	15	
46	9.22 952	73	9.23 586	75	0.76 414	9.99 366	14	
47	9.23 025	73	9.23 661	76	0.76 339	9.99 364	13	
48	9.23 098	73	9.23 737	75	0.76 263	9.99 362	12	
49	9.23 171	73	9.23 812	75	0.76 188	9.99 359	11	
50	9.23 244	73	9.23 887	75	0.76 113	9.99 357	10	
51	9.23 317	73	9.23 962	75	0.76 038	9.99 355	9	
52	9.23 390	72	9.24 037	75	0.75 963	9.99 353	8	
53	9.23 462	73	9.24 112	74	0.75 888	9.99 351	7	
54	9.23 535	72	9.24 186	75	0.75 814	9.99 348	6	
55	9.23 607	72	9.24 261	74	0.75 739	9.99 346	5	
56	9.23 679	73	9.24 335	75	0.75 665	9.99 344	4	
57	9.23 752	71	9.24 410	74	0.75 590	9.99 342	3	
58	9.23 823	72	9.24 484	74	0.75 516	9.99 340	2	
59	9.23 895	72	9.24 558	74	0.75 442	9.99 337	1	
60	9.23 967		9.24 632		0.75 368	9.99 335	0	

	82	81	80	79
2	16.4	16.2	16.0	15.8
3	24.6	24.3	24.0	23.7
4	32.8	32.4	32.0	31.6
5	41.0	40.5	40.0	39.5
6	49.2	48.6	48.0	47.4
7	57.4	56.7	56.0	55.3
8	65.6	64.8	64.0	63.2
9	73.8	72.9	72.0	71.1

	78	77	76	75
2	15.6	15.4	15.2	15.0
3	23.4	23.1	22.8	22.5
4	31.2	30.8	30.4	30.0
5	39.0	38.5	38.0	37.5
6	46.8	46.2	45.6	45.0
7	54.6	53.9	53.2	52.5
8	62.4	61.6	60.8	60.0
9	70.2	69.3	68.4	67.5

	74	73	72	71
2	14.8	14.6	14.4	14.2
3	22.2	21.9	21.6	21.3
4	29.6	29.2	28.8	28.4
5	37.0	36.5	36.0	35.5
6	44.4	43.8	43.2	42.6
7	51.8	51.1	50.4	49.7
8	59.2	58.4	57.6	56.8
9	66.6	65.7	64.8	63.9

	L Cos	d	L Cot	c d	L Tan	L Sin	′

′	L Sin	d	L Tan	c d	L Cot	L Cos	d	
0	9.23 967	72	9.24 632	74	0.75 368	9.99 335	2	60
1	9.24 039	71	9.24 706	73	0.75 294	9.99 333	2	59
2	9.24 110	71	9.24 779	74	0.75 221	9.99 331	3	58
3	9.24 181	72	9.24 853	73	0.75 147	9.99 328	2	57
4	9.24 253	71	9.24 926	74	0.75 074	9.99 326	2	56
5	9.24 324	71	9.25 000	73	0.75 000	9.99 324	2	55
6	9.24 395	71	9.25 073	73	0.74 927	9.99 322	3	54
7	9.24 466	70	9.25 146	73	0.74 854	9.99 319	2	53
8	9.24 536	71	9.25 219	73	0.74 781	9.99 317	2	52
9	9.24 607	70	9.25 292	73	0.74 708	9.99 315	2	51
10	9.24 677	71	9.25 365	72	0.74 635	9.99 313	3	50
11	9.24 748	70	9.25 437	73	0.74 563	9.99 310	2	49
12	9.24 818	70	9.25 510	72	0.74 490	9.99 308	2	48
13	9.24 888	70	9.25 582	73	0.74 418	9.99 306	2	47
14	9.24 958	70	9.25 655	72	0.74 345	9.99 304	3	46
15	9.25 028	70	9.25 727	72	0.74 273	9.99 301	2	45
16	9.25 098	70	9.25 799	72	0.74 201	9.99 299	2	44
17	9.25 168	69	9.25 871	72	0.74 129	9.99 297	3	43
18	9.25 237	70	9.25 943	72	0.74 057	9.99 294	2	42
19	9.25 307	69	9.26 015	71	0.73 985	9.99 292	2	41
20	9.25 376	69	9.26 086	72	0.73 914	9.99 290	2	40
21	9.25 445	69	9.26 158	71	0.73 842	9.99 288	3	39
22	9.25 514	69	9.26 229	72	0.73 771	9.99 285	2	38
23	9.25 583	69	9.26 301	71	0.73 699	9.99 283	2	37
24	9.25 652	69	9.26 372	71	0.73 628	9.99 281	3	36
25	9.25 721	69	9.26 443	71	0.73 557	9.99 278	2	35
26	9.25 790	68	9.26 514	71	0.73 486	9.99 276	2	34
27	9.25 858	69	9.26 585	70	0.73 415	9.99 274	3	33
28	9.25 927	68	9.26 655	71	0.73 345	9.99 271	2	32
29	9.25 995	68	9.26 726	71	0.73 274	9.99 269	2	31
30	9.26 063	68	9.26 797	70	0.73 203	9.99 267	3	30
31	9.26 131	68	9.26 867	70	0.73 133	9.99 264	2	29
32	9.26 199	68	9.26 937	71	0.73 063	9.99 262	2	28
33	9.26 267	68	9.27 008	70	0.72 992	9.99 260	3	27
34	9.26 335	68	9.27 078	70	0.72 922	9.99 257	2	26
35	9.26 403	67	9.27 148	70	0.72 852	9.99 255	3	25
36	9.26 470	68	9.27 218	70	0.72 782	9.99 252	2	24
37	9.26 538	67	9.27 288	69	0.72 712	9.99 250	2	23
38	9.26 605	67	9.27 357	70	0.72 643	9.99 248	3	22
39	9.26 672	67	9.27 427	69	0.72 573	9.99 245	2	21
40	9.26 739	67	9.27 496	70	0.72 504	9.99 243	2	20
41	9.26 806	67	9.27 566	69	0.72 434	9.99 241	3	19
42	9.26 873	67	9.27 635	69	0.72 365	9.99 238	2	18
43	9.26 940	67	9.27 704	69	0.72 296	9.99 236	3	17
44	9.27 007	66	9.27 773	69	0.72 227	9.99 233	2	16
45	9.27 073	67	9.27 842	69	0.72 158	9.99 231	2	15
46	9.27 140	66	9.27 911	69	0.72 089	9.99 229	3	14
47	9.27 206	67	9.27 980	69	0.72 020	9.99 226	2	13
48	9.27 273	66	9.28 049	68	0.71 951	9.99 224	3	12
49	9.27 339	66	9.28 117	69	0.71 883	9.99 221	2	11
50	9.27 405	66	9.28 186	68	0.71 814	9.99 219	2	10
51	9.27 471	66	9.28 254	69	0.71 746	9.99 217	3	9
52	9.27 537	65	9.28 323	68	0.71 677	9.99 214	2	8
53	9.27 602	66	9.28 391	68	0.71 609	9.99 212	3	7
54	9.27 668	66	9.28 459	68	0.71 541	9.99 209	2	6
55	9.27 734	65	9.28 527	68	0.71 473	9.99 207	3	5
56	9.27 799	65	9.28 595	67	0.71 405	9.99 204	2	4
57	9.27 864	66	9.28 662	68	0.71 338	9.99 202	2	3
58	9.27 930	65	9.28 730	68	0.71 270	9.99 200	3	2
59	9.27 995	65	9.28 798	67	0.71 202	9.99 197	2	1
60	9.28 060		9.28 865		0.71 135	9.99 195		0
	L Cos	d	L Cot	c d	L Tan	L Sin	d	′

Prop. Parts

	74	73	72
2	14.8	14.6	14.4
3	22.2	21.9	21.6
4	29.6	29.2	28.8
5	37.0	36.5	36.0
6	44.4	43.8	43.2
7	51.8	51.1	50.4
8	59.2	58.4	57.6
9	66.6	65.7	64.8

	71	70	69
2	14.2	14.0	13.8
3	21.3	21.0	20.7
4	28.4	28.0	27.6
5	35.5	35.0	34.5
6	42.6	42.0	41.4
7	49.7	49.0	48.3
8	56.8	56.0	55.2
9	63.9	63.0	62.1

	68	67	66
2	13.6	13.4	13.2
3	20.4	20.1	19.8
4	27.2	26.8	26.4
5	34.0	33.5	33.0
6	40.8	40.2	39.6
7	47.6	46.9	46.2
8	54.4	53.6	52.8
9	61.2	60.3	59.4

	65	3	2
2	13.0	0.6	0.4
3	19.5	0.9	0.6
4	26.0	1.2	0.8
5	32.5	1.5	1.0
6	39.0	1.8	1.2
7	45.5	2.1	1.4
8	52.0	2.4	1.6
9	58.5	2.7	1.8

101° 191° 281°

′	L Sin	d	L Tan	c d	L Cot	L Cos	d	′
0	9.28 060	65	9.28 865	68	0.71 135	9.99 195	3	60
1	9.28 125	65	9.28 933	67	0.71 067	9.99 192	2	59
2	9.28 190	64	9.29 000	67	0.71 000	9.99 190	3	58
3	9.28 254	65	9.29 067	67	0.70 933	9.99 187	2	57
4	9.28 319	65	9.29 134	67	0.70 866	9.99 185	3	56
5	9.28 384	64	9.29 201	67	0.70 799	9.99 182	2	55
6	9.28 448	64	9.29 268	67	0.70 732	9.99 180	3	54
7	9.28 512	65	9.29 335	67	0.70 665	9.99 177	2	53
8	9.28 577	64	9.29 402	66	0.70 598	9.99 175	3	52
9	9.28 641	64	9.29 468	67	0.70 532	9.99 172	2	51
10	9.28 705	64	9.29 535	66	0.70 465	9.99 170	3	50
11	9.28 769	64	9.29 601	67	0.70 399	9.99 167	2	49
12	9.28 833	63	9.29 668	66	0.70 332	9.99 165	3	48
13	9.28 896	64	9.29 734	66	0.70 266	9.99 162	2	47
14	9.28 960	64	9.29 800	66	0.70 200	9.99 160	3	46
15	9.29 024	63	9.29 866	66	0.70 134	9.99 157	2	45
16	9.29 087	63	9.29 932	66	0.70 068	9.99 155	3	44
17	9.29 150	64	9.29 998	66	0.70 002	9.99 152	2	43
18	9.29 214	63	9.30 064	66	0.69 936	9.99 150	3	42
19	9.29 277	63	9.30 130	65	0.69 870	9.99 147	2	41
20	9.29 340	63	9.30 195	66	0.69 805	9.99 145	3	40
21	9.29 403	63	9.30 261	65	0.69 739	9.99 142	2	39
22	9.29 466	63	9.30 326	65	0.69 674	9.99 140	3	38
23	9.29 529	62	9.30 391	66	0.69 609	9.99 137	2	37
24	9.29 591	63	9.30 457	65	0.69 543	9.99 135	3	36
25	9.29 654	62	9.30 522	65	0.69 478	9.99 132	2	35
26	9.29 716	63	9.30 587	65	0.69 413	9.99 130	3	34
27	9.29 779	62	9.30 652	65	0.69 348	9.99 127	3	33
28	9.29 841	62	9.30 717	65	0.69 283	9.99 124	2	32
29	9.29 903	63	9.30 782	64	0.69 218	9.99 122	3	31
30	9.29 966	62	9.30 846	65	0.69 154	9.99 119	2	30
31	9.30 028	62	9.30 911	64	0.69 089	9.99 117	3	29
32	9.30 090	61	9.30 975	65	0.69 025	9.99 114	2	28
33	9.30 151	62	9.31 040	64	0.68 960	9.99 112	3	27
34	9.30 213	62	9.31 104	64	0.68 896	9.99 109	3	26
35	9.30 275	61	9.31 168	65	0.68 832	9.99 106	2	25
36	9.30 336	62	9.31 233	64	0.68 767	9.99 104	3	24
37	9.30 398	61	9.31 297	64	0.68 703	9.99 101	2	23
38	9.30 459	62	9.31 361	64	0.68 639	9.99 099	3	22
39	9.30 521	61	9.31 425	64	0.68 575	9.99 096	3	21
40	9.30 582	61	9.31 489	63	0.68 511	9.99 093	2	20
41	9.30 643	61	9.31 552	64	0.68 448	9.99 091	3	19
42	9.30 704	61	9.31 616	63	0.68 384	9.99 088	2	18
43	9.30 765	61	9.31 679	64	0.68 321	9.99 086	3	17
44	9.30 826	61	9.31 743	63	0.68 257	9.99 083	3	16
45	9.30 887	60	9.31 806	64	0.68 194	9.99 080	2	15
46	9.30 947	61	9.31 870	63	0.68 130	9.99 078	3	14
47	9.31 008	60	9.31 933	63	0.68 067	9.99 075	3	13
48	9.31 068	61	9.31 996	63	0.68 004	9.99 072	2	12
49	9.31 129	60	9.32 059	63	0.67 941	9.99 070	3	11
50	9.31 189	61	9.32 122	63	0.67 878	9.99 067	3	10
51	9.31 250	60	9.32 185	63	0.67 815	9.99 064	3	9
52	9.31 310	60	9.32 248	63	0.67 752	9.99 062	2	8
53	9.31 370	60	9.32 311	63	0.67 689	9.99 059	3	7
54	9.31 430	60	9.32 373	62	0.67 627	9.99 056	3	6
55	9.31 490	59	9.32 436	63	0.67 564	9.99 054	2	5
56	9.31 549	60	9.32 498	62	0.67 502	9.99 051	3	4
57	9.31 609	60	9.32 561	62	0.67 439	9.99 048	3	3
58	9.31 669	59	9.32 623	62	0.67 377	9.99 046	3	2
59	9.31 728	60	9.32 685	62	0.67 315	9.99 043	3	1
60	9.31 788		9.32 747		0.67 253	9.99 040		0
	L Cos	d	L Cot	c d	L Tan	L Sin	d	′

Prop. Parts			
	68	67	66
2	13.6	13.4	13.2
3	20.4	20.1	19.8
4	27.2	26.8	26.4
5	34.0	33.5	33.0
6	40.8	40.2	39.6
7	47.6	46.9	46.2
8	54.4	53.6	52.8
9	61.2	60.3	59.4
	65	64	63
2	13.0	12.8	12.6
3	19.5	19.2	18.9
4	26.0	25.6	25.2
5	32.5	32.0	31.5
6	39.0	38.4	37.8
7	45.5	44.8	44.1
8	52.0	51.2	50.4
9	58.5	57.6	56.7
	62	61	60
2	12.4	12.2	12.0
3	18.6	18.3	18.0
4	24.8	24.4	24.0
5	31.0	30.5	30.0
6	37.2	36.6	36.0
7	43.4	42.7	42.0
8	49.6	48.8	48.0
9	55.8	54.9	54.0
	59	3	2
2	11.8	0.6	0.4
3	17.7	0.9	0.6
4	23.6	1.2	0.8
5	29.5	1.5	1.0
6	35.4	1.8	1.2
7	41.3	2.1	1.4
8	47.2	2.4	1.6
9	53.1	2.7	1.8

168° 258° 348° 78°

′	L Sin	d	L Tan	c d	L Cot	L Cos	d		Prop. Parts
0	9.31 788		9.32 747		0.67 253	9.99 040		60	
1	9.31 847	59	9.32 810	63	0.67 190	9.99 038	2	59	
2	9.31 907	60	9.32 872	62	0.67 128	9.99 035	3	58	
3	9.31 966	59	9.32 933	61	0.67 067	9.99 032	3	57	
4	9.32 025	59	9.32 995	62	0.67 005	9.99 030	2	56	
5	9.32 084		9.33 057	62	0.66 943	9.99 027	3	55	
6	9.32 143	59	9.33 119	61	0.66 881	9.99 024	3	54	
7	9.32 202	59	9.33 180	62	0.66 820	9.99 022	2	53	
8	9.32 261	59	9.33 242	61	0.66 758	9.99 019	3	52	63 62 61
9	9.32 319	58	9.33 303	62	0.66 697	9.99 016	3	51	2 12.6 12.4 12.2
10	9.32 378	59	9.33 365	61	0.66 635	9.99 013	3	50	3 18.9 18.6 18.3
11	9.32 437	58	9.33 426	61	0.66 574	9.99 011	2	49	4 25.2 24.8 24.4
12	9.32 495	58	9.33 487	61	0.66 513	9.99 008	3	48	5 31.5 31.0 30.5
13	9.32 553	59	9.33 548	61	0.66 452	9.99 005	3	47	6 37.8 37.2 36.6
14	9.32 612	58	9.33 609	61	0.66 391	9.99 002	3	46	7 44.1 43.4 42.7
15	9.32 670	58	9.33 670	61	0.66 330	9.99 000	2	45	8 50.4 49.6 48.8
16	9.32 728	58	9.33 731	61	0.66 269	9.98 997	3	44	9 56.7 55.8 54.9
17	9.32 786	58	9.33 792	61	0.66 208	9.98 994	3	43	
18	9.32 844	58	9.33 853	60	0.66 147	9.98 991	3	42	
19	9.32 902	58	9.33 913	61	0.66 087	9.98 989	2	41	
20	9.32 960	58	9.33 974	60	0.66 026	9.98 986	3	40	60 59 58
21	9.33 018	57	9.34 034	61	0.65 966	9.98 983	3	39	2 12.0 11.8 11.6
22	9.33 075	58	9.34 095	60	0.65 905	9.98 980	3	38	3 18.0 17.7 17.4
23	9.33 133	57	9.34 155	60	0.65 845	9.98 978	2	37	4 24.0 23.6 23.2
24	9.33 190	58	9.34 215	60	0.65 785	9.98 975	3	36	5 30.0 29.5 29.0
25	9.33 248	57	9.34 276	61	0.65 724	9.98 972	3	35	6 36.0 35.4 34.8
26	9.33 305	57	9.34 336	60	0.65 664	9.98 969	3	34	7 42.0 41.3 40.6
27	9.33 362	58	9.34 396	60	0.65 604	9.98 967	2	33	8 48.0 47.2 46.4
28	9.33 420	57	9.34 456	60	0.65 544	9.98 964	3	32	9 54.0 53.1 52.2
29	9.33 477	57	9.34 516	60	0.65 484	9.98 961	3	31	
30	9.33 534	57	9.34 576	59	0.65 424	9.98 958	3	30	
31	9.33 591	56	9.34 635	60	0.65 365	9.98 955	2	29	
32	9.33 647	57	9.34 695	60	0.65 305	9.98 953	3	28	
33	9.33 704	57	9.34 755	59	0.65 245	9.98 950	3	27	57 56
34	9.33 761	57	9.34 814	60	0.65 186	9.98 947	3	26	2 11.4 11.2
35	9.33 818	56	9.34 874	59	0.65 126	9.98 944	3	25	3 17.1 16 8
36	9.33 874	57	9.34 933	59	0.65 067	9.98 941	3	24	4 22.8 22.4
37	9.33 931	56	9.34 992	59	0.65 008	9.98 938	3	23	5 28.5 28.0
38	9.33 987	56	9.35 051	60	0.64 949	9.98 936	2	22	6 34.2 33.6
39	9.34 043	57	9.35 111	59	0.64 889	9.98 933	3	21	7 39.9 39.2
40	9.34 100	56	9.35 170	59	0.64 830	9.98 930	3	20	8 45.6 44.8
41	9.34 156	56	9.35 229	59	0.64 771	9.98 927	3	19	9 51.3 50.4
42	9.34 212	56	9.35 288	59	0.64 712	9.98 924	3	18	
43	9.34 268	56	9.35 347	58	0.64 653	9.98 921	2	17	
44	9.34 324	56	9.35 405	59	0.64 595	9.98 919	3	16	
45	9.34 380	56	9.35 464	59	0.64 536	9.98 916	3	15	55 3 2
46	9.34 436	55	9.35 523	58	0.64 477	9.98 913	3	14	2 11.0 0.6 0.4
47	9.34 491	56	9.35 581	59	0.64 419	9.98 910	3	13	3 16.5 0.9 0.6
48	9.34 547	55	9.35 640	58	0.64 360	9.98 907	3	12	4 22.0 1.2 0.8
49	9.34 602	56	9.35 698	59	0.64 302	9.98 904	3	11	5 27.5 1.5 1.0
50	9.34 658	55	9.35 757	58	0.64 243	9.98 901	3	10	6 33.0 1.8 1.2
51	9.34 713	56	9.35 815	58	0.64 185	9.98 898	2	9	7 38.5 2.1 1.4
52	9.34 769	55	9.35 873	58	0.64 127	9.98 896	3	8	8 44.0 2.4 1.6
53	9.34 824	55	9.35 931	58	0.64 069	9.98 893	3	7	9 49.5 2.7 1.8
54	9.34 879	55	9.35 989	58	0.64 011	9.98 890	3	6	
55	9.34 934	55	9.36 047	58	0.63 953	9.98 887	3	5	
56	9.34 989	55	9.36 105	58	0.63 895	9.98 884	3	4	
57	9.35 044	55	9.36 163	58	0.63 837	9.98 881	3	3	
58	9.35 099	55	9.36 221	58	0.63 779	9.98 878	3	2	
59	9.35 154	55	9.36 279	57	0.63 721	9.98 875	3	1	
60	9.35 209		9.36 336		0.63 664	9.98 872		0	
	L Cos	d	L Cot	c d	L Tan	L Sin	d	′	

103° 193° 283°

′	L Sin	d	L Tan	c d	L Cot	L Cos	d		Prop. Parts
0	9.35 209	54	9.36 336	58	0.63 664	9.98 872	3	60	
1	9.35 263	55	9.36 394	58	0.63 606	9.98 869	2	59	
2	9.35 318	55	9.36 452	57	0.63 548	9.98 867	3	58	
3	9.35 373	54	9.36 509	57	0.63 491	9.98 864	3	57	
4	9.35 427	54	9.36 566	58	0.63 434	9.98 861	3	56	
5	9.35 481	55	9.36 624	57	0.63 376	9.98 858	3	55	
6	9.35 536	54	9.36 681	57	0.63 319	9.98 855	3	54	
7	9.35 590	54	9.36 738	57	0.63 262	9.98 852	3	53	
8	9.35 644	54	9.36 795	57	0.63 205	9.98 849	3	52	
9	9.35 698	54	9.36 852	57	0.63 148	9.98 846	3	51	
10	9.35 752	54	9.36 909	57	0.63 091	9.98 843	3	50	
11	9.35 806	54	9.36 966	57	0.63 034	9.98 840	3	49	
12	9.35 860	54	9.37 023	57	0.62 977	9.98 837	3	48	
13	9.35 914	54	9.37 080	57	0.62 920	9.98 834	3	47	
14	9.35 968	54	9.37 137	56	0.62 863	9.98 831	3	46	
15	9.36 022	53	9.37 193	57	0.62 807	9.98 828	3	45	
16	9.36 075	54	9.37 250	56	0.62 750	9.98 825	3	44	
17	9.36 129	53	9.37 306	57	0.62 694	9.98 822	3	43	
18	9.36 182	54	9.37 363	56	0.62 637	9.98 819	3	42	
19	9.36 236	53	9.37 419	57	0.62 581	9.98 816	3	41	
20	9.36 289	53	9.37 476	56	0.62 524	9.98 813	3	40	
21	9.36 342	53	9.37 532	56	0.62 468	9.98 810	3	39	
22	9.36 395	54	9.37 588	56	0.62 412	9.98 807	3	38	
23	9.36 449	53	9.37 644	56	0.62 356	9.98 804	3	37	
24	9.36 502	53	9.37 700	56	0.62 300	9.98 801	3	36	
25	9.36 555	53	9.37 756	56	0.62 244	9.98 798	3	35	
26	9.36 608	52	9.37 812	56	0.62 188	9.98 795	3	34	
27	9.36 660	53	9.37 868	56	0.62 132	9.98 792	3	33	
28	9.36 713	53	9.37 924	56	0.62 076	9.98 789	3	32	
29	9.36 766	53	9.37 980	55	0.62 020	9.98 786	3	31	
30	9.36 819	52	9.38 035	56	0.61 965	9.98 783	3	30	
31	9.36 871	53	9.38 091	56	0.61 909	9.98 780	3	29	
32	9.36 924	52	9.38 147	55	0.61 853	9.98 777	3	28	
33	9.36 976	52	9.38 202	55	0.61 798	9.98 774	3	27	
34	9.37 028	53	9.38 257	56	0.61 743	9.98 771	3	26	
35	9.37 081	52	9.38 313	55	0.61 687	9.98 768	3	25	
36	9.37 133	52	9.38 368	55	0.61 632	9.98 765	3	24	
37	9.37 185	52	9.38 423	56	0.61 577	9.98 762	3	23	
38	9.37 237	52	9.38 479	55	0.61 521	9.98 759	3	22	
39	9.37 289	52	9.38 534	55	0.61 466	9.98 756	3	21	
40	9.37 341	52	9.38 589	55	0.61 411	9.98 753	3	20	
41	9.37 393	52	9.38 644	55	0.61 356	9.98 750	4	19	
42	9.37 445	52	9.38 699	55	0.61 301	9.98 746	3	18	
43	9.37 497	52	9.38 754	54	0.61 246	9.98 743	3	17	
44	9.37 549	51	9.38 808	55	0.61 192	9.98 740	3	16	
45	9.37 600	52	9.38 863	55	0.61 137	9.98 737	3	15	
46	9.37 652	51	9.38 918	54	0.61 082	9.98 734	3	14	
47	9.37 703	52	9.38 972	55	0.61 028	9.98 731	3	13	
48	9.37 755	51	9.39 027	55	0.60 973	9.98 728	3	12	
49	9.37 806	52	9.39 082	54	0.60 918	9.98 725	3	11	
50	9.37 858	51	9.39 136	54	0.60 864	9.98 722	3	10	
51	9.37 909	51	9.39 190	55	0.60 810	9.98 719	4	9	
52	9.37 960	51	9.39 245	54	0.60 755	9.98 715	3	8	
53	9.38 011	51	9.39 299	54	0.60 701	9.98 712	3	7	
54	9.38 062	51	9.39 353	54	0.60 647	9.98 709	3	6	
55	9.38 113	51	9.39 407	54	0.60 593	9.98 706	3	5	
56	9.38 164	51	9.39 461	54	0.60 539	9.98 703	3	4	
57	9.38 215	51	9.39 515	54	0.60 485	9.98 700	3	3	
58	9.38 266	51	9.39 569	54	0.60 431	9.98 697	3	2	
59	9.38 317	51	9.39 623	54	0.60 377	9.98 694	4	1	
60	9.38 368		9.39 677		0.60 323	9.98 690		0	

	58	57	56
2	11.6	11.4	11.2
3	17.4	17.1	16.8
4	23.2	22.8	22.4
5	29.0	28.5	28.0
6	34.8	34.2	33.6
7	40.6	39.9	39.2
8	46.4	45.6	44.8
9	52.2	51.3	50.4

	55	54	53
2	11.0	10.8	10.6
3	16.5	16.2	15.9
4	22.0	21.6	21.2
5	27.5	27.0	26.5
6	33.0	32.4	31.8
7	38.5	37.8	37.1
8	44.0	43.2	42.4
9	49.5	48.6	47.7

	52	51
2	10.4	10.2
3	15.6	15.3
4	20.8	20.4
5	26.0	25.5
6	31.2	30.6
7	36.4	35.7
8	41.6	40.8
9	46.8	45.9

	4	3	2
2	0.8	0.6	0.4
3	1.2	0.9	0.6
4	1.6	1.2	0.8
5	2.0	1.5	1.0
6	2.4	1.8	1.2
7	2.8	2.1	1.4
8	3.2	2.4	1.6
9	3.6	2.7	1.8

′	L Cos	d	L Cot	c d	L Tan	L Sin	d	′

166° 256° 346° 76°

′	L Sin	d	L Tan	c d	L Cot	L Cos	d		Prop. Parts
0	9.38 368	50	9.39 677	54	0.60 323	9.98 690	3	60	
1	9.38 418	51	9.39 731	54	0.60 269	9.98 687	3	59	
2	9.38 469	50	9.39 785	53	0.60 215	9.98 684	3	58	
3	9.38 519	51	9.39 838	54	0.60 162	9.98 681	3	57	
4	9.38 570	50	9.39 892	53	0.60 108	9.98 678	3	56	
5	9.38 620	50	9.39 945	54	0.60 055	9.98 675	4	55	
6	9.38 670	51	9.39 999	53	0.60 001	9.98 671	3	54	
7	9.38 721	50	9.40 052	54	0.59 948	9.98 668	3	53	
8	9.38 771	50	9.40 106	53	0.59 894	9.98 665	3	52	
9	9.38 821	50	9.40 159	53	0.59 841	9.98 662	3	51	
10	9.38 871	50	9.40 212	54	0.59 788	9.98 659	3	50	
11	9.38 921	50	9.40 266	53	0.59 734	9.98 656	4	49	
12	9.38 971	50	9.40 319	53	0.59 681	9.98 652	3	48	
13	9.39 021	50	9.40 372	53	0.59 628	9.98 649	3	47	
14	9.39 071	50	9.40 425	53	0.59 575	9.98 646	3	46	
15	9.39 121	49	9.40 478	53	0.59 522	9.98 643	3	45	
16	9.39 170	50	9.40 531	53	0.59 469	9.98 640	4	44	
17	9.39 220	50	9.40 584	52	0.59 416	9.98 636	3	43	
18	9.39 270	49	9.40 636	53	0.59 364	9.98 633	3	42	
19	9.39 319	50	9.40 689	53	0.59 311	9.98 630	3	41	
20	9.39 369	49	9.40 742	53	0.59 258	9.98 627	4	40	
21	9.39 418	49	9.40 795	52	0.59 205	9.98 623	3	39	
22	9.39 467	50	9.40 847	53	0.59 153	9.98 620	3	38	
23	9.39 517	49	9.40 900	52	0.59 100	9.98 617	3	37	
24	9.39 566	49	9.40 952	53	0.59 048	9.98 614	4	36	
25	9.39 615	49	9.41 005	52	0.58 995	9.98 610	3	35	
26	9.39 664	49	9.41 057	52	0.58 943	9.98 607	3	34	
27	9.39 713	49	9.41 109	52	0.58 891	9.98 604	3	33	
28	9.39 762	49	9.41 161	53	0.58 839	9.98 601	4	32	
29	9.39 811	49	9.41 214	52	0.58 786	9.98 597	3	31	
30	9.39 860	49	9.41 266	52	0.58 734	9.98 594	3	30	
31	9.39 909	49	9.41 318	52	0.58 682	9.98 591	3	29	
32	9.39 958	48	9.41 370	52	0.58 630	9.98 588	4	28	
33	9.40 006	49	9.41 422	52	0.58 578	9.98 584	3	27	
34	9.40 055	48	9.41 474	52	0.58 526	9.98 581	3	26	
35	9.40 103	49	9.41 526	52	0.58 474	9.98 578	4	25	
36	9.40 152	48	9.41 578	51	0.58 422	9.98 574	3	24	
37	9.40 200	49	9.41 629	52	0.58 371	9.98 571	3	23	
38	9.40 249	48	9.41 681	52	0.58 319	9.98 568	3	22	
39	9.40 297	49	9.41 733	51	0.58 267	9.98 565	4	21	
40	9.40 346	48	9.41 784	52	0.58 216	9.98 561	3	20	
41	9.40 394	48	9.41 836	51	0.58 164	9.98 558	3	19	
42	9.40 442	48	9.41 887	52	0.58 113	9.98 555	4	18	
43	9.40 490	48	9.41 939	51	0.58 061	9.98 551	3	17	
44	9.40 538	48	9.41 990	51	0.58 010	9.98 548	3	16	
45	9.40 586	48	9.42 041	52	0.57 959	9.98 545	4	15	
46	9.40 634	48	9.42 093	51	0.57 907	9.98 541	3	14	
47	9.40 682	48	9.42 144	51	0.57 856	9.98 538	3	13	
48	9.40 730	48	9.42 195	51	0.57 805	9.98 535	4	12	
49	9.40 778	47	9.42 246	51	0.57 754	9.98 531	3	11	
50	9.40 825	48	9.42 297	51	0.57 703	9.98 528	3	10	
51	9.40 873	48	9.42 348	51	0.57 652	9.98 525	4	9	
52	9.40 921	47	9.42 399	51	0.57 601	9.98 521	3	8	
53	9.40 968	48	9.42 450	51	0.57 550	9.98 518	3	7	
54	9.41 016	47	9.42 501	51	0.57 499	9.98 515	4	6	
55	9.41 063	48	9.42 552	51	0.57 448	9.98 511	3	5	
56	9.41 111	47	9.42 603	50	0.57 397	9.98 508	3	4	
57	9.41 158	47	9.42 653	51	0.57 347	9.98 505	4	3	
58	9.41 205	47	9.42 704	51	0.57 296	9.98 501	3	2	
59	9.41 252	48	9.42 755	50	0.57 245	9.98 498	4	1	
60	9.41 300		9.42 805		0.57 195	9.98 494		0	
	L Cos	d	L Cot	c d	L Tan	L Sin	d	′	

	54	53	52
2	10.8	10.6	10.4
3	16.2	15.9	15.6
4	21.6	21.2	20.8
5	27.0	26.5	26.0
6	32.4	31.8	31.2
7	37.8	37.1	36.4
8	43.2	42.4	41.6
9	48.6	47.7	46.8

	51	50	49
2	10.2	10.0	9.8
3	15.3	15.0	14.7
4	20.4	20.0	19.6
5	25.5	25.0	24.5
6	30.6	30.0	29.4
7	35.7	35.0	34.3
8	40.8	40.0	39.2
9	45.9	45.0	44.1

	48	47
2	9.6	9.4
3	14.4	14.1
4	19.2	18.8
5	24.0	23.5
6	28.8	28.2
7	33.6	32.9
8	38.4	37.6
9	43.2	42.3

	4	3
2	0.8	0.6
3	1.2	0.9
4	1.6	1.2
5	2.0	1.5
6	2.4	1.8
7	2.8	2.1
8	3.2	2.4
9	3.6	2.7

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105° 195° 285°

′	L Sin	d	L Tan	c d	L Cot	L Cos	d	
0	9.41 300	47	9.42 805	51	0.57 195	9.98 494	3	60
1	9.41 347	47	9.42 856	50	0.57 144	9.98 491	3	59
2	9.41 394	47	9.42 906	51	0.57 094	9.98 488	4	58
3	9.41 441	47	9.42 957	50	0.57 043	9.98 484	3	57
4	9.41 488	47	9.43 007	50	0.56 993	9.98 481	4	56
5	9.41 535	47	9.43 057	51	0.56 943	9.98 477	3	55
6	9.41 582	46	9.43 108	50	0.56 892	9.98 474	3	54
7	9.41 628	47	9.43 158	50	0.56 842	9.98 471	4	53
8	9.41 675	47	9.43 208	50	0.56 792	9.98 467	3	52
9	9.41 722	46	9.43 258	50	0.56 742	9.98 464	4	51
10	9.41 768	47	9.43 308	50	0.56 692	9.98 460	3	50
11	9.41 815	46	9.43 358	50	0.56 642	9.98 457	4	49
12	9.41 861	47	9.43 408	50	0.56 592	9.98 453	3	48
13	9.41 908	46	9.43 458	50	0.56 542	9.98 450	3	47
14	9.41 954	47	9.43 508	50	0.56 492	9.98 447	4	46
15	9.42 001	46	9.43 558	49	0.56 442	9.98 443	3	45
16	9.42 047	46	9.43 607	50	0.56 393	9.98 440	4	44
17	9.42 093	47	9.43 657	50	0.56 343	9.98 436	3	43
18	9.42 140	46	9.43 707	49	0.56 293	9.98 433	4	42
19	9.42 186	46	9.43 756	50	0.56 244	9.98 429	3	41
20	9.42 232	46	9.43 806	49	0.56 194	9.98 426	4	40
21	9.42 278	46	9.43 855	50	0.56 145	9.98 422	3	39
22	9.42 324	46	9.43 905	49	0.56 095	9.98 419	4	38
23	9.42 370	46	9.43 954	50	0.56 046	9.98 415	3	37
24	9.42 416	45	9.44 004	49	0.55 996	9.98 412	3	36
25	9.42 461	46	9.44 053	49	0.55 947	9.98 409	4	35
26	9.42 507	46	9.44 102	49	0.55 898	9.98 405	3	34
27	9.42 553	46	9.44 151	50	0.55 849	9.98 402	4	33
28	9.42 599	45	9.44 201	49	0.55 799	9.98 398	3	32
29	9.42 644	46	9.44 250	49	0.55 750	9.98 395	4	31
30	9.42 690	45	9.44 299	49	0.55 701	9.98 391	3	30
31	9.42 735	46	9.44 348	49	0.55 652	9.98 388	4	29
32	9.42 781	45	9.44 397	49	0.55 603	9.98 384	3	28
33	9.42 826	46	9.44 446	49	0.55 554	9.98 381	4	27
34	9.42 872	45	9.44 495	49	0.55 505	9.98 377	4	26
35	9.42 917	45	9.44 544	48	0.55 456	9.98 373	3	25
36	9.42 962	46	9.44 592	49	0.55 408	9.98 370	4	24
37	9.43 008	45	9.44 641	49	0.55 359	9.98 366	3	23
38	9.43 053	45	9.44 690	48	0.55 310	9.98 363	4	22
39	9.43 098	45	9.44 738	49	0.55 262	9.98 359	3	21
40	9.43 143	45	9.44 787	49	0.55 213	9.98 356	4	20
41	9.43 188	45	9.44 836	48	0.55 164	9.98 352	3	19
42	9.43 233	45	9.44 884	49	0.55 116	9.98 349	4	18
43	9.43 278	45	9.44 933	48	0.55 067	9.98 345	3	17
44	9.43 323	44	9.44 981	48	0.55 019	9.98 342	4	16
45	9.43 367	45	9.45 029	49	0.54 971	9.98 338	4	15
46	9.43 412	45	9.45 078	48	0.54 922	9.98 334	3	14
47	9.43 457	45	9.45 126	48	0.54 874	9.98 331	4	13
48	9.43 502	44	9.45 174	48	0.54 826	9.98 327	3	12
49	9.43 546	45	9.45 222	49	0.54 778	9.98 324	4	11
50	9.43 591	44	9.45 271	48	0.54 729	9.98 320	3	10
51	9.43 635	45	9.45 319	48	0.54 681	9.98 317	4	9
52	9.43 680	44	9.45 367	48	0.54 633	9.98 313	4	8
53	9.43 724	45	9.45 415	48	0.54 585	9.98 309	3	7
54	9.43 769	44	9.45 463	48	0.54 537	9.98 306	4	6
55	9.43 813	44	9.45 511	48	0.54 489	9.98 302	3	5
56	9.43 857	44	9.45 559	47	0.54 441	9.98 299	4	4
57	9.43 901	45	9.45 606	48	0.54 394	9.98 295	4	2
58	9.43 946	44	9.45 654	48	0.54 346	9.98 291	3	2
59	9.43 990	44	9.45 702	48	0.54 298	9.98 288	4	1
60	9.44 034		9.45 750		0.54 250	9.98 284		0
	L Cos	d	L Cot	c d	L Tan	L Sin	d	′

Prop. Parts

	51	50	49
2	10.2	10.0	9.8
3	15.3	15.0	14.7
4	20.4	20.0	19.6
5	25.5	25.0	24.5
6	30.6	30.0	29.4
7	35.7	35.0	34.3
8	40.8	40.0	39.2
9	45.9	45.0	44.1

	48	47	46
2	9.6	9.4	9.2
3	14.4	14.1	13.8
4	19.2	18.8	18.4
5	24.0	23.5	23.0
6	28.8	28.2	27.6
7	33.6	32.9	32.2
8	38.4	37.6	36.8
9	43.2	42.3	41.4

	45	44
2	9.0	8.8
3	13.5	13.2
4	18.0	17.6
5	22.5	22.0
6	27.0	26.4
7	31.5	30.8
8	36.0	35.2
9	40.5	39.6

	4	3
2	0.8	0.6
3	1.2	0.9
4	1.6	1.2
5	2.0	1.5
6	2.4	1.8
7	2.8	2.1
8	3.2	2.4
9	3.6	2.7

164° 254° 344° 74°

′	L Sin	d	L Tan	c d	L Cot	L Cos	d		′
0	9.44 034	44	9.45 750	47	0.54 250	9.98 284	3	60	
1	9.44 078	44	9.45 797	48	0.54 203	9.98 281	4	59	
2	9.44 122	44	9.45 845	47	0.54 155	9.98 277	4	58	
3	9.44 166	44	9.45 892	48	0.54 108	9.98 273	3	57	
4	9.44 210	43	9.45 940	47	0.54 060	9.98 270	4	56	
5	9.44 253	44	9.45 987	48	0.54 013	9.98 266	4	55	
6	9.44 297	44	9.46 035	47	0.53 965	9.98 262	3	54	
7	9.44 341	44	9.46 082	48	0.53 918	9.98 259	4	53	
8	9.44 385	43	9.46 130	47	0.53 870	9.98 255	4	52	
9	9.44 428	44	9.46 177	47	0.53 823	9.98 251	3	51	
10	9.44 472	44	9.46 224	47	0.53 776	9.98 248	4	50	
11	9.44 516	43	9.46 271	48	0.53 729	9.98 244	4	49	
12	9.44 559	43	9.46 319	47	0.53 681	9.98 240	3	48	
13	9.44 602	44	9.46 366	47	0.53 634	9.98 237	4	47	
14	9.44 646	43	9.46 413	47	0.53 587	9.98 233	4	46	
15	9.44 689	44	9.46 460	47	0.53 540	9.98 229	3	45	
16	9.44 733	43	9.46 507	47	0.53 493	9.98 226	4	44	
17	9.44 776	43	9.46 554	47	0.53 446	9.98 222	4	43	
18	9.44 819	43	9.46 601	47	0.53 399	9.98 218	3	42	
19	9.44 862	43	9.46 648	46	0.53 352	9.98 215	4	41	
20	9.44 905	43	9.46 694	47	0.53 306	9.98 211	4	40	
21	9.44 948	44	9.46 741	47	0.53 259	9.98 207	3	39	
22	9.44 992	43	9.46 788	47	0.53 212	9.98 204	4	38	
23	9.45 035	42	9.46 835	46	0.53 165	9.98 200	4	37	
24	9.45 077	43	9.46 881	47	0.53 119	9.98 196	4	36	
25	9.45 120	43	9.46 928	47	0.53 072	9.98 192	3	35	
26	9.45 163	43	9.46 975	46	0.53 025	9.98 189	4	34	
27	9.45 206	43	9.47 021	47	0.52 979	9.98 185	4	33	
28	9.45 249	43	9.47 068	46	0.52 932	9.98 181	4	32	
29	9.45 292	42	9.47 114	46	0.52 886	9.98 177	3	31	
30	9.45 334	43	9.47 160	47	0.52 840	9.98 174	4	30	
31	9.45 377	42	9.47 207	46	0.52 793	9.98 170	4	29	
32	9.45 419	43	9.47 253	46	0.52 747	9.98 166	4	28	
33	9.45 462	42	9.47 299	47	0.52 701	9.98 162	3	27	
34	9.45 504	43	9.47 346	46	0.52 654	9.98 159	4	26	
35	9.45 547	42	9.47 392	46	0.52 608	9.98 155	4	25	
36	9.45 589	43	9.47 438	46	0.52 562	9.98 151	4	24	
37	9.45 632	42	9.47 484	46	0.52 516	9.98 147	3	23	
38	9.45 674	42	9.47 530	46	0.52 470	9.98 144	4	22	
39	9.45 716	42	9.47 576	46	0.52 424	9.98 140	4	21	
40	9.45 758	43	9.47 622	46	0.52 378	9.98 136	4	20	
41	9.45 801	42	9.47 668	46	0.52 332	9.98 132	3	19	
42	9.45 843	42	9.47 714	46	0.52 286	9.98 129	4	18	
43	9.45 885	42	9.47 760	46	0.52 240	9.98 125	4	17	
44	9.45 927	42	9.47 806	46	0.52 194	9.98 121	4	16	
45	9.45 969	42	9.47 852	45	0.52 148	9.98 117	4	15	
46	9.46 011	42	9.47 897	46	0.52 103	9.98 113	3	14	
47	9.46 053	42	9.47 943	46	0.52 057	9.98 110	4	13	
48	9.46 095	41	9.47 989	46	0.52 011	9.98 106	4	12	
49	9.46 136	42	9.48 035	45	0.51 965	9.98 102	4	11	
50	9.46 178	42	9.48 080	46	0.51 920	9.98 098	4	10	
51	9.46 220	42	9.48 126	45	0.51 874	9.98 094	4	9	
52	9.46 262	41	9.48 171	46	0.51 829	9.98 090	3	8	
53	9.46 303	42	9.48 217	45	0.51 783	9.98 087	4	7	
54	9.46 345	41	9.48 262	45	0.51 738	9.98 083	4	6	
55	9.46 386	42	9.48 307	46	0.51 693	9.98 079	4	5	
56	9.46 428	41	9.48 353	45	0.51 647	9.98 075	4	4	
57	9.46 469	42	9.48 398	45	0.51 602	9.98 071	4	3	
58	9.46 511	41	9.48 443	46	0.51 557	9.98 067	4	2	
59	9.46 552	42	9.48 489	45	0.51 511	9.98 063	3	1	
60	9.46 594		9.48 534		0.51 466	9.98 060		0	
	L Cos	d	L Cot	c d	L Tan	L Sin	d	′	

Prop. Parts

	48	47	46
2	9.6	9.4	9.2
3	14.4	14.1	13.8
4	19.2	18.8	18.4
5	24.0	23.5	23.0
6	28.8	28.2	27.6
7	33.6	32.9	32.2
8	38.4	37.6	36.8
9	43.2	42.3	41.4

	45	44	43
2	9.0	8.8	8.6
3	13.5	13.2	12.9
4	18.0	17.6	17.2
5	22.5	22.0	21.5
6	27.0	26.4	25.8
7	31.5	30.8	30.1
8	36.0	35.2	34.4
9	40.5	39.6	38.7

	42	41
2	8.4	8.2
3	12.6	12.3
4	16.8	16.4
5	21.0	20.5
6	25.2	24.6
7	29.4	28.7
8	33.6	32.8
9	37.8	36.9

	4	3
2	0.8	0.6
3	1.2	0.9
4	1.6	1.2
5	2.0	1.5
6	2.4	1.8
7	2.8	2.1
8	3.2	2.4
9	3.6	2.7

′	L Sin	d	L Tan	c d	L Cot	L Cos	d	
0	9.46 594	41	9.48 534	45	0.51 466	9.98 060	4	60
1	9.46 635	41	9.48 579	45	0.51 421	9.98 056	4	59
2	9.46 676	41	9.48 624	45	0.51 376	9.98 052	4	58
3	9.46 717	41	9.48 669	45	0.51 331	9.98 048	4	57
4	9.46 758	42	9.48 714	45	0.51 286	9.98 044	4	56
5	9.46 800	41	9.48 759	45	0.51 241	9.98 040	4	55
6	9.46 841	41	9.48 804	45	0.51 196	9.98 036	4	54
7	9.46 882	41	9.48 849	45	0.51 151	9.98 032	3	53
8	9.46 923	41	9.48 894	45	0.51 106	9.98 029	4	52
9	9.46 964	41	9.48 939	45	0.51 061	9.98 025	4	51
10	9.47 005	40	9.48 984	45	0.51 016	9.98 021	4	50
11	9.47 045	41	9.49 029	44	0.50 971	9.98 017	4	49
12	9.47 086	41	9.49 073	45	0.50 927	9.98 013	4	48
13	9.47 127	41	9.49 118	45	0.50 882	9.98 009	4	47
14	9.47 168	41	9.49 163	44	0.50 837	9.98 005	4	46
15	9.47 209	40	9.49 207	45	0.50 793	9.98 001	4	45
16	9.47 249	41	9.49 252	44	0.50 748	9.97 997	4	44
17	9.47 290	40	9.49 296	45	0.50 704	9.97 993	4	43
18	9.47 330	41	9.49 341	44	0.50 659	9.97 989	3	42
19	9.47 371	40	9.49 385	45	0.50 615	9.97 986	4	41
20	9.47 411	41	9.49 430	44	0.50 570	9.97 982	4	40
21	9.47 452	40	9.49 474	45	0.50 526	9.97 978	4	39
22	9.47 492	41	9.49 519	44	0.50 481	9.97 974	4	38
23	9.47 533	40	9.49 563	44	0.50 437	9.97 970	4	37
24	9.47 573	40	9.49 607	45	0.50 393	9.97 966	4	36
25	9.47 613	41	9.49 652	44	0.50 348	9.97 962	4	35
26	9.47 654	40	9.49 696	44	0.50 304	9.97 958	4	34
27	9.47 694	40	9.49 740	44	0.50 260	9.97 954	4	33
28	9.47 734	40	9.49 784	44	0.50 216	9.97 950	4	32
29	9.47 774	40	9.49 828	44	0.50 172	9.97 946	4	31
30	9.47 814	40	9.49 872	44	0.50 128	9.97 942	4	30
31	9.47 854	40	9.49 916	44	0.50 084	9.97 938	4	29
32	9.47 894	40	9.49 960	44	0.50 040	9.97 934	4	28
33	9.47 934	40	9.50 004	44	0.49 996	9.97 930	4	27
34	9.47 974	40	9.50 048	44	0.49 952	9.97 926	4	26
35	9.48 014	40	9.50 092	44	0.49 908	9.97 922	4	25
36	9.48 054	40	9.50 136	44	0.49 864	9.97 918	4	24
37	9.48 094	39	9.50 180	43	0.49 820	9.97 914	4	23
38	9.48 133	40	9.50 223	44	0.49 777	9.97 910	4	22
39	9.48 173	40	9.50 267	44	0.49 733	9.97 906	4	21
40	9.48 213	39	9.50 311	44	0.49 689	9.97 902	4	20
41	9.48 252	40	9.50 355	43	0.49 645	9.97 898	4	19
42	9.48 292	40	9.50 398	44	0.49 602	9.97 894	4	18
43	9.48 332	39	9.50 442	43	0.49 558	9.97 890	4	17
44	9.48 371	40	9.50 485	44	0.49 515	9.97 886	4	16
45	9.48 411	39	9.50 529	43	0.49 471	9.97 882	4	15
46	9.48 450	40	9.50 572	44	0.49 428	9.97 878	4	14
47	9.48 490	39	9.50 616	43	0.49 384	9.97 874	4	13
48	9.48 529	39	9.50 659	44	0.49 341	9.97 870	4	12
49	9.48 568	39	9.50 703	43	0.49 297	9.97 866	5	11
50	9.48 607	40	9.50 746	43	0.49 254	9.97 861	4	10
51	9.48 647	39	9.50 789	44	0.49 211	9.97 857	4	9
52	9.48 686	39	9.50 833	43	0.49 167	9.97 853	4	8
53	9.48 725	39	9.50 876	43	0.49 124	9.97 849	4	7
54	9.48 764	39	9.50 919	43	0.49 081	9.97 845	4	6
55	9.48 803	39	9.50 962	43	0.49 038	9.97 841	4	5
56	9.48 842	39	9.51 005	43	0.48 995	9.97 837	4	4
57	9.48 881	39	9.51 048	44	0.48 952	9.97 833	4	3
58	9.48 920	39	9.51 092	43	0.48 908	9.97 829	4	2
59	9.48 959	39	9.51 135	43	0.48 865	9.97 825	4	1
60	9.48 998		9.51 178		0.48 822	9.97 821		0
	L Cos	d	L Cot	c d	L Tan	L Sin	d	′

Prop. Parts

	45	44	43
2	9.0	8.8	8.6
3	13.5	13.2	12.9
4	18.0	17.6	17.2
5	22.5	22.0	21.5
6	27.0	26.4	25.8
7	31.5	30.8	30.1
8	36.0	35.2	34.4
9	40.5	39.6	38.7

	42	41	40
2	8.4	8.2	8.0
3	12.6	12.3	12.0
4	16.8	16.4	16.0
5	21.0	20.5	20.0
6	25.2	24.6	24.0
7	29.4	28.7	28.0
8	33.6	32.8	32.0
9	37.8	36.9	36.0

	39	5
2	7.8	1.0
3	11.7	1.5
4	15.6	2.0
5	19.5	2.5
6	23.4	3.0
7	27.3	3.5
8	31.2	4.0
9	35.1	4.5

	4	3
2	0.8	0.6
3	1.2	0.9
4	1.6	1.2
5	2.0	1.5
6	2.4	1.8
7	2.8	2.1
8	3.2	2.4
9	3.6	2.7

′	L Sin	d	L Tan	c d	L Cot	L Cos	d		Prop. Parts
0	9.48 998	39	9.51 178	43	0.48 822	9.97 821	4	60	
1	9.49 037	39	9.51 221	43	0.48 779	9.97 817	5	59	
2	9.49 076	39	9.51 264	42	0.48 736	9.97 812	4	58	
3	9.49 115	38	9.51 306	43	0.48 694	9.97 808	4	57	
4	9.49 153	39	9.51 349	43	0.48 651	9.97 804	4	56	
5	9.49 192	39	9.51 392	43	0.48 608	9.97 800	4	55	
6	9.49 231	38	9.51 435	43	0.48 565	9.97 796	4	54	
7	9.49 269	39	9.51 478	42	0.48 522	9.97 792	4	53	
8	9.49 308	39	9.51 520	43	0.48 480	9.97 788	4	52	
9	9.49 347	38	9.51 563	43	0.48 437	9.97 784	5	51	
10	9.49 385	39	9.51 606	42	0.48 394	9.97 779	4	50	
11	9.49 424	38	9.51 648	43	0.48 352	9.97 775	4	49	
12	9.49 462	38	9.51 691	43	0.48 309	9.97 771	4	48	
13	9.49 500	39	9.51 734	42	0.48 266	9.97 767	4	47	**43 42 41**
14	9.49 539	38	9.51 776	43	0.48 224	9.97 763	4	46	2 8.6 8.4 8.2
15	9.49 577	38	9.51 819	42	0.48 181	9.97 759	5	45	3 12.9 12.6 12.3
16	9.49 615	39	9.51 861	42	0.48 139	9.97 754	4	44	4 17.2 16.8 16.4
17	9.49 654	38	9.51 903	43	0.48 097	9.97 750	4	43	5 21.5 21.0 20.5
18	9.49 692	38	9.51 946	42	0.48 054	9.97 746	4	42	6 25.8 25.2 24.6
19	9.49 730	38	9.51 988	43	0.48 012	9.97 742	4	41	7 30.1 29.4 28.7
20	9.49 768	38	9.52 031	42	0.47 969	9.97 738	4	40	8 34.4 33.6 32.8
21	9.49 806	38	9.52 073	42	0.47 927	9.97 734	5	39	9 38.7 37.8 36.9
22	9.49 844	38	9.52 115	42	0.47 885	9.97 729	4	38	
23	9.49 882	38	9.52 157	43	0.47 843	9.97 725	4	37	
24	9.49 920	38	9.52 200	42	0.47 800	9.97 721	4	36	
25	9.49 958	38	9.52 242	42	0.47 758	9.97 717	4	35	
26	9.49 996	38	9.52 284	42	0.47 716	9.97 713	5	34	**39 38 37**
27	9.50 034	38	9.52 326	42	0.47 674	9.97 708	4	33	2 7.8 7.6 7.4
28	9.50 072	38	9.52 368	42	0.47 632	9.97 704	4	32	3 11.7 11.4 11.1
29	9.50 110	38	9.52 410	42	0.47 590	9.97 700	4	31	4 15.6 15.2 14.8
30	9.50 148	37	9.52 452	42	0.47 548	9.97 696	5	30	5 19.5 19.0 18.5
31	9.50 185	38	9.52 494	42	0.47 506	9.97 691	4	29	6 23.4 22.8 22.2
32	9.50 223	38	9.52 536	42	0.47 464	9.97 687	4	28	7 27.3 26.6 25.9
33	9.50 261	37	9.52 578	42	0.47 422	9.97 683	4	27	8 31.2 30.4 29.6
34	9.50 298	38	9.52 620	41	0.47 380	9.97 679	5	26	9 35.1 34.2 33.3
35	9.50 336	38	9.52 661	42	0.47 339	9.97 674	4	25	
36	9.50 374	37	9.52 703	42	0.47 297	9.97 670	4	24	
37	9.50 411	38	9.52 745	42	0.47 255	9.97 666	4	23	
38	9.50 449	37	9.52 787	42	0.47 213	9.97 662	5	22	
39	9.50 486	37	9.52 829	41	0.47 171	9.97 657	4	21	
40	9.50 523	38	9.52 870	42	0.47 130	9.97 653	4	20	**36 5 4**
41	9.50 561	37	9.52 912	41	0.47 088	9.97 649	4	19	2 7.2 1.0 0.8
42	9.50 598	37	9.52 953	42	0.47 047	9.97 645	5	18	3 10.8 1.5 1.2
43	9.50 635	38	9.52 995	42	0.47 005	9.97 640	4	17	4 14.4 2.0 1.6
44	9.50 673	37	9.53 037	41	0.46 963	9.97 636	4	16	5 18.0 2.5 2.0
45	9.50 710	37	9.53 078	42	0.46 922	9.97 632	4	15	6 21.6 3.0 2.4
46	9.50 747	37	9.53 120	41	0.46 880	9.97 628	5	14	7 25.2 3.5 2.8
47	9.50 784	37	9.53 161	41	0.46 839	9.97 623	4	13	8 28.8 4.0 3.2
48	9.50 821	37	9.53 202	42	0.46 798	9.97 619	4	12	9 32.4 4.5 3.6
49	9.50 858	38	9.53 244	41	0.46 756	9.97 615	5	11	
50	9.50 896	37	9.53 285	42	0.46 715	9.97 610	4	10	
51	9.50 933	37	9.53 327	41	0.46 673	9.97 606	4	9	
52	9.50 970	37	9.53 368	41	0.46 632	9.97 602	5	8	
53	9.51 007	36	9.53 409	41	0.46 591	9.97 597	4	7	
54	9.51 043	37	9.53 450	42	0.46 550	9.97 593	4	6	
55	9.51 080	37	9.53 492	41	0.46 508	9.97 589	5	5	
56	9.51 117	37	9.53 533	41	0.46 467	9.97 584	4	4	
57	9.51 154	37	9.53 574	41	0.46 426	9.97 580	4	3	
58	9.51 191	36	9.53 615	41	0.46 385	9.97 576	5	2	
59	9.51 227	37	9.53 656	41	0.46 344	9.97 571	4	1	
60	9.51 264		9.53 697		0.46 303	9.97 567		0	
	L Cos	d	L Cot	c d	L Tan	L Sin	d	′	

′	L Sin	d	L Tan	c d	L Cot	L Cos	d	
0	9.51 264	37	9.53 697	41	0.46 303	9.97 567	4	**60**
1	9.51 301	37	9.53 738	41	0.46 262	9.97 563	5	59
2	9.51 338	36	9.53 779	41	0.46 221	9.97 558	4	58
3	9.51 374	37	9.53 820	41	0.46 180	9.97 554	4	57
4	9.51 411	36	9.53 861	41	0.46 139	9.97 550	5	56
5	9.51 447	37	9.53 902	41	0.46 098	9.97 545	4	**55**
6	9.51 484	36	9.53 943	41	0.46 057	9.97 541	5	54
7	9.51 520	37	9.53 984	41	0.46 016	9.97 536	4	53
8	9.51 557	36	9.54 025	40	0.45 975	9.97 532	4	52
9	9.51 593	36	9.54 065	41	0.45 935	9.97 528	5	51
10	9.51 629	37	9.54 106	41	0.45 894	9.97 523	4	**50**
11	9.51 666	36	9.54 147	40	0.45 853	9.97 519	4	49
12	9.51 702	36	9.54 187	41	0.45 813	9.97 515	5	48
13	9.51 738	36	9.54 228	41	0.45 772	9.97 510	4	47
14	9.51 774	37	9.54 269	40	0.45 731	9.97 506	5	46
15	9.51 811	36	9.54 309	41	0.45 691	9.97 501	4	**45**
16	9.51 847	36	9.54 350	40	0.45 650	9.97 497	5	44
17	9.51 883	36	9.54 390	41	0.45 610	9.97 492	4	43
18	9.51 919	36	9.54 431	40	0.45 569	9.97 488	4	42
19	9.51 955	36	9.54 471	41	0.45 529	9.97 484	5	41
20	9.51 991	36	9.54 512	40	0.45 488	9.97 479	4	**40**
21	9.52 027	36	9.54 552	41	0.45 448	9.97 475	5	39
22	9.52 063	36	9.54 593	40	0.45 407	9.97 470	4	38
23	9.52 099	36	9.54 633	40	0.45 367	9.97 466	5	37
24	9.52 135	36	9.54 673	41	0.45 327	9.97 461	4	36
25	9.52 171	36	9.54 714	40	0.45 286	9.97 457	4	**35**
26	9.52 207	35	9.54 754	40	0.45 246	9.97 453	5	34
27	9.52 242	36	9.54 794	41	0.45 206	9.97 448	4	33
28	9.52 278	36	9.54 835	40	0.45 165	9.97 444	5	32
29	9.52 314	36	9.54 875	40	0.45 125	9.97 439	4	31
30	9.52 350	35	9.54 915	40	0.45 085	9.97 435	5	**30**
31	9.52 385	36	9.54 955	40	0.45 045	9.97 430	4	29
32	9.52 421	35	9.54 995	40	0.45 005	9.97 426	5	28
33	9.52 456	36	9.55 035	40	0.44 965	9.97 421	4	27
34	9.52 492	35	9.55 075	40	0.44 925	9.97 417	5	26
35	9.52 527	36	9.55 115	40	0.44 885	9.97 412	4	**25**
36	9.52 563	35	9.55 155	40	0.44 845	9.97 408	5	24
37	9.52 598	36	9.55 195	40	0.44 805	9.97 403	4	23
38	9.52 634	35	9.55 235	40	0.44 765	9.97 399	5	22
39	9.52 669	36	9.55 275	40	0.44 725	9.97 394	4	21
40	9.52 705	35	9.55 315	40	0.44 685	9.97 390	5	**20**
41	9.52 740	35	9.55 355	40	0.44 645	9.97 385	4	19
42	9.52 775	36	9.55 395	39	0.44 605	9.97 381	5	18
43	9.52 811	35	9.55 434	40	0.44 566	9.97 376	4	17
44	9.52 846	35	9.55 474	40	0.44 526	9.97 372	5	16
45	9.52 881	35	9.55 514	40	0.44 486	9.97 367	4	**15**
46	9.52 916	35	9.55 554	39	0.44 446	9.97 363	5	14
47	9.52 951	35	9.55 593	40	0.44 407	9.97 358	5	13
48	9.52 986	35	9.55 633	40	0.44 367	9.97 353	4	12
49	9.53 021	35	9.55 673	39	0.44 327	9.97 349	5	11
50	9.53 056	36	9.55 712	40	0.44 288	9.97 344	4	**10**
51	9.53 092	34	9.55 752	39	0.44 248	9.97 340	5	9
52	9.53 126	35	9.55 791	40	0.44 209	9.97 335	4	8
53	9.53 161	35	9.55 831	40	0.44 169	9.97 331	5	7
54	9.53 196	35	9.55 870	40	0.44 130	9.97 326	4	6
55	9.53 231	35	9.55 910	39	0.44 090	9.97 322	5	**5**
56	9.53 266	35	9.55 949	40	0.44 051	9.97 317	5	4
57	9.53 301	35	9.55 989	39	0.44 011	9.97 312	4	3
58	9.53 336	34	9.56 028	39	0.43 972	9.97 308	5	2
59	9.53 370	35	9.56 067	40	0.43 933	9.97 303	4	1
60	9.53 405		9.56 107		0.43 893	9.97 299		**0**
	L Cos	d	L Cot	c d	L Tan	L Sin	d	′

Prop. Parts

	41	40	39
2	8.2	8.0	7.8
3	12.3	12.0	11.7
4	16.4	16.0	15.6
5	20.5	20.0	19.5
6	24.6	24.0	23.4
7	28.7	28.0	27.3
8	32.8	32.0	31.2
9	36.9	36.0	35.1

	37	36	35
2	7.4	7.2	7.0
3	11.1	10.8	10.5
4	14.8	14.4	14.0
5	18.5	18.0	17.5
6	22.2	21.6	21.0
7	25.9	25.2	24.5
8	29.6	28.8	28.0
9	33.3	32.4	31.5

	34	5	4
2	6.8	1.0	0.8
3	10.2	1.5	1.2
4	13.6	2.0	1.6
5	17.0	2.5	2.0
6	20.4	3.0	2.4
7	23.8	3.5	2.8
8	27.2	4.0	3.2
9	30.6	4.5	3.6

′	L Sin	d	L Tan	c d	L Cot	L Cos	d		Prop. Parts
0	9.53 405	35	9.56 107	39	0.43 893	9.97 299	5	**60**	
1	9.53 440	35	9.56 146	39	0.43 854	9.97 294	5	59	
2	9.53 475	34	9.56 185	39	0.43 815	9.97 289	4	58	
3	9.53 509	35	9.56 224	40	0.43 776	9.97 285	5	57	
4	9.53 544	34	9.56 264	39	0.43 736	9.97 280	4	56	
5	9.53 578	35	9.56 303	39	0.43 697	9.97 276	5	**55**	
6	9.53 613	34	9.56 342	39	0.43 658	9.97 271	5	54	
7	9.53 647	35	9.56 381	39	0.43 619	9.97 266	4	53	
8	9.53 682	34	9.56 420	39	0.43 580	9.97 262	5	52	
9	9.53 716	35	9.56 459	39	0.43 541	9.97 257	5	51	
10	9.53 751	34	9.56 498	39	0.43 502	9.97 252	4	**50**	
11	9.53 785	34	9.56 537	39	0.43 463	9.97 248	5	49	
12	9.53 819	34	9.56 576	39	0.43 424	9.97 243	5	48	
13	9.53 854	35	9.56 615	39	0.43 385	9.97 238	4	47	
14	9.53 888	34	9.56 654	39	0.43 346	9.97 234	5	46	
15	9.53 922	34	9.56 693	39	0.43 307	9.97 229	5	**45**	
16	9.53 957	35	9.56 732	39	0.43 268	9.97 224	4	44	
17	9.53 991	34	9.56 771	39	0.43 229	9.97 220	5	43	
18	9.54 025	34	9.56 810	39	0.43 190	9.97 215	5	42	
19	9.54 059	34	9.56 849	38	0.43 151	9.97 210	4	41	
20	9.54 093	34	9.56 887	39	0.43 113	9.97 206	5	**40**	
21	9.54 127	34	9.56 926	39	0.43 074	9.97 201	5	39	
22	9.54 161	34	9.56 965	39	0.43 035	9.97 196	4	38	
23	9.54 195	34	9.57 004	38	0.42 996	9.97 192	5	37	
24	9.54 229	34	9.57 042	39	0.42 958	9.97 187	5	36	
25	9.54 263	34	9.57 081	39	0.42 919	9.97 182	4	**35**	
26	9.54 297	34	9.57 120	38	0.42 880	9.97 178	5	34	
27	9.54 331	34	9.57 158	39	0.42 842	9.97 173	5	33	
28	9.54 365	34	9.57 197	38	0.42 803	9.97 168	5	32	
29	9.54 399	34	9.57 235	39	0.42 765	9.97 163	4	31	
30	9.54 433	33	9.57 274	38	0.42 726	9.97 159	5	**30**	
31	9.54 466	34	9.57 312	39	0.42 688	9.97 154	5	29	
32	9.54 500	34	9.57 351	38	0.42 649	9.97 149	4	28	
33	9.54 534	33	9.57 389	39	0.42 611	9.97 145	5	27	
34	9.54 567	34	9.57 428	38	0.42 572	9.97 140	5	26	
35	9.54 601	34	9.57 466	38	0.42 534	9.97 135	5	**25**	
36	9.54 635	33	9.57 504	39	0.42 496	9.97 130	4	24	
37	9.54 668	34	9.57 543	38	0.42 457	9.97 126	5	23	
38	9.54 702	33	9.57 581	38	0.42 419	9.97 121	5	22	
39	9.54 735	34	9.57 619	39	0.42 381	9.97 116	5	21	
40	9.54 769	33	9.57 658	38	0.42 342	9.97 111	4	**20**	
41	9.54 802	34	9.57 696	38	0.42 304	9.97 107	5	19	
42	9.54 836	33	9.57 734	38	0.42 266	9.97 102	5	18	
43	9.54 869	34	9.57 772	38	0.42 228	9.97 097	5	17	
44	9.54 903	33	9.57 810	39	0.42 190	9.97 092	5	16	
45	9.54 936	33	9.57 849	38	0.42 151	9.97 087	4	**15**	
46	9.54 969	34	9.57 887	38	0.42 113	9.97 083	5	14	
47	9.55 003	33	9.57 925	38	0.42 075	9.97 078	5	13	
48	9.55 036	33	9.57 963	38	0.42 037	9.97 073	5	12	
49	9.55 069	33	9.58 001	38	0.41 999	9.97 068	5	11	
50	9.55 102	34	9.58 039	38	0.41 961	9.97 063	4	**10**	
51	9.55 136	33	9.58 077	38	0.41 923	9.97 059	5	9	
52	9.55 169	33	9.58 115	38	0.41 885	9.97 054	5	8	
53	9.55 202	33	9.58 153	38	0.41 847	9.97 049	5	7	
54	9.55 235	33	9.58 191	38	0.41 809	9.97 044	5	6	
55	9.55 268	33	9.58 229	38	0.41 771	9.97 039	4	**5**	
56	9.55 301	33	9.58 267	37	0.41 733	9.97 035	5	4	
57	9.55 334	33	9.58 304	38	0.41 696	9.97 030	5	3	
58	9.55 367	33	9.58 342	38	0.41 658	9.97 025	5	2	
59	9.55 400	33	9.58 380	38	0.41 620	9.97 020	5	1	
60	9.55 433		9.58 418		0.41 582	9.97 015		**0**	

	40	39	38
2	8.0	7.8	7.6
3	12.0	11.7	11.4
4	16.0	15.6	15.2
5	20.0	19.5	19.0
6	24.0	23.4	22.8
7	28.0	27.3	26.6
8	32.0	31.2	30.4
9	36.0	35.1	34.2

	37	35	34
2	7.4	7.0	6.8
3	11.1	10.5	10.2
4	14.8	14.0	13.6
5	18.5	17.5	17.0
6	22.2	21.0	20.4
7	25.9	24.5	23.8
8	29.6	28.0	27.2
9	33.3	31.5	30.6

	33	5	4
2	6.6	1.0	0.8
3	9.9	1.5	1.2
4	13.2	2.0	1.6
5	16.5	2.5	2.0
6	19.8	3.0	2.4
7	23.1	3.5	2.8
8	26.4	4.0	3.2
9	29.7	4.5	3.6

′	L Cos	d	L Cot	c d	L Tan	L Sin	d

159° 249° 339° 69°

′	L Sin	d	L Tan	c d	L Cot	L Cos	d		Prop. Parts
0	9.55 433	33	9.58 418	37	0.41 582	9.97 015	5	60	
1	9.55 466	33	9.58 455	38	0.41 545	9.97 010	5	59	
2	9.55 499	33	9.58 493	38	0.41 507	9.97 005	4	58	
3	9.55 532	32	9.58 531	38	0.41 469	9.97 001	5	57	
4	9.55 564	33	9.58 569	37	0.41 431	9.96 996	5	56	
5	9.55 597	33	9.58 606	38	0.41 394	9.96 991	5	55	
6	9.55 630	33	9.58 644	37	0.41 356	9.96 986	5	54	
7	9.55 663	32	9.58 681	38	0.41 319	9.96 981	5	53	
8	9.55 695	33	9.58 719	38	0.41 281	9.96 976	5	52	
9	9.55 728	33	9.58 757	37	0.41 243	9.96 971	5	51	
10	9.55 761	32	9.58 794	38	0.41 206	9.96 966	4	50	
11	9.55 793	33	9.58 832	37	0.41 168	9.96 962	5	49	
12	9.55 826	32	9.58 869	38	0.41 131	9.96 957	5	48	
13	9.55 858	33	9.58 907	37	0.41 093	9.96 952	5	47	**38 37 36**
14	9.55 891	32	9.58 944	37	0.41 056	9.96 947	5	46	2 7.6 7.4 7.2
15	9.55 923	33	9.58 981	38	0.41 019	9.96 942	5	45	3 11.4 11.1 10.8
16	9.55 956	32	9.59 019	37	0.40 981	9.96 937	5	44	4 15.2 14.8 14.4
17	9.55 988	33	9.59 056	38	0.40 944	9.96 932	5	43	5 19.0 18.5 18.0
18	9.56 021	32	9.59 094	37	0.40 906	9.96 927	5	42	6 22.8 22.2 21.6
19	9.56 053	32	9.59 131	37	0.40 869	9.96 922	5	41	7 26.6 25.9 25.2
20	9.56 085	33	9.59 168	37	0.40 832	9.96 917	5	40	8 30.4 29.6 28.8
21	9.56 118	32	9.59 205	38	0.40 795	9.96 912	5	39	9 34.2 33.3 32.4
22	9.56 150	32	9.59 243	37	0.40 757	9.96 907	4	38	
23	9.56 182	33	9.59 280	37	0.40 720	9.96 903	5	37	
24	9.56 213	32	9.59 317	37	0.40 683	9.96 898	5	36	
25	9.56 247	32	9.59 354	37	0.40 646	9.96 893	5	35	
26	9.56 279	32	9.59 391	38	0.40 609	9.96 888	5	34	**33 32 31**
27	9.56 311	32	9.59 429	37	0.40 571	9.96 883	5	33	
28	9.56 343	32	9.59 466	37	0.40 534	9.96 878	5	32	2 6.6 6.4 6.2
29	9.56 375	33	9.59 503	37	0.40 497	9.96 873	5	31	3 9.9 9.6 9.3
30	9.56 408	32	9.59 540	37	0.40 460	9.96 868	5	30	4 13.2 12.8 12.4
31	9.56 440	32	9.59 577	37	0.40 423	9.96 863	5	29	5 16.5 16.0 15.5
32	9.56 472	32	9.59 614	37	0.40 386	9.96 858	5	28	6 19.8 19.2 18.6
33	9.56 504	32	9.59 651	37	0.40 349	9.96 853	5	27	7 23.1 22.4 21.7
34	9.56 536	32	9.59 688	37	0.40 312	9.96 848	5	26	8 26.4 25.6 24.8
35	9.56 568	31	9.59 725	37	0.40 275	9.96 843	5	25	9 29.7 28.8 27.9
36	9.56 599	32	9.59 762	37	0.40 238	9.96 838	5	24	
37	9.56 631	32	9.59 799	36	0.40 201	9.96 833	5	23	
38	9.56 663	32	9.59 835	37	0.40 165	9.96 828	5	22	
39	9.56 695	32	9.59 872	37	0.40 128	9.96 823	5	21	
40	9.56 727	32	9.59 909	37	0.40 091	9.96 818	5	20	**6 5 4**
41	9.56 759	31	9.59 946	37	0.40 054	9.96 813	5	19	2 1.2 1.0 0.8
42	9.56 790	32	9.59 983	36	0.40 017	9.96 808	5	18	3 1.8 1.5 1.2
43	9.56 822	32	9.60 019	37	0.39 981	9.96 803	5	17	4 2.4 2.0 1.6
44	9.56 854	32	9.60 056	37	0.39 944	9.96 798	5	16	5 3.0 2.5 2.0
45	9.56 886	31	9.60 093	37	0.39 907	9.96 793	5	15	6 3.6 3.0 2.4
46	9.56 917	32	9.60 130	36	0.39 870	9.96 788	5	14	7 4.2 3.5 2.8
47	9.56 949	31	9.60 166	37	0.39 834	9.96 783	5	13	8 4.8 4.0 3.2
48	9.56 980	32	9.60 203	37	0.39 797	9.96 778	6	12	9 5.4 4.5 3.6
49	9.57 012	32	9.60 240	36	0.39 760	9.96 772	5	11	
50	9.57 044	31	9.60 276	37	0.39 724	9.96 767	5	10	
51	9.57 075	32	9.60 313	36	0.39 687	9.96 762	5	9	
52	9.57 107	31	9.60 349	37	0.39 651	9.96 757	5	8	
53	9.57 138	31	9.60 386	36	0.39 614	9.96 752	5	7	
54	9.57 169	32	9.60 422	37	0.39 578	9.96 747	5	6	
55	9.57 201	31	9.60 459	36	0.39 541	9.96 742	5	5	
56	9.57 232	32	9.60 495	37	0.39 505	9.96 737	5	4	
57	9.57 264	31	9.60 532	36	0.39 468	9.96 732	5	3	
58	9.57 295	31	9.60 568	37	0.39 432	9.96 727	5	2	
59	9.57 326	32	9.60 605	36	0.39 395	9.96 722	5	1	
60	9.57 358		9.60 641		0.39 359	9.96 717		0	
	L Cos	d	L Cot	c d	L Tan	L Sin	d	′	

′	L Sin	d	L Tan	c d	L Cot	L Cos	d	
0	9.57 358	31	9.60 641	36	0.39 359	9.96 717	6	60
1	9.57 389	31	9.60 677	37	0.39 323	9.96 711	5	59
2	9.57 420	31	9.60 714	36	0.39 286	9.96 706	5	58
3	9.57 451	31	9.60 750	36	0.39 250	9.96 701	5	57
4	9.57 482	32	9.60 786	37	0.39 214	9.96 696	5	56
5	9.57 514	31	9.60 823	36	0.39 177	9.96 691	5	55
6	9.57 545	31	9.60 859	36	0.39 141	9.96 686	5	54
7	9.57 576	31	9.60 895	36	0.39 105	9.96 681	5	53
8	9.57 607	31	9.60 931	36	0.39 069	9.96 676	6	52
9	9.57 638	31	9.60 967	37	0.39 033	9.96 670	5	51
10	9.57 669	31	9.61 004	36	0.38 996	9.96 665	5	50
11	9.57 700	31	9.61 040	36	0.38 960	9.96 660	5	49
12	9.57 731	31	9.61 076	36	0.38 924	9.96 655	5	48
13	9.57 762	31	9.61 112	36	0.38 888	9.96 650	5	47
14	9.57 793	31	9.61 148	36	0.38 852	9.96 645	5	46
15	9.57 824	31	9.61 184	36	0.38 816	9.96 640	6	45
16	9.57 855	30	9.61 220	36	0.38 780	9.96 634	5	44
17	9.57 885	31	9.61 256	36	0.38 744	9.96 629	5	43
18	9.57 916	31	9.61 292	36	0.38 708	9.96 624	5	42
19	9.57 947	31	9.61 328	36	0.38 672	9.96 619	5	41
20	9.57 978	30	9.61 364	36	0.38 636	9.96 614	6	40
21	9.58 008	31	9.61 400	36	0.38 600	9.96 608	5	39
22	9.58 039	31	9.61 436	36	0.38 564	9.96 603	5	38
23	9.58 070	31	9.61 472	36	0.38 528	9.96 598	5	37
24	9.58 101	30	9.61 508	36	0.38 492	9.96 593	5	36
25	9.58 131	31	9.61 544	35	0.38 456	9.96 588	6	35
26	9.58 162	30	9.61 579	36	0.38 421	9.96 582	5	34
27	9.58 192	31	9.61 615	36	0.38 385	9.96 577	5	33
28	9.58 223	30	9.61 651	36	0.38 349	9.96 572	5	32
29	9.58 253	31	9.61 687	35	0.38 313	9.96 567	5	31
30	9.58 284	30	9.61 722	36	0.38 278	9.96 562	6	30
31	9.58 314	31	9.61 758	36	0.38 242	9.96 556	5	29
32	9.58 345	30	9.61 794	36	0.38 206	9.96 551	5	28
33	9.58 375	31	9.61 830	35	0.38 170	9.96 546	5	27
34	9.58 406	30	9.61 865	36	0.38 135	9.96 541	6	26
35	9.58 436	31	9.61 901	35	0.38 099	9.96 535	5	25
36	9.58 467	30	9.61 936	36	0.38 064	9.96 530	5	24
37	9.58 497	30	9.61 972	36	0.38 028	9.96 525	5	23
38	9.58 527	30	9.62 008	35	0.37 992	9.96 520	6	22
39	9.58 557	31	9.62 043	36	0.37 957	9.96 514	5	21
40	9.58 588	30	9.62 079	35	0.37 921	9.96 509	5	20
41	9.58 618	30	9.62 114	36	0.37 886	9.96 504	6	19
42	9.58 648	30	9.62 150	35	0.37 850	9.96 498	5	18
43	9.58 678	31	9.62 185	36	0.37 815	9.96 493	5	17
44	9.58 709	30	9.62 221	35	0.37 779	9.96 488	5	16
45	9.58 739	30	9.62 256	36	0.37 744	9.96 483	6	15
46	9.58 769	30	9.62 292	35	0.37 708	9.96 477	5	14
47	9.58 799	30	9.62 327	35	0.37 673	9.96 472	5	13
48	9.58 829	30	9.62 362	36	0.37 638	9.96 467	6	12
49	9.58 859	30	9.62 398	35	0.37 602	9.96 461	5	11
50	9.58 889	30	9.62 433	35	0.37 567	9.96 456	5	10
51	9.58 919	30	9.62 468	36	0.37 532	9.96 451	6	9
52	9.58 949	30	9.62 504	35	0.37 496	9.96 445	5	8
53	9.58 979	30	9.62 539	35	0.37 461	9.96 440	5	7
54	9.59 009	30	9.62 574	35	0.37 426	9.96 435	6	6
55	9.59 039	30	9.62 609	36	0.37 391	9.96 429	5	5
56	9.59 069	29	9.62 645	35	0.37 355	9.96 424	5	4
57	9.59 098	30	9.62 680	35	0.37 320	9.96 419	6	3
58	9.59 128	30	9.62 715	35	0.37 285	9.96 413	5	2
59	9.59 158	30	9.62 750	35	0.37 250	9.96 408	5	1
60	9.59 188		9.62 785		0.37 215	9.96 403		0
	L Cos	d	L Cot	c d	L Tan	L Sin	d	′

Prop. Parts

	37	36	35
2	7.4	7.2	7.0
3	11.1	10.8	10.5
4	14.8	14.4	14.0
5	18.5	18.0	17.5
6	22.2	21.6	21.0
7	25.9	25.2	24.5
8	29.6	28.8	28.0
9	33.3	32.4	31.5

	32	31	30
2	6.4	6.2	6.0
3	9.6	9.3	9.0
4	12.8	12.4	12.0
5	16.0	15.5	15.0
6	19.2	18.6	18.0
7	22.4	21.7	21.0
8	25.6	24.8	24.0
9	28.8	27.9	27.0

	29	6	5
2	5.8	1.2	1.0
3	8.7	1.8	1.5
4	11.6	2.4	2.0
5	14.5	3.0	2.5
6	17.4	3.6	3.0
7	20.3	4.2	3.5
8	23.2	4.8	4.0
9	26.1	5.4	4.5

′	L Sin	d	L Tan	c d	L Cot	L Cos	d	
0	9.59 188	30	9.62 785	35	0.37 215	9.96 403	6	60
1	9.59 218	29	9.62 820	35	0.37 180	9.96 397	5	59
2	9.59 247	30	9.62 855	35	0.37 145	9.96 392	5	58
3	9.59 277	30	9.62 890	36	0.37 110	9.96 387	6	57
4	9.59 307	29	9.62 926	35	0.37 074	9.96 381	5	56
5	9.59 336	30	9.62 961	35	0.37 039	9.96 376	6	55
6	9.59 366	30	9.62 996	35	0.37 004	9.96 370	5	54
7	9.59 396	29	9.63 031	35	0.36 969	9.96 365	5	53
8	9.59 425	30	9.63 066	35	0.36 934	9.96 360	6	52
9	9.59 455	29	9.63 101	34	0.36 899	9.96 354	5	51
10	9.59 484	30	9.63 135	35	0.36 865	9.96 349	6	50
11	9.59 514	29	9.63 170	35	0.36 830	9.96 343	5	49
12	9.59 543	30	9.63 205	35	0.36 795	9.96 338	5	48
13	9.59 573	29	9.63 240	35	0.36 760	9.96 333	6	47
14	9.59 602	30	9.63 275	35	0.36 725	9.96 327	5	46
15	9.59 632	29	9.63 310	35	0.36 690	9.96 322	6	45
16	9.59 661	29	9.63 345	34	0.36 655	9.96 316	5	44
17	9.59 690	30	9.63 379	35	0.36 621	9.96 311	6	43
18	9.59 720	29	9.63 414	35	0.36 586	9.96 305	5	42
19	9.59 749	29	9.63 449	35	0.36 551	9.96 300	6	41
20	9.59 778	30	9.63 484	35	0.36 516	9.96 294	5	40
21	9.59 808	29	9.63 519	34	0.36 481	9.96 289	5	39
22	9.59 837	29	9.63 553	35	0.36 447	9.96 284	6	38
23	9.59 866	29	9.63 588	35	0.36 412	9.96 278	5	37
24	9.59 895	29	9.63 623	34	0.36 377	9.96 273	6	36
25	9.59 924	30	9.63 657	35	0.36 343	9.96 267	5	35
26	9.59 954	29	9.63 692	34	0.36 308	9.96 262	6	34
27	9.59 983	29	9.63 726	35	0.36 274	9.96 256	5	33
28	9.60 012	29	9.63 761	35	0.36 239	9.96 251	6	32
29	9.60 041	29	9.63 796	34	0.36 204	9.96 245	5	31
30	9.60 070	29	9.63 830	35	0.36 170	9.96 240	6	30
31	9.60 099	29	9.63 865	34	0.36 135	9.96 234	5	29
32	9.60 128	29	9.63 899	35	0.36 101	9.96 229	6	28
33	9.60 157	29	9.63 934	34	0.36 066	9.96 223	5	27
34	9.60 186	29	9.63 968	35	0.36 032	9.96 218	6	26
35	9.60 215	29	9.64 003	34	0.35 997	9.96 212	5	25
36	9.60 244	29	9.64 037	35	0.35 963	9.96 207	6	24
37	9.60 273	29	9.64 072	34	0.35 928	9.96 201	5	23
38	9.60 302	29	9.64 106	34	0.35 894	9.96 196	6	22
39	9.60 331	28	9.64 140	35	0.35 860	9.96 190	5	21
40	9.60 359	29	9.64 175	34	0.35 825	9.96 185	6	20
41	9.60 388	29	9.64 209	34	0.35 791	9.96 179	5	19
42	9.60 417	29	9.64 243	35	0.35 757	9.96 174	6	18
43	9.60 446	28	9.64 278	34	0.35 722	9.96 168	6	17
44	9.60 474	29	9.64 312	34	0.35 688	9.96 162	5	16
45	9.60 503	29	9.64 346	35	0.35 654	9.96 157	6	15
46	9.60 532	29	9.64 381	34	0.35 619	9.96 151	5	14
47	9.60 561	28	9.64 415	34	0.35 585	9.96 146	6	13
48	9.60 589	29	9.64 449	34	0.35 551	9.96 140	5	12
49	9.60 618	28	9.64 483	34	0.35 517	9.96 135	6	11
50	9.60 646	29	9.64 517	35	0.35 483	9.96 129	6	10
51	9.60 675	29	9.64 552	34	0.35 448	9.96 123	5	9
52	9.60 704	28	9.64 586	34	0.35 414	9.96 118	6	8
53	9.60 732	29	9.64 620	34	0.35 380	9.96 112	5	7
54	9.60 761	28	9.64 654	34	0.35 346	9.96 107	6	6
55	9.60 789	29	9.64 688	34	0.35 312	9.96 101	6	5
56	9.60 818	28	9.64 722	34	0.35 278	9.96 095	5	4
57	9.60 846	29	9.64 756	34	0.35 244	9.96 090	6	3
58	9.60 875	28	9.64 790	34	0.35 210	9.96 084	5	2
59	9.60 903	28	9.64 824	34	0.35 176	9.96 079	6	1
60	9.60 931		9.64 858		0.35 142	9.96 073		0
	L Cos	d	L Cot	c d	L Tan	L Sin	d	′

Prop. Parts

	36	35	34
2	7.2	7.0	6.8
3	10.8	10.5	10.2
4	14.4	14.0	13.6
5	18.0	17.5	17.0
6	21.6	21.0	20.4
7	25.2	24.5	23.8
8	28.8	28.0	27.2
9	32.4	31.5	30.6

	30	29	28
2	6.0	5.8	5.6
3	9.0	8.7	8.4
4	12.0	11.6	11.2
5	15.0	14.5	14.0
6	18.0	17.4	16.8
7	21.0	20.3	19.6
8	24.0	23.2	22.4
9	27.0	26.1	25.2

	5	5
2	1.2	1.0
3	1.8	1.5
4	2.4	2.0
5	3.0	2.5
6	3.6	3.0
7	4.2	3.5
8	4.8	4.0
9	5.4	4.5

′	L Sin	d	L Tan	c d	L Cot	L Cos	d		′
0	9.60 931	29	9.64 858	34	0.35 142	9.96 073	6	60	
1	9.60 960	28	9.64 892	34	0.35 108	9.96 067	5	59	
2	9.60 988	28	9.64 926	34	0.35 074	9.96 062	6	58	
3	9.61 016	29	9.64 960	34	0.35 040	9.96 056	6	57	
4	9.61 045	28	9.64 994	34	0.35 006	9.96 050	5	56	
5	9.61 073	28	9.65 028	34	0.34 972	9.96 045	6	55	
6	9.61 101	28	9.65 062	34	0.34 938	9.96 039	5	54	
7	9.61 129	29	9.65 096	34	0.34 904	9.96 034	6	53	
8	9.61 158	28	9.65 130	34	0.34 870	9.96 028	6	52	
9	9.61 186	28	9.65 164	33	0.34 836	9.96 022	5	51	
10	9.61 214	28	9.65 197	34	0.34 803	9.96 017	6	50	
11	9.61 242	28	9.65 231	34	0.34 769	9.96 011	6	49	
12	9.61 270	28	9.65 265	34	0.34 735	9.96 005	5	48	
13	9.61 298	28	9.65 299	34	0.34 701	9.96 000	6	47	
14	9.61 326	28	9.65 333	33	0.34 667	9.95 994	6	46	
15	9.61 354	28	9.65 366	34	0.34 634	9.95 988	6	45	
16	9.61 382	29	9.65 400	34	0.34 600	9.95 982	5	44	
17	9.61 411	27	9.65 434	34	0.34 566	9.95 977	6	43	
18	9.61 438	28	9.65 467	33	0.34 533	9.95 971	6	42	
19	9.61 466	28	9.65 501	34	0.34 499	9.95 965	5	41	
20	9.61 494	28	9.65 535	33	0.34 465	9.95 960	6	40	
21	9.61 522	28	9.65 568	34	0.34 432	9.95 954	6	39	
22	9.61 550	28	9.65 602	34	0.34 398	9.95 948	6	38	
23	9.61 578	28	9.65 636	33	0.34 364	9.95 942	5	37	
24	9.61 606	28	9.65 669	34	0.34 331	9.95 937	6	36	
25	9.61 634	28	9.65 703	33	0.34 297	9.95 931	6	35	
26	9.61 662	27	9.65 736	34	0.34 264	9.95 925	5	34	
27	9.61 689	28	9.65 770	34	0.34 230	9.95 920	6	33	
28	9.61 717	28	9.65 803	33	0.34 197	9.95 914	6	32	
29	9.61 745	28	9.65 837	34	0.34 163	9.95 908	6	31	
30	9.61 773	27	9.65 870	33	0.34 130	9.95 902	5	30	
31	9.61 800	28	9.65 904	34	0.34 096	9.95 897	6	29	
32	9.61 828	28	9.65 937	33	0.34 063	9.95 891	6	28	
33	9.61 856	27	9.65 971	34	0.34 029	9.95 885	6	27	
34	9.61 883	28	9.66 004	33	0.33 996	9.95 879	6	26	
35	9.61 911	28	9.66 038	33	0.33 962	9.95 873	5	25	
36	9.61 939	27	9.66 071	33	0.33 929	9.95 868	6	24	
37	9.61 966	28	9.66 104	33	0.33 896	9.95 862	6	23	
38	9.61 994	27	9.66 138	34	0.33 862	9.95 856	6	22	
39	9.62 021	28	9.66 171	33	0.33 829	9.95 850	6	21	
40	9.62 049	27	9.66 204	33	0.33 796	9.95 844	5	20	
41	9.62 076	28	9.66 238	34	0.33 762	9.95 839	6	19	
42	9.62 104	27	9.66 271	33	0.33 729	9.95 833	6	18	
43	9.62 131	28	9.66 304	33	0.33 696	9.95 827	6	17	
44	9.62 159	27	9.66 337	33	0.33 663	9.95 821	6	16	
45	9.62 186	28	9.66 371	33	0.33 629	9.95 815	5	15	
46	9.62 214	27	9.66 404	33	0.33 596	9.95 810	6	14	
47	9.62 241	27	9.66 437	33	0.33 563	9.95 804	6	13	
48	9.62 268	28	9.66 470	33	0.33 530	9.95 798	6	12	
49	9.62 296	27	9.66 503	34	0.33 497	9.95 792	6	11	
50	9.62 323	27	9.66 537	33	0.33 463	9.95 786	6	10	
51	9.62 350	27	9.66 570	33	0.33 430	9.95 780	5	9	
52	9.62 377	28	9.66 603	33	0.33 397	9.95 775	6	8	
53	9.62 405	27	9.66 636	33	0.33 364	9.95 769	6	7	
54	9.62 432	27	9.66 669	33	0.33 331	9.95 763	6	6	
55	9.62 459	27	9.66 702	33	0.33 298	9.95 757	6	5	
56	9.62 486	27	9.66 735	33	0.33 265	9.95 751	6	4	
57	9.62 513	28	9.66 768	33	0.33 232	9.95 745	6	3	
58	9.62 541	27	9.66 801	33	0.33 199	9.95 739	6	2	
59	9.62 568	27	9.66 834	33	0.33 166	9.95 733	6	1	
60	9.62 595		9.66 867		0.33 133	9.95 728	5	0	
	L Cos	d	L Cot	c d	L Tan	L Sin	d	′	

Prop. Parts

	34	33	29
2	6.8	6.6	5.8
3	10.2	9.9	8.7
4	13.6	13.2	11.6
5	17.0	16.5	14.5
6	20.4	19.8	17.4
7	23.8	23.1	20.3
8	27.2	26.4	23.2
9	30.6	29.7	26.1

	28	27
2	5.6	5.4
3	8.4	8.1
4	11.2	10.8
5	14.0	13.5
6	16.8	16.2
7	19.6	18.9
8	22.4	21.6
9	25.2	24.3

	6	5
2	1.2	1.0
3	1.8	1.5
4	2.4	2.0
5	3.0	2.5
6	3.6	3.0
7	4.2	3.5
8	4.8	4.0
9	5.4	4.5

′	L Sin	d	L Tan	c d	L Cot	L Cos	d		Prop. Parts
0	9.62 595		9.66 867		0.33 133	9.95 728		60	
1	9.62 622	27	9.66 900	33	0.33 100	9.95 722	6	59	
2	9.62 649	27	9.66 933	33	0.33 067	9.95 716	6	58	
3	9.62 676	27	9.66 966	33	0.33 034	9.95 710	6	57	
4	9.62 703	27	9.66 999	33	0.33 001	9.95 704	6	56	
5	9.62 730	27	9.67 032	33	0.32 968	9.95 698	6	55	
6	9.62 757	27	9.67 065	33	0.32 935	9.95 692	6	54	
7	9.62 784	27	9.67 098	33	0.32 902	9.95 686	6	53	
8	9.62 811	27	9.67 131	33	0.32 869	9.95 680	6	52	
9	9.62 838	27	9.67 163	32	0.32 837	9.95 674	6	51	
10	9.62 865	27	9.67 196	33	0.32 804	9.95 668	5	50	
11	9.62 892	26	9.67 229	33	0.32 771	9.95 663	6	49	
12	9.62 918	27	9.67 262	33	0.32 738	9.95 657	6	48	
13	9.62 945	27	9.67 295	33	0.32 705	9.95 651	6	47	
14	9.62 972	27	9.67 327	32	0.32 673	9.95 645	6	46	
15	9.62 999	27	9.67 360	33	0.32 640	9.95 639	6	45	
16	9.63 026	26	9.67 393	33	0.32 607	9.95 633	6	44	
17	9.63 052	27	9.67 426	33	0.32 574	9.95 627	6	43	
18	9.63 079	27	9.67 458	32	0.32 542	9.95 621	6	42	
19	9.63 106	27	9.67 491	33	0.32 509	9.95 615	6	41	
20	9.63 133	26	9.67 524	33	0.32 476	9.95 609	6	40	
21	9.63 159	27	9.67 556	32	0.32 444	9.95 603	6	39	
22	9.63 186	27	9.67 589	33	0.32 411	9.95 597	6	38	
23	9.63 213	26	9.67 622	33	0.32 378	9.95 591	6	37	
24	9.63 239	27	9.67 654	32	0.32 346	9.95 585	6	36	
25	9.63 266	26	9.67 687	33	0.32 313	9.95 579	6	35	
26	9.63 292	27	9.67 719	32	0.32 281	9.95 573	6	34	
27	9.63 319	26	9.67 752	33	0.32 248	9.95 567	6	33	
28	9.63 345	27	9.67 785	33	0.32 215	9.95 561	6	32	
29	9.63 372	26	9.67 817	32	0.32 183	9.95 555	6	31	
30	9.63 398	27	9.67 850	33	0.32 150	9.95 549	6	30	
31	9.63 425	26	9.67 882	32	0.32 118	9.95 543	6	29	
32	9.63 451	27	9.67 915	33	0.32 085	9.95 537	6	28	
33	9.63 478	26	9.67 947	32	0.32 053	9.95 531	6	27	
34	9.63 504	27	9.67 980	33	0.32 020	9.95 525	6	26	
35	9.63 531	26	9.68 012	32	0.31 988	9.95 519	6	25	
36	9.63 557	26	9.68 044	33	0.31 956	9.95 513	6	24	
37	9.63 583	27	9.68 077	32	0.31 923	9.95 507	7	23	
38	9.63 610	26	9.68 109	33	0.31 891	9.95 500	6	22	
39	9.63 636	26	9.68 142	32	0.31 858	9.95 494	6	21	
40	9.63 662	27	9.68 174	32	0.31 826	9.95 488	6	20	
41	9.63 689	26	9.68 206	33	0.31 794	9.95 482	6	19	
42	9.63 715	26	9.68 239	32	0.31 761	9.95 476	6	18	
43	9.63 741	26	9.68 271	32	0.31 729	9.95 470	6	17	
44	9.63 767	27	9.68 303	33	0.31 697	9.95 464	6	16	
45	9.63 794	26	9.68 336	32	0.31 664	9.95 458	6	15	
46	9.63 820	26	9.68 368	32	0.31 632	9.95 452	6	14	
47	9.63 846	26	9.68 400	32	0.31 600	9.95 446	6	13	
48	9.63 872	26	9.68 432	33	0.31 568	9.95 440	6	12	
49	9.63 898	26	9.68 465	32	0.31 535	9.95 434	7	11	
50	9.63 924	26	9.68 497	32	0.31 503	9.95 427	6	10	
51	9.63 950	26	9.68 529	32	0.31 471	9.95 421	6	9	
52	9.63 976	26	9.68 561	32	0.31 439	9.95 415	6	8	
53	9.64 002	26	9.68 593	33	0.31 407	9.95 409	6	7	
54	9.64 028	26	9.68 626	32	0.31 374	9.95 403	6	6	
55	9.64 054	26	9.68 658	32	0.31 342	9.95 397	6	5	
56	9.64 080	26	9.68 690	32	0.31 310	9.95 391	7	4	
57	9.64 106	26	9.68 722	32	0.31 278	9.95 384	6	3	
58	9.64 132	26	9.68 754	32	0.31 246	9.95 378	6	2	
59	9.64 158	26	9.68 786	32	0.31 214	9.95 372	6	1	
60	9.64 184		9.68 818		0.31 182	9.95 366		0	
	L Cos	d	L Cot	c d	L Tan	L Sin	d	′	

	33	32	27
2	6.6	6.4	5.4
3	9.9	9.6	8.1
4	13.2	12.8	10.8
5	16.5	16.0	13.5
6	19.8	19.2	16.2
7	23.1	22.4	18.9
8	26.4	25.6	21.6
9	29.7	28.8	24.3

	26	7
2	5.2	1.4
3	7.8	2.1
4	10.4	2.8
5	13.0	3.5
6	15.6	4.2
7	18.2	4.9
8	20.8	5.6
9	23.4	6.3

	6	5
2	1.2	1.0
3	1.8	1.5
4	2.4	2.0
5	3.0	2.5
6	3.6	3.0
7	4.2	3.5
8	4.8	4.0
9	5.4	4.5

′	L Sin	d	L Tan	c d	L Cot	L Cos	d	
0	9.64 181	26	9.68 818	32	0.31 182	9.95 366	6	60
1	9.64 210	26	9.68 850	32	0.31 150	9.95 360	6	59
2	9.64 236	26	9.68 882	32	0.31 118	9.95 354	6	58
3	9.64 262	26	9.68 914	32	0.31 086	9.95 348	7	57
4	9.64 288	25	9.68 946	32	0.31 054	9.95 341	6	56
5	9.64 313	26	9.68 978	32	0.31 022	9.95 335	6	55
6	9.64 339	26	9.69 010	32	0.30 990	9.95 329	6	54
7	9.64 365	26	9.69 042	32	0.30 958	9.95 323	6	53
8	9.64 391	26	9.69 074	32	0.30 926	9.95 317	7	52
9	9.64 417	25	9.69 106	32	0.30 894	9.95 310	6	51
10	9.64 442	26	9.69 138	32	0.30 862	9.95 304	6	50
11	9.64 468	26	9.69 170	32	0.30 830	9.95 298	6	49
12	9.64 494	25	9.69 202	32	0.30 798	9.95 292	6	48
13	9.64 519	26	9.69 234	32	0.30 766	9.95 286	7	47
14	9.64 545	26	9.69 266	32	0.30 734	9.95 279	6	46
15	9.64 571	25	9.69 298	31	0.30 702	9.95 273	6	45
16	9.64 596	26	9.69 329	32	0.30 671	9.95 267	6	44
17	9.64 622	25	9.69 361	32	0.30 639	9.95 261	7	43
18	9.64 647	26	9.69 393	32	0.30 607	9.95 254	6	42
19	9.64 673	25	9.69 425	32	0.30 575	9.95 248	6	41
20	9.64 698	26	9.69 457	31	0.30 543	9.95 242	6	40
21	9.64 724	25	9.69 488	32	0.30 512	9.95 236	7	39
22	9.64 749	26	9.69 520	32	0.30 480	9.95 229	6	38
23	9.64 775	25	9.69 552	32	0.30 448	9.95 223	6	37
24	9.64 800	26	9.69 584	31	0.30 416	9.95 217	6	36
25	9.64 826	25	9.69 615	32	0.30 385	9.95 211	6	35
26	9.64 851	26	9.69 647	32	0.30 353	9.95 204	7	34
27	9.64 877	25	9.69 679	31	0.30 321	9.95 198	6	33
28	9.64 902	25	9.69 710	32	0.30 290	9.95 192	6	32
29	9.64 927	26	9.69 742	32	0.30 258	9.95 185	7	31
30	9.64 953	25	9.69 774	31	0.30 226	9.95 179	6	30
31	9.64 978	25	9.69 805	32	0.30 195	9.95 173	6	29
32	9.65 003	26	9.69 837	31	0.30 163	9.95 167	6	28
33	9.65 029	25	9.69 868	32	0.30 132	9.95 160	7	27
34	9.65 054	25	9.69 900	32	0.30 100	9.95 154	6	26
35	9.65 079	25	9.69 932	31	0.30 068	9.95 148	6	25
36	9.65 104	26	9.69 963	32	0.30 037	9.95 141	7	24
37	9.65 130	25	9.69 995	31	0.30 005	9.95 135	6	23
38	9.65 155	25	9.70 026	32	0.29 974	9.95 129	6	22
39	9.65 180	25	9.70 058	31	0.29 942	9.95 122	7	21
40	9.65 205	25	9.70 089	32	0.29 911	9.95 116	6	20
41	9.65 230	25	9.70 121	31	0.29 879	9.95 110	6	19
42	9.65 255	26	9.70 152	32	0.29 848	9.95 103	7	18
43	9.65 281	25	9.70 184	31	0.29 816	9.95 097	6	17
44	9.65 306	25	9.70 215	32	0.29 785	9.95 090	7	16
45	9.65 331	25	9.70 247	31	0.29 753	9.95 084	6	15
46	9.65 356	25	9.70 278	31	0.29 722	9.95 078	6	14
47	9.65 381	25	9.70 309	32	0.29 691	9.95 071	7	13
48	9.65 406	25	9.70 341	31	0.29 659	9.95 065	6	12
49	9.65 431	25	9.70 372	32	0.29 628	9.95 059	6	11
50	9.65 456	25	9.70 404	31	0.29 596	9.95 052	7	10
51	9.65 481	25	9.70 435	31	0.29 565	9.95 046	6	9
52	9.65 506	25	9.70 466	32	0.29 534	9.95 039	7	8
53	9.65 531	25	9.70 498	31	0.29 502	9.95 033	6	7
54	9.65 556	24	9.70 529	31	0.29 471	9.95 027	6	6
55	9.65 580	25	9.70 560	32	0.29 440	9.95 020	7	5
56	9.65 605	25	9.70 592	31	0.29 408	9.95 014	6	4
57	9.65 630	25	9.70 623	31	0.29 377	9.95 007	7	3
58	9.65 655	25	9.70 654	31	0.29 346	9.95 001	6	2
59	9.65 680	25	9.70 685	32	0.29 315	9.94 995	6	1
60	9.65 705		9.70 717		0.29 283	9.94 988	7	0
	L Cos	d	L Cot	c d	L Tan	L Sin	d	′

Prop. Parts

	32	31	26
2	6.4	6.2	5.2
3	9.6	9.3	7.8
4	12.8	12.4	10.4
5	16.0	15.5	13.0
6	19.2	18.6	15.6
7	22.4	21.7	18.2
8	25.6	24.8	20.8
9	28.8	27.9	23.4

	25	24
2	5.0	4.8
3	7.5	7.2
4	10.0	9.6
5	12.5	12.0
6	15.0	14.4
7	17.5	16.8
8	20.0	19.2
9	22.5	21.6

	7	6
2	1.4	1.2
3	2.1	1.8
4	2.8	2.4
5	3.5	3.0
6	4.2	3.6
7	4.9	4.2
8	5.6	4.8
9	6.3	5.4

′	L Sin	d	L Tan	c d	L Cot	L Cos	d	
0	9.65 705	24	9.70 717	31	0.29 283	9.94 988	6	60
1	9.65 729	25	9.70 748	31	0.29 252	9.94 982	7	59
2	9.65 754	25	9.70 779	31	0.29 221	9.94 975	6	58
3	9.65 779	25	9.70 810	31	0.29 190	9.94 969	7	57
4	9.65 804	24	9.70 841	32	0.29 159	9.94 962	6	56
5	9.65 828	25	9.70 873	31	0.29 127	9.94 956	7	55
6	9.65 853	25	9.70 904	31	0.29 096	9.94 949	6	54
7	9.65 878	24	9.70 935	31	0.29 065	9.94 943	7	53
8	9.65 902	25	9.70 966	31	0.29 034	9.94 936	6	52
9	9.65 927	25	9.70 997	31	0.29 003	9.94 930	7	51
10	9.65 952	24	9.71 028	31	0.28 972	9.94 923	6	50
11	9.65 976	25	9.71 059	31	0.28 941	9.94 917	6	49
12	9.66 001	24	9.71 090	31	0.28 910	9.94 911	7	48
13	9.66 025	25	9.71 121	32	0.28 879	9.94 904	6	47
14	9.66 050	25	9.71 153	31	0.28 847	9.94 898	7	46
15	9.66 075	24	9.71 184	31	0.28 816	9.94 891	6	45
16	9.66 099	25	9.71 215	31	0.28 785	9.94 885	7	44
17	9.66 124	24	9.71 246	31	0.28 754	9.94 878	7	43
18	9.66 148	25	9.71 277	31	0.28 723	9.94 871	6	42
19	9.66 173	24	9.71 308	31	0.28 692	9.94 865	7	41
20	9.66 197	24	9.71 339	31	0.28 661	9.94 858	6	40
21	9.66 221	25	9.71 370	31	0.28 630	9.94 852	7	39
22	9.66 246	24	9.71 401	30	0.28 599	9.94 845	6	38
23	9.66 270	25	9.71 431	31	0.28 569	9.94 839	7	37
24	9.66 295	24	9.71 462	31	0.28 538	9.94 832	6	36
25	9.66 319	24	9.71 493	31	0.28 507	9.94 826	7	35
26	9.66 343	25	9.71 524	31	0.28 476	9.94 819	6	34
27	9.66 368	24	9.71 555	31	0.28 445	9.94 813	7	33
28	9.66 392	24	9.71 586	31	0.28 414	9.94 806	7	32
29	9.66 416	25	9.71 617	31	0.28 383	9.94 799	6	31
30	9.66 441	24	9.71 648	31	0.28 352	9.94 793	7	30
31	9.66 465	24	9.71 679	30	0.28 321	9.94 786	6	29
32	9.66 489	24	9.71 709	31	0.28 291	9.94 780	7	28
33	9.66 513	24	9.71 740	31	0.28 260	9.94 773	6	27
34	9.66 537	25	9.71 771	31	0.28 229	9.94 767	7	26
35	9.66 562	24	9.71 802	31	0.28 198	9.94 760	7	25
36	9.66 586	24	9.71 833	30	0.28 167	9.94 753	6	24
37	9.66 610	24	9.71 863	31	0.28 137	9.94 747	7	23
38	9.66 634	24	9.71 894	31	0.28 106	9.94 740	6	22
39	9.66 658	24	9.71 925	30	0.28 075	9.94 734	7	21
40	9.66 682	24	9.71 955	31	0.28 045	9.94 727	7	20
41	9.66 706	25	9.71 986	31	0.28 014	9.94 720	6	19
42	9.66 731	24	9.72 017	31	0.27 983	9.94 714	7	18
43	9.66 755	24	9.72 048	30	0.27 952	9.94 707	7	17
44	9.66 779	24	9.72 078	31	0.27 922	9.94 700	6	16
45	9.66 803	24	9.72 109	31	0.27 891	9.94 694	7	15
46	9.66 827	24	9.72 140	30	0.27 860	9.94 687	7	14
47	9.66 851	24	9.72 170	31	0.27 830	9.94 680	6	13
48	9.66 875	24	9.72 201	30	0.27 799	9.94 674	7	12
49	9.66 899	23	9.72 231	31	0.27 769	9.94 667	7	11
50	9.66 922	24	9.72 262	31	0.27 738	9.94 660	6	10
51	9.66 946	24	9.72 293	30	0.27 707	9.94 654	7	9
52	9.66 970	24	9.72 323	31	0.27 677	9.94 647	7	8
53	9.66 994	24	9.72 354	30	0.27 646	9.94 640	6	7
54	9.67 018	24	9.72 384	31	0.27 616	9.94 634	7	6
55	9.67 042	24	9.72 415	30	0.27 585	9.94 627	7	5
56	9.67 066	24	9.72 445	31	0.27 555	9.94 620	6	4
57	9.67 090	24	9.72 476	30	0.27 524	9.94 614	7	3
58	9.67 113	23	9.72 506	31	0.27 494	9.94 607	7	2
59	9.67 137	24	9.72 537	30	0.27 463	9.94 600	7	1
60	9.67 161		9.72 567		0.27 433	9.94 593		0
	L Cos	d	L Cot	c d	L Tan	L Sin	d	′

Prop. Parts

	32	31	30
2	6.4	6.2	6.0
3	9.6	9.3	9.0
4	12.8	12.4	12.0
5	16.0	15.5	15.0
6	19.2	18.6	18.0
7	22.4	21.7	21.0
8	25.6	24.8	24.0
9	28.8	27.9	27.0

	25	24	23
2	5.0	4.8	4.6
3	7.5	7.2	6.9
4	10.0	9.6	9.2
5	12.5	12.0	11.5
6	15.0	14.4	13.8
7	17.5	16.8	16.1
8	20.0	19.2	18.4
9	22.5	21.6	20.7

	7	6
2	1.4	1.2
3	2.1	1.8
4	2.8	2.4
5	3.5	3.0
6	4.2	3.6
7	4.9	4.2
8	5.6	4.8
9	6.3	5.4

′	L Sin	d	L Tan	c d	L Cot	L Cos	d		Prop. Parts
0	9.67 161	24	9.72 567	31	0.27 433	9.94 593	6	60	
1	9.67 185	23	9.72 598	30	0.27 402	9.94 587	7	59	
2	9.67 208	24	9.72 628	31	0.27 372	9.94 580	7	58	
3	9.67 232	24	9.72 659	30	0.27 341	9.94 573	6	57	
4	9.67 256	24	9.72 689	31	0.27 311	9.94 567	7	56	
5	9.67 280	23	9.72 720	30	0.27 280	9.94 560	7	55	
6	9.67 303	24	9.72 750	30	0.27 250	9.94 553	7	54	
7	9.67 327	23	9.72 780	31	0.27 220	9.94 546	6	53	
8	9.67 350	24	9.72 811	30	0.27 189	9.94 540	7	52	
9	9.67 374	24	9.72 841	31	0.27 159	9.94 533	7	51	
10	9.67 398	23	9.72 872	30	0.27 128	9.94 526	7	50	
11	9.67 421	24	9.72 902	30	0.27 098	9.94 519	6	49	
12	9.67 445	23	9.72 932	31	0.27 068	9.94 513	7	48	
13	9.67 468	24	9.72 963	30	0.27 037	9.94 506	7	47	31 30 29
14	9.67 492	23	9.72 993	30	0.27 007	9.94 499	7	46	2 6.2 6.0 5.8
15	9.67 515	24	9.73 023	31	0.26 977	9.94 492	7	45	3 9.3 9.0 8.7
16	9.67 539	23	9.73 054	30	0.26 946	9.94 485	7	44	4 12.4 12.0 11.6
17	9.67 562	24	9.73 084	30	0.26 916	9.94 479	6	43	5 15.5 15.0 14.5
18	9.67 586	23	9.73 114	30	0.26 886	9.94 472	7	42	6 18.6 18.0 17.4
19	9.67 609	24	9.73 144	31	0.26 856	9.94 465	7	41	7 21.7 21.0 20.3
20	9.67 633	23	9.73 175	30	0.26 825	9.94 458	7	40	8 24.8 24.0 23.2
21	9.67 656	24	9.73 205	30	0.26 795	9.94 451	7	39	9 27.9 27.0 26.1
22	9.67 680	23	9.73 235	30	0.26 765	9.94 445	6	38	
23	9.67 703	23	9.73 265	30	0.26 735	9.94 438	7	37	
24	9.67 726	24	9.73 295	31	0.26 705	9.94 431	7	36	
25	9.67 750	23	9.73 326	30	0.26 674	9.94 424	7	35	
26	9.67 773	23	9.73 356	30	0.26 644	9.94 417	7	34	24 23 22
27	9.67 796	24	9.73 386	30	0.26 614	9.94 410	6	33	2 4.8 4.6 4.4
28	9.67 820	23	9.73 416	30	0.26 584	9.94 404	7	32	3 7.2 6.9 6.6
29	9.67 843	23	9.73 446	30	0.26 554	9.94 397	7	31	4 9.6 9.2 8.8
30	9.67 866	24	9.73 476	31	0.26 524	9.94 390	7	30	5 12.0 11.5 11.0
31	9.67 890	23	9.73 507	30	0.26 493	9.94 383	7	29	6 14.4 13.8 13.2
32	9.67 913	23	9.73 537	30	0.26 463	9.94 376	7	28	7 16.8 16.1 15.4
33	9.67 936	23	9.73 567	30	0.26 433	9.94 369	7	27	8 19.2 18.4 17.6
34	9.67 959	23	9.73 597	30	0.26 403	9.94 362	7	26	9 21.6 20.7 19.8
35	9.67 982	24	9.73 627	30	0.26 373	9.94 355	6	25	
36	9.68 006	23	9.73 657	30	0.26 343	9.94 349	7	24	
37	9.68 029	23	9.73 687	30	0.26 313	9.94 342	7	23	
38	9.68 052	23	9.73 717	30	0.26 283	9.94 335	7	22	
39	9.68 075	23	9.73 747	30	0.26 253	9.94 328	7	21	7 6
40	9.68 098	23	9.73 777	30	0.26 223	9.94 321	7	20	2 1.4 1.2
41	9.68 121	23	9.73 807	30	0.26 193	9.94 314	7	19	3 2.1 1.8
42	9.68 144	23	9.73 837	30	0.26 163	9.94 307	7	18	4 2.8 2.4
43	9.68 167	23	9.73 867	30	0.26 133	9.94 300	7	17	5 3.5 3.0
44	9.68 190	23	9.73 897	30	0.26 103	9.94 293	7	16	6 4.2 3.6
45	9.68 213	24	9.73 927	30	0.26 073	9.94 286	7	15	7 4.9 4.2
46	9.68 237	23	9.73 957	30	0.26 043	9.94 279	6	14	8 5.6 4.8
47	9.68 260	23	9.73 987	30	0.26 013	9.94 273	7	13	9 6.3 5.4
48	9.68 283	22	9.74 017	30	0.25 983	9.94 266	7	12	
49	9.68 305	23	9.74 047	30	0.25 953	9.94 259	7	11	
50	9.68 328	23	9.74 077	30	0.25 923	9.94 252	7	10	
51	9.68 351	23	9.74 107	30	0.25 893	9.94 245	7	9	
52	9.68 374	23	9.74 137	30	0.25 863	9.94 238	7	8	
53	9.68 397	23	9.74 166	29	0.25 834	9.94 231	7	7	
54	9.68 420	23	9.74 196	30	0.25 804	9.94 224	7	6	
55	9.68 443	23	9.74 226	30	0.25 774	9.94 217	7	5	
56	9.68 466	23	9.74 256	30	0.25 744	9.94 210	7	4	
57	9.68 489	23	9.74 286	30	0.25 714	9.94 203	7	3	
58	9.68 512	22	9.74 316	30	0.25 684	9.94 196	7	2	
59	9.68 534	23	9.74 345	29	0.25 655	9.94 189	7	1	
60	9.68 557		9.74 375	30	0.25 625	9.94 182		0	
	L Cos	d	L Cot	c d	L Tan	L Sin	d	′	

′	L Sin	d	L Tan	c d	L Cot	L Cos	d		Prop. Parts
0	9.68 557		9.74 375		0.25 625	9.94 182		**60**	
1	9.68 580	23	9.74 405	30	0.25 595	9.94 175	7	59	
2	9.68 603	23	9.74 435	30	0.25 565	9.94 168	7	58	
3	9.68 625	22	9.74 465	30	0.25 535	9.94 161	7	57	
4	9.68 648	23	9.74 494	29	0.25 506	9.94 154	7	56	
		23		30			7		
5	9.68 671		9.74 524		0.25 476	9.94 147		**55**	
6	9.68 694	23	9.74 554	30	0.25 446	9.94 140	7	54	
7	9.68 716	22	9.74 583	29	0.25 417	9.94 133	7	53	
8	9.68 739	23	9.74 613	30	0.25 387	9.94 126	7	52	
9	9.68 762	23	9.74 643	30	0.25 357	9.94 119	7	51	
		22		30			7		
10	9.68 784		9.74 673		0.25 327	9.94 112		**50**	
11	9.68 807	23	9.74 702	29	0.25 298	9.94 105	7	49	
12	9.68 829	22	9.74 732	30	0.25 268	9.94 098	7	48	
13	9.68 852	23	9.74 762	30	0.25 238	9.94 090	8	47	
14	9.68 875	23	9.74 791	29	0.25 209	9.94 083	7	46	
		22		30			7		
15	9.68 897		9.74 821		0.25 179	9.94 076		**45**	
16	9.68 920	23	9.74 851	30	0.25 149	9.94 069	7	44	
17	9.68 942	22	9.74 880	29	0.25 120	9.94 062	7	43	
18	9.68 965	23	9.74 910	30	0.25 090	9.94 055	7	42	
19	9.68 987	22	9.74 939	29	0.25 061	9.94 048	7	41	
		23		30			7		
20	9.69 010		9.74 969		0.25 031	9.94 041		**40**	
21	9.69 032	22	9.74 998	29	0.25 002	9.94 034	7	39	
22	9.69 055	23	9.75 028	30	0.24 972	9.94 027	7	38	
23	9.69 077	22	9.75 058	30	0.24 942	9.94 020	7	37	
24	9.69 100	23	9.75 087	29	0.24 913	9.94 012	8	36	
		22		30			7		
25	9.69 122		9.75 117		0.24 883	9.94 005		**35**	
26	9.69 144	22	9.75 146	29	0.24 854	9.93 998	7	34	
27	9.69 167	23	9.75 176	30	0.24 824	9.93 991	7	33	
28	9.69 189	22	9.75 205	29	0.24 795	9.93 984	7	32	
29	9.69 212	23	9.75 235	30	0.24 765	9.93 977	7	31	
		22		29			7		
30	9.69 234		9.75 264		0.24 736	9.93 970		**30**	
31	9.69 256	22	9.75 294	30	0.24 706	9.93 963	7	29	
32	9.69 279	23	9.75 323	29	0.24 677	9.93 955	8	28	
33	9.69 301	22	9.75 353	30	0.24 647	9.93 948	7	27	
34	9.69 323	22	9.75 382	29	0.24 618	9.93 941	7	26	
		22		29			7		
35	9.69 345		9.75 411		0.24 589	9.93 934		**25**	
36	9.69 368	23	9.75 441	30	0.24 559	9.93 927	7	24	
37	9.69 390	22	9.75 470	29	0.24 530	9.93 920	7	23	
38	9.69 412	22	9.75 500	30	0.24 500	9.93 912	8	22	
39	9.69 434	22	9.75 529	29	0.24 471	9.93 905	7	21	
		22		29			7		
40	9.69 456		9.75 558		0.24 412	9.93 898		**20**	
41	9.69 479	23	9.75 588	30	0.24 412	9.93 891	7	19	
42	9.69 501	22	9.75 617	29	0.24 383	9.93 884	7	18	
43	9.69 523	22	9.75 647	30	0.24 353	9.93 876	8	17	
44	9.69 545	22	9.75 676	29	0.24 324	9.93 869	7	16	
		22		29			7		
45	9.69 567		9.75 705		0.24 295	9.93 862		**15**	
46	9.69 589	22	9.75 735	30	0.24 265	9.93 855	7	14	
47	9.69 611	22	9.75 764	29	0.24 236	9.93 847	8	13	
48	9.69 633	22	9.75 793	29	0.24 207	9.93 840	7	12	
49	9.69 655	22	9.75 822	29	0.24 178	9.93 833	7	11	
		22		30			7		
50	9.69 677		9.75 852		0.24 148	9.93 826		**10**	
51	9.69 699	22	9.75 881	29	0.24 119	9.93 819	7	9	
52	9.69 721	22	9.75 910	29	0.24 090	9.93 811	8	8	
53	9.69 743	22	9.75 939	29	0.24 061	9.93 804	7	7	
54	9.69 765	22	9.75 969	30	0.24 031	9.93 797	7	6	
		22		29			8		
55	9.69 787		9.75 998		0.24 002	9.93 789		**5**	
56	9.69 809	22	9.76 027	29	0.23 973	9.93 782	7	4	
57	9.69 831	22	9.76 056	29	0.23 944	9.93 775	7	3	
58	9.69 853	22	9.76 086	30	0.23 914	9.93 768	7	2	
59	9.69 875	22	9.76 115	29	0.23 885	9.93 760	8	1	
		22		29			7		
60	9.69 897		9.76 144		0.23 856	9.93 753		**0**	

	30	29	23
2	6.0	5.8	4.6
3	9.0	8.7	6.9
4	12.0	11.6	9.2
5	15.0	14.5	11.5
6	18.0	17.4	13.8
7	21.0	20.3	16.1
8	24.0	23.2	18.4
9	27.0	26.1	20.7

	22	8	7
2	4.4	1.6	1.4
3	6.6	2.4	2.1
4	8.8	3.2	2.8
5	11.0	4.0	3.5
6	13.2	4.8	4.2
7	15.4	5.6	4.9
8	17.6	6.4	5.6
9	19.8	7.2	6.3

′	L Cos	d	L Cot	c d	L Tan	L Sin	d

′	L Sin	d	L Tan	c d	L Cot	L Cos	d	
0	9.69 897		9.76 144		0.23 856	9.93 753		**60**
1	9.69 919	22	9.76 173	29	0.23 827	9.93 746	7	59
2	9.69 941	22	9.76 202	29	0.23 798	9.93 738	8	58
3	9.69 963	22	9.76 231	29	0.23 769	9.93 731	7	57
4	9.69 984	21	9.76 261	30	0.23 739	9.93 724	7	56
5	9.70 006	22	9.76 290	29	0.23 710	9.93 717	7	**55**
6	9.70 028	22	9.76 319	29	0.23 681	9.93 709	8	54
7	9.70 050	22	9.76 348	29	0.23 652	9.93 702	7	53
8	9.70 072	21	9.76 377	29	0.23 623	9.93 695	7	52
9	9.70 093	22	9.76 406	29	0.23 594	9.93 687	8	51
10	9.70 115	22	9.76 435	29	0.23 565	9.93 680	7	**50**
11	9.70 137	22	9.76 464	29	0.23 536	9.93 673	7	49
12	9.70 159	21	9.76 493	29	0.23 507	9.93 665	8	48
13	9.70 180	22	9.76 522	29	0.23 478	9.93 658	7	47
14	9.70 202	22	9.76 551	29	0.23 449	9.93 650	8	46
15	9.70 224	21	9.76 580	29	0.23 420	9.93 643	7	**45**
16	9.70 245	22	9.76 609	30	0.23 391	9.93 636	7	44
17	9.70 267	21	9.76 639	29	0.23 361	9.93 628	8	43
18	9.70 288	22	9.76 668	29	0.23 332	9.93 621	7	42
19	9.70 310	22	9.76 697	28	0.23 303	9.93 614	7	41
20	9.70 332	21	9.76 725	29	0.23 275	9.93 606	8	**40**
21	9.70 353	22	9.76 754	29	0.23 246	9.93 599	7	39
22	9.70 375	21	9.76 783	29	0.23 217	9.93 591	8	38
23	9.70 396	22	9.76 812	29	0.23 188	9.93 584	7	37
24	9.70 418	21	9.76 841	29	0.23 159	9.93 577	7	36
25	9.70 439	22	9.76 870	29	0.23 130	9.93 569	8	**35**
26	9.70 461	21	9.76 899	29	0.23 101	9.93 562	7	34
27	9.70 482	22	9.76 928	29	0.23 072	9.93 554	8	33
28	9.70 504	21	9.76 957	29	0.23 043	9.93 547	7	32
29	9.70 525	22	9.76 986	29	0.23 014	9.93 539	8	31
30	9.70 547	21	9.77 015	29	0.22 985	9.93 532	7	**30**
31	9.70 568	22	9.77 044	29	0.22 956	9.93 525	7	29
32	9.70 590	21	9.77 073	28	0.22 927	9.93 517	8	28
33	9.70 611	22	9.77 101	29	0.22 899	9.93 510	7	27
34	9.70 633	21	9.77 130	29	0.22 870	9.93 502	8	26
35	9.70 654	21	9.77 159	29	0.22 841	9.93 495	7	**25**
36	9.70 675	22	9.77 188	29	0.22 812	9.93 487	8	24
37	9.70 697	21	9.77 217	29	0.22 783	9.93 480	7	23
38	9.70 718	21	9.77 246	29	0.22 754	9.93 472	8	22
39	9.70 739	22	9.77 274	28	0.22 726	9.93 465	7	21
40	9.70 761	21	9.77 303	29	0.22 697	9.93 457	8	**20**
41	9.70 782	21	9.77 332	29	0.22 668	9.93 450	7	19
42	9.70 803	21	9.77 361	29	0.22 639	9.93 442	8	18
43	9.70 824	22	9.77 390	28	0.22 610	9.93 435	7	17
44	9.70 846	21	9.77 418	29	0.22 582	9.93 427	8	16
45	9.70 867	21	9.77 447	29	0.22 553	9.93 420	7	**15**
46	9.70 888	21	9.77 476	29	0.22 524	9.93 412	8	14
47	9.70 909	22	9.77 505	28	0.22 495	9.93 405	7	13
48	9.70 931	21	9.77 533	29	0.22 467	9.93 397	8	12
49	9.70 952	21	9.77 562	29	0.22 438	9.93 390	7	11
50	9.70 973	21	9.77 591	28	0.22 409	9.93 382	8	**10**
51	9.70 994	21	9.77 619	29	0.22 381	9.93 375	7	9
52	9.71 015	21	9.77 648	29	0.22 352	9.93 367	8	8
53	9.71 036	21	9.77 677	29	0.22 323	9.93 360	7	7
54	9.71 058	22	9.77 706	29	0.22 294	9.93 352	8	6
55	9.71 079	21	9.77 734	28	0.22 266	9.93 344	8	**5**
56	9.71 100	21	9.77 763	29	0.22 237	9.93 337	7	4
57	9.71 121	21	9.77 791	28	0.22 209	9.93 329	8	3
58	9.71 142	21	9.77 820	29	0.22 180	9.93 322	7	2
59	9.71 163	21	9.77 849	29	0.22 151	9.93 314	8	1
60	9.71 184		9.77 877	28	0.22 123	9.93 307	7	**0**
	L Cos	d	L Cot	c d	L Tan	L Sin	d	′

Prop. Parts

	30	29	28
2	6.0	5.8	5.6
3	9.0	8.7	8.4
4	12.0	11.6	11.2
5	15.0	14.5	14.0
6	18.0	17.4	16.8
7	21.0	20.3	19.6
8	24.0	23.2	22.4
9	27.0	26.1	25.2

	22	21
2	4.4	4.2
3	6.6	6.3
4	8.8	8.4
5	11.0	10.5
6	13.2	12.6
7	15.4	14.7
8	17.6	16.8
9	19.8	18.9

	8	7
2	1.6	1.4
3	2.4	2.1
4	3.2	2.8
5	4.0	3.5
6	4.8	4.2
7	5.6	4.9
8	6.4	5.6
9	7.2	6.3

121° 211° 301°

′	L Sin	d	L Tan	c d	L Cot	L Cos	d		Prop. Parts
0	9.71 184	21	9.77 877	29	0.22 123	9.93 307	8	**60**	
1	9.71 205	21	9.77 906	29	0.22 094	9.93 299	8	59	
2	9.71 226	21	9.77 935	28	0.22 065	9.93 291	7	58	
3	9.71 247	21	9.77 963	29	0.22 037	9.93 284	8	57	
4	9.71 268	21	9.77 992	28	0.22 008	9.93 276	7	56	
5	9.71 289	21	9.78 020	29	0.21 980	9.93 269	8	**55**	
6	9.71 310	21	9.78 049	28	0.21 951	9.93 261	8	54	
7	9.71 331	21	9.78 077	29	0.21 923	9.93 253	7	53	
8	9.71 352	21	9.78 106	29	0.21 894	9.93 246	8	52	
9	9.71 373	20	9.78 135	28	0.21 865	9.93 238	8	51	
10	9.71 393	21	9.78 163	29	0.21 837	9.93 230	7	**50**	
11	9.71 414	21	9.78 192	28	0.21 808	9.93 223	8	49	
12	9.71 435	21	9.78 220	29	0.21 780	9.93 215	8	48	
13	9.71 456	21	9.78 249	28	0.21 751	9.93 207	7	47	
14	9.71 477	21	9.78 277	29	0.21 723	9.93 200	8	46	
15	9.71 498	21	9.78 306	28	0.21 694	9.93 192	8	**45**	
16	9.71 519	20	9.78 334	29	0.21 666	9.93 184	7	44	
17	9.71 539	21	9.78 363	28	0.21 637	9.93 177	8	43	**29 28 21**
18	9.71 560	21	9.78 391	28	0.21 609	9.93 169	8	42	2 5.8 5.6 4.2
19	9.71 581	21	9.78 419	29	0.21 581	9.93 161	7	41	3 8.7 8.4 6.3
20	9.71 602	20	9.78 448	28	0.21 552	9.93 154	8	**40**	4 11.6 11.2 8.4
21	9.71 622	21	9.78 476	29	0.21 524	9.93 146	8	39	5 14.5 14.0 10.5
22	9.71 643	21	9.78 505	28	0.21 495	9.93 138	7	38	6 17.4 16.8 12.6
23	9.71 664	21	9.78 533	29	0.21 467	9.93 131	8	37	7 20.3 19.6 14.7
24	9.71 685	20	9.78 562	28	0.21 438	9.93 123	8	36	8 23.2 22.4 16.8
25	9.71 705	21	9.78 590	28	0.21 410	9.93 115	7	**35**	9 26.1 25.2 18.9
26	9.71 726	21	9.78 618	29	0.21 382	9.93 108	8	34	
27	9.71 747	20	9.78 647	28	0.21 353	9.93 100	8	33	
28	9.71 767	21	9.78 675	29	0.21 325	9.93 092	8	32	
29	9.71 788	21	9.78 704	28	0.21 296	9.93 084	7	31	
30	9.71 809	20	9.78 732	28	0.21 268	9.93 077	8	**30**	
31	9.71 829	21	9.78 760	29	0.21 240	9.93 069	8	29	
32	9.71 850	20	9.78 789	28	0.21 211	9.93 061	8	28	
33	9.71 870	21	9.78 817	28	0.21 183	9.93 053	7	27	
34	9.71 891	20	9.78 845	29	0.21 155	9.93 046	8	26	**20 8 7**
35	9.71 911	21	9.78 874	28	0.21 126	9.93 038	8	**25**	2 4.0 1.6 1.4
36	9.71 932	20	9.78 902	28	0.21 098	9.93 030	8	24	3 6.0 2.4 2.1
37	9.71 952	21	9.78 930	29	0.21 070	9.93 022	8	23	4 8.0 3.2 2.8
38	9.71 973	21	9.78 959	28	0.21 041	9.93 014	7	22	5 10.0 4.0 3.5
39	9.71 994	20	9.78 987	28	0.21 013	9.93 007	8	21	6 12.0 4.8 4.2
40	9.72 014	20	9.79 015	28	0.20 985	9.92 999	8	**20**	7 14.0 5.6 4.9
41	9.72 034	21	9.79 043	29	0.20 957	9.92 991	8	19	8 16.0 6.4 5.6
42	9.72 055	20	9.79 072	28	0.20 928	9.92 983	7	18	9 18.0 7.2 6.3
43	9.72 075	21	9.79 100	28	0.20 900	9.92 976	8	17	
44	9.72 096	20	9.79 128	28	0.20 872	9.92 968	8	16	
45	9.72 116	21	9.79 156	29	0.20 844	9.92 960	8	**15**	
46	9.72 137	20	9.79 185	28	0.20 815	9.92 952	8	14	
47	9.72 157	20	9.79 213	28	0.20 787	9.92 944	8	13	
48	9.72 177	21	9.79 241	28	0.20 759	9.92 936	7	12	
49	9.72 198	20	9.79 269	28	0.20 731	9.92 929	8	11	
50	9.72 218	20	9.79 297	29	0.20 703	9.92 921	8	**10**	
51	9.72 238	21	9.79 326	28	0.20 674	9.92 913	8	9	
52	9.72 259	20	9.79 354	28	0.20 646	9.92 905	8	8	
53	9.72 279	20	9.79 382	28	0.20 618	9.92 897	8	7	
54	9.72 299	21	9.79 410	28	0.20 590	9.92 889	8	6	
55	9.72 320	20	9.79 438	28	0.20 562	9.92 881	7	**5**	
56	9.72 340	20	9.79 466	29	0.20 534	9.92 874	8	4	
57	9.72 360	21	9.79 495	28	0.20 505	9.92 866	8	3	
58	9.72 381	20	9.79 523	28	0.20 477	9.92 858	8	2	
59	9.72 401	20	9.79 551	28	0.20 449	9.92 850	8	1	
60	9.72 421		9.79 579		0.20 421	9.92 842		**0**	
	L Cos	d	L Cot	c d	L Tan	L Sin	d	′	

148° 238° 328° 58°

′	L Sin	d	L Tan	c d	L Cot	L Cos	d		Prop. Parts
0	9.72 421	20	9.79 579	28	0.20 421	9.92 842	8	60	
1	9.72 441	20	9.79 607	28	0.20 393	9.92 834	8	59	
2	9.72 461	21	9.79 635	28	0.20 365	9.92 826	8	58	
3	9.72 482	20	9.79 663	28	0.20 337	9.92 818	8	57	
4	9.72 502	20	9.79 691	28	0.20 309	9.92 810	7	56	
5	9.72 522	20	9.79 719	28	0.20 281	9.92 803	8	55	
6	9.72 542	20	9.79 747	29	0.20 253	9.92 795	8	54	
7	9.72 562	20	9.79 776	28	0.20 224	9.92 787	8	53	
8	9.72 582	20	9.79 804	28	0.20 196	9.92 779	8	52	
9	9.72 602	20	9.79 832	28	0.20 168	9.92 771	8	51	
10	9.72 622	21	9.79 860	28	0.20 140	9.92 763	8	50	
11	9.72 643	20	9.79 888	28	0.20 112	9.92 755	8	49	
12	9.72 663	20	9.79 916	28	0.20 084	9.92 747	8	48	
13	9.72 683	20	9.79 944	28	0.20 056	9.92 739	8	47	**29 28 27**
14	9.72 703	20	9.79 972	28	0.20 028	9.92 731	8	46	2 5.8 5.6 5.4
15	9.72 723	20	9.80 000	28	0.20 000	9.92 723	8	45	3 8.7 8.4 8.1
16	9.72 743	20	9.80 028	28	0.19 972	9.92 715	8	44	4 11.6 11.2 10.8
17	9.72 763	20	9.80 056	28	0.19 944	9.92 707	8	43	5 14.5 14.0 13.5
18	9.72 783	20	9.80 084	28	0.19 916	9.92 699	8	42	6 17.4 16.8 16.2
19	9.72 803	20	9.80 112	28	0.19 888	9.92 691	8	41	7 20.3 19.6 18.9
20	9.72 823	20	9.80 140	28	0.19 860	9.92 683	8	40	8 23.2 22.4 21.6
21	9.72 843	20	9.80 168	27	0.19 832	9.92 675	8	39	9 26.1 25.2 24.3
22	9.72 863	20	9.80 195	28	0.19 805	9.92 667	8	38	
23	9.72 883	19	9.80 223	28	0.19 777	9.92 659	8	37	
24	9.72 902	20	9.80 251	28	0.19 749	9.92 651	8	36	
25	9.72 922	20	9.80 279	28	0.19 721	9.92 643	8	35	
26	9.72 942	20	9.80 307	28	0.19 693	9.92 635	8	34	**21 20 19**
27	9.72 962	20	9.80 335	28	0.19 665	9.92 627	8	33	
28	9.72 982	20	9.80 363	28	0.19 637	9.92 619	8	32	2 4.2 4.0 3.8
29	9.73 002	20	9.80 391	28	0.19 609	9.92 611	8	31	3 6.3 6.0 5.7
30	9.73 022	19	9.80 419	28	0.19 581	9.92 603	8	30	4 8.4 8.0 7.6
31	9.73 041	20	9.80 447	27	0.19 553	9.92 595	8	29	5 10.5 10.0 9.5
32	9.73 061	20	9.80 474	28	0.19 526	9.92 587	8	28	6 12.6 12.0 11.4
33	9.73 081	20	9.80 502	28	0.19 498	9.92 579	8	27	7 14.7 14.0 13.3
34	9.73 101	20	9.80 530	28	0.19 470	9.92 571	8	26	8 16.8 16.0 15.2
35	9.73 121	19	9.80 558	28	0.19 442	9.92 563	8	25	9 18.9 18.0 17.1
36	9.73 140	20	9.80 586	28	0.19 414	9.92 555	9	24	
37	9.73 160	20	9.80 614	28	0.19 386	9.92 546	8	23	
38	9.73 180	20	9.80 642	27	0.19 358	9.92 538	8	22	
39	9.73 200	19	9.80 669	28	0.19 331	9.92 530	8	21	
40	9.73 219	20	9.80 697	28	0.19 303	9.92 522	8	20	**9 8 7**
41	9.73 239	20	9.80 725	28	0.19 275	9.92 514	8	19	2 1.8 1.6 1.4
42	9.73 259	19	9.80 753	28	0.19 247	9.92 506	8	18	3 2.7 2.4 2.1
43	9.73 278	20	9.80 781	27	0.19 219	9.92 498	8	17	4 3.6 3.2 2.8
44	9.73 298	20	9.80 808	28	0.19 192	9.92 490	8	16	5 4.5 4.0 3.5
45	9.73 318	19	9.80 836	28	0.19 164	9.92 482	9	15	6 5.4 4.8 4.2
46	9.73 337	20	9.80 864	28	0.19 136	9.92 473	8	14	7 6.3 5.6 4.9
47	9.73 357	20	9.80 892	27	0.19 108	9.92 465	8	13	8 7.2 6.4 5.6
48	9.73 377	19	9.80 919	28	0.19 081	9.92 457	8	12	9 8.1 7.2 6.3
49	9.73 396	20	9.80 947	28	0.19 053	9.92 449	8	11	
50	9.73 416	19	9.80 975	28	0.19 025	9.92 441	8	10	
51	9.73 435	20	9.81 003	27	0.18 997	9.92 433	8	9	
52	9.73 455	19	9.81 030	28	0.18 970	9.92 425	9	8	
53	9.73 474	20	9.81 058	28	0.18 942	9.92 416	8	7	
54	9.73 494	19	9.81 086	27	0.18 914	9.92 408	8	6	
55	9.73 513	20	9.81 113	28	0.18 887	9.92 400	8	5	
56	9.73 533	19	9.81 141	28	0.18 859	9.92 392	8	4	
57	9.73 552	20	9.81 169	27	0.18 831	9.92 384	8	3	
58	9.73 572	19	9.81 196	28	0.18 804	9.92 376	9	2	
59	9.73 591	20	9.81 224	28	0.18 776	9.92 367	8	1	
60	9.73 611		9.81 252		0.18 748	9.92 359		0	
	L Cos	d	L Cot	c d	L Tan	L Sin	d	′	

′	L Sin	d	L Tan	c d	L Cot	L Cos	d	
0	9.73 611	19	9.81 252	27	0.18 748	9.92 359	8	**60**
1	9.73 630	20	9.81 279	28	0.18 721	9.92 351	8	59
2	9.73 650	19	9.81 307	28	0.18 693	9.92 343	8	58
3	9.73 669	20	9.81 335	27	0.18 665	9.92 335	9	57
4	9.73 689	19	9.81 362	28	0.18 638	9.92 326	8	56
5	9.73 708	19	9.81 390	28	0.18 610	9.92 318	8	**55**
6	9.73 727	20	9.81 418	27	0.18 582	9.92 310	8	54
7	9.73 747	19	9.81 445	28	0.18 555	9.92 302	9	53
8	9.73 766	19	9.81 473	27	0.18 527	9.92 293	8	52
9	9.73 785	20	9.81 500	28	0.18 500	9.92 285	8	51
10	9.73 805	19	9.81 528	28	0.18 472	9.92 277	8	**50**
11	9.73 824	19	9.81 556	27	0.18 444	9.92 269	9	49
12	9.73 843	20	9.81 583	28	0.18 417	9.92 260	8	48
13	9.73 863	19	9.81 611	27	0.18 389	9.92 252	8	47
14	9.73 882	19	9.81 638	28	0.18 362	9.92 244	9	46
15	9.73 901	20	9.81 666	27	0.18 334	9.92 235	8	**45**
16	9.73 921	19	9.81 693	28	0.18 307	9.92 227	8	44
17	9.73 940	19	9.81 721	27	0.18 279	9.92 219	8	43
18	9.73 959	19	9.81 748	28	0.18 252	9.92 211	9	42
19	9.73 978	19	9.81 776	27	0.18 224	9.92 202	8	41
20	9.73 997	20	9.81 803	28	0.18 197	9.92 194	8	**40**
21	9.74 017	19	9.81 831	27	0.18 169	9.92 186	9	39
22	9.74 036	19	9.81 858	28	0.18 142	9.92 177	8	38
23	9.74 055	19	9.81 886	27	0.18 114	9.92 169	8	37
24	9.74 074	19	9.81 913	28	0.18 087	9.92 161	9	36
25	9.74 093	20	9.81 941	27	0.18 059	9.92 152	8	**35**
26	9.74 113	19	9.81 968	28	0.18 032	9.92 144	8	34
27	9.74 132	19	9.81 996	27	0.18 004	9.92 136	9	33
28	9.74 151	19	9.82 023	28	0.17 977	9.92 127	8	32
29	9.74 170	19	9.82 051	27	0.17 949	9.92 119	8	31
30	9.74 189	19	9.82 078	28	0.17 922	9.92 111	9	**30**
31	9.74 208	19	9.82 106	27	0.17 894	9.92 102	8	29
32	9.74 227	19	9.82 133	28	0.17 867	9.92 094	8	28
33	9.74 246	19	9.82 161	27	0.17 839	9.92 086	9	27
34	9.74 265	19	9.82 188	27	0.17 812	9.92 077	8	26
35	9.74 284	19	9.82 215	28	0.17 785	9.92 069	9	**25**
36	9.74 303	19	9.82 243	27	0.17 757	9.92 060	8	24
37	9.74 322	19	9.82 270	28	0.17 730	9.92 052	8	23
38	9.74 341	19	9.82 298	27	0.17 702	9.92 044	9	22
39	9.74 360	19	9.82 325	27	0.17 675	9.92 035	8	21
40	9.74 379	19	9.82 352	28	0.17 648	9.92 027	9	**20**
41	9.74 398	19	9.82 380	27	0.17 620	9.92 018	8	19
42	9.74 417	19	9.82 407	28	0.17 593	9.92 010	8	18
43	9.74 436	19	9.82 435	27	0.17 565	9.92 002	9	17
44	9.74 455	19	9.82 462	27	0.17 538	9.91 993	8	16
45	9.74 474	19	9.82 489	28	0.17 511	9.91 985	9	**15**
46	9.74 493	19	9.82 517	27	0.17 483	9.91 976	8	14
47	9.74 512	19	9.82 544	27	0.17 456	9.91 968	9	13
48	9.74 531	18	9.82 571	28	0.17 429	9.91 959	8	12
49	9.74 549	19	9.82 599	27	0.17 401	9.91 951	9	11
50	9.74 568	19	9.82 626	27	0.17 374	9.91 942	8	**10**
51	9.74 587	19	9.82 653	28	0.17 347	9.91 934	9	9
52	9.74 606	19	9.82 681	27	0.17 319	9.91 925	8	8
53	9.74 625	19	9.82 708	27	0.17 292	9.91 917	9	7
54	9.74 644	18	9.82 735	27	0.17 265	9.91 908	9	6
55	9.74 662	19	9.82 762	28	0.17 238	9.91 900	9	**5**
56	9.74 681	19	9.82 790	27	0.17 210	9.91 891	8	4
57	9.74 700	19	9.82 817	27	0.17 183	9.91 883	9	3
58	9.74 719	18	9.82 844	27	0.17 156	9.91 874	8	2
59	9.74 737	19	9.82 871	28	0.17 129	9.91 866	9	1
60	9.74 756		9.82 899		0.17 101	9.91 857		**0**
	L Cos	d	L Cot	c d	L Tan	L Sin	d	′

Prop. Parts

	28	27	20
2	5.6	5.4	4.0
3	8.4	8.1	6.0
4	11.2	10.8	8.0
5	14.0	13.5	10.0
6	16.8	16.2	12.0
7	19.6	18.9	14.0
8	22.4	21.6	16.0
9	25.2	24.3	18.0

	19	18
2	3.8	3.6
3	5.7	5.4
4	7.6	7.2
5	9.5	9.0
6	11.4	10.8
7	13.3	12.6
8	15.2	14.4
9	17.1	16.2

	9	8
2	1.8	1.6
3	2.7	2.4
4	3.6	3.2
5	4.5	4.0
6	5.4	4.8
7	6.3	5.6
8	7.2	6.4
9	8.1	7.2

′	L Sin	d	L Tan	c d	L Cot	L Cos	d		′
0	9.74 756	19	9.82 899	27	0.17 101	9.91 857	8	60	
1	9.74 775	19	9.82 926	27	0.17 074	9.91 849	9	59	
2	9.74 794	18	9.82 953	27	0.17 047	9.91 840	8	58	
3	9.74 812	19	9.82 980	28	0.17 020	9.91 832	9	57	
4	9.74 831	19	9.83 008	27	0.16 992	9.91 823	8	56	
5	9.74 850	18	9.83 035	27	0.16 965	9.91 815	9	55	
6	9.74 868	19	9.83 062	27	0.16 938	9.91 806	8	54	
7	9.74 887	19	9.83 089	28	0.16 911	9.91 798	9	53	
8	9.74 906	18	9.83 117	27	0.16 883	9.91 789	8	52	
9	9.74 924	19	9.83 144	27	0.16 856	9.91 781	9	51	
10	9.74 943	18	9.83 171	27	0.16 829	9.91 772	9	50	
11	9.74 961	19	9.83 198	27	0.16 802	9.91 763	8	49	
12	9.74 980	19	9.83 225	27	0.16 775	9.91 755	9	48	
13	9.74 999	18	9.83 252	28	0.16 748	9.91 746	8	47	
14	9.75 017	19	9.83 280	27	0.16 720	9.91 738	9	46	
15	9.75 036	18	9.83 307	27	0.16 693	9.91 729	9	45	
16	9.75 054	19	9.83 334	27	0.16 666	9.91 720	8	44	
17	9.75 073	18	9.83 361	27	0.16 639	9.91 712	9	43	
18	9.75 091	19	9.83 388	27	0.16 612	9.91 703	8	42	
19	9.75 110	18	9.83 415	27	0.16 585	9.91 695	9	41	
20	9.75 128	19	9.83 442	28	0.16 558	9.91 686	9	40	
21	9.75 147	18	9.83 470	27	0.16 530	9.91 677	8	39	
22	9.75 165	19	9.83 497	27	0.16 503	9.91 669	9	38	
23	9.75 184	18	9.83 524	27	0.16 476	9.91 660	9	37	
24	9.75 202	19	9.83 551	27	0.16 449	9.91 651	8	36	
25	9.75 221	18	9.83 578	27	0.16 422	9.91 643	9	35	
26	9.75 239	19	9.83 605	27	0.16 395	9.91 634	9	34	
27	9.75 258	18	9.83 632	27	0.16 368	9.91 625	8	32	
28	9.75 276	18	9.83 659	27	0.16 341	9.91 617	9	32	
29	9.75 294	19	9.83 686	27	0.16 314	9.91 608	9	31	
30	9.75 313	18	9.83 713	27	0.16 287	9.91 599	8	30	
31	9.75 331	19	9.83 740	28	0.16 260	9.91 591	9	29	
32	9.75 350	18	9.83 768	27	0.16 232	9.91 582	9	28	
33	9.75 368	18	9.83 795	27	0.16 205	9.91 573	8	27	
34	9.75 386	19	9.83 822	27	0.16 178	9.91 565	9	26	
35	9.75 405	18	9.83 849	27	0.16 151	9.91 556	9	25	
36	9.75 423	18	9.83 876	27	0.16 124	9.91 547	9	24	
37	9.75 441	18	9.83 903	27	0.16 097	9.91 538	8	23	
38	9.75 459	19	9.83 930	27	0.16 070	9.91 530	9	22	
39	9.75 478	18	9.83 957	27	0.16 043	9.91 521	9	21	
40	9.75 496	18	9.83 984	27	0.16 016	9.91 512	8	20	
41	9.75 514	19	9.84 011	27	0.15 989	9.91 504	9	19	
42	9.75 533	18	9.84 038	27	0.15 962	9.91 495	9	18	
43	9.75 551	18	9.84 065	27	0.15 935	9.91 486	9	17	
44	9.75 569	18	9.84 092	27	0.15 908	9.91 477	9	16	
45	9.75 587	18	9.84 119	27	0.15 881	9.91 469	8	15	
46	9.75 605	19	9.84 146	27	0.15 854	9.91 460	9	14	
47	9.75 624	18	9.84 173	27	0.15 827	9.91 451	9	13	
48	9.75 642	18	9.84 200	27	0.15 800	9.91 442	9	12	
49	9.75 660	18	9.84 227	27	0.15 773	9.91 433	9	11	
50	9.75 678	18	9.84 254	26	0.15 746	9.91 425	8	10	
51	9.75 696	18	9.84 280	27	0.15 720	9.91 416	9	9	
52	9.75 714	19	9.84 307	27	0.15 693	9.91 407	9	8	
53	9.75 733	18	9.84 334	27	0.15 666	9.91 398	9	7	
54	9.75 751	18	9.84 361	27	0.15 639	9.91 389	9	6	
55	9.75 769	18	9.84 388	27	0.15 612	9.91 381	8	5	
56	9.75 787	18	9.84 415	27	0.15 585	9.91 372	9	4	
57	9.75 805	18	9.84 442	27	0.15 558	9.91 363	9	3	
58	9.75 823	18	9.84 469	27	0.15 531	9.91 354	9	2	
59	9.75 841	18	9.84 496	27	0.15 504	9.91 345	9	1	
60	9.75 859		9.84 523		0.15 477	9.91 336		0	
	L Cos	d	L Cot	c d	L Tan	L Sin	d	′	

Prop. Parts

	28	27	26
2	5.6	5.4	5.2
3	8.4	8.1	7.8
4	11.2	10.8	10.4
5	14.0	13.5	13.0
6	16.8	16.2	15.6
7	19.6	18.9	18.2
8	22.4	21.6	20.8
9	25.2	24.3	23.4

	19	18
2	3.8	3.6
3	5.7	5.4
4	7.6	7.2
5	9.5	9.0
6	11.4	10.8
7	13.3	12.6
8	15.2	14.4
9	17.1	16.2

	9	8
2	1.8	1.6
3	2.7	2.4
4	3.6	3.2
5	4.5	4.0
6	5.4	4.8
7	6.3	5.6
8	7.2	6.4
9	8.1	7.2

′	L Sin	d	L Tan	c d	L Cot	L Cos	d		Prop. Parts
0	9.75 859	18	9.84 523	27	0.15 477	9.91 336	8	60	
1	9.75 877	18	9.84 550	26	0.15 450	9.91 328	9	59	
2	9.75 895	18	9.84 576	27	0.15 424	9.91 319	9	58	
3	9.75 913	18	9.84 603	27	0.15 397	9.91 310	9	57	
4	9.75 931	18	9.84 630	27	0.15 370	9.91 301	9	56	
5	9.75 949	18	9.84 657	27	0.15 343	9.91 292	9	55	
6	9.75 967	18	9.84 684	27	0.15 316	9.91 283	9	54	
7	9.75 985	18	9.84 711	27	0.15 289	9.91 274	8	53	
8	9.76 003	18	9.84 738	26	0.15 262	9.91 266	9	52	
9	9.76 021	18	9.84 764	27	0.15 236	9.91 257	9	51	
10	9.76 039	18	9.84 791	27	0.15 209	9.91 248	9	50	
11	9.76 057	18	9.84 818	27	0.15 182	9.91 239	9	49	
12	9.76 075	18	9.84 845	27	0.15 155	9.91 230	9	48	
13	9.76 093	18	9.84 872	27	0.15 128	9.91 221	9	47	
14	9.76 111	18	9.84 899	26	0.15 101	9.91 212	9	46	
15	9.76 129	17	9.84 925	27	0.15 075	9.91 203	9	45	
16	9.76 146	18	9.84 952	27	0.15 048	9.91 194	9	44	
17	9.76 164	18	9.84 979	27	0.15 021	9.91 185	9	43	
18	9.76 182	18	9.85 006	27	0.14 994	9.91 176	9	42	
19	9.76 200	18	9.85 033	26	0.14 967	9.91 167	9	41	
20	9.76 218	18	9.85 059	27	0.14 941	9.91 158	9	40	
21	9.76 236	17	9.85 086	27	0.14 914	9.91 149	8	39	
22	9.76 253	18	9.85 113	27	0.14 887	9.91 141	9	38	
23	9.76 271	18	9.85 140	26	0.14 860	9.91 132	9	37	
24	9.76 289	18	9.85 166	27	0.14 834	9.91 123	9	36	
25	9.76 307	17	9.85 193	27	0.14 807	9.91 114	9	35	
26	9.76 324	18	9.85 220	27	0.14 780	9.91 105	9	34	
27	9.76 342	18	9.85 247	27	0.14 753	9.91 096	9	33	
28	9.76 360	18	9.85 273	26	0.14 727	9.91 087	9	32	
29	9.76 378	17	9.85 300	27	0.14 700	9.91 078	9	31	
30	9.76 395	18	9.85 327	27	0.14 673	9.91 069	9	30	
31	9.76 413	18	9.85 354	26	0.14 646	9.91 060	9	29	
32	9.76 431	17	9.85 380	27	0.14 620	9.91 051	9	28	
33	9.76 448	18	9.85 407	27	0.14 593	9.91 042	9	27	
34	9.76 466	18	9.85 434	26	0.14 566	9.91 033	10	26	
35	9.76 484	17	9.85 460	27	0.14 540	9.91 023	9	25	
36	9.76 501	18	9.85 487	27	0.14 513	9.91 014	9	24	
37	9.76 519	18	9.85 514	26	0.14 486	9.91 005	9	23	
38	9.76 537	17	9.85 540	27	0.14 460	9.90 996	9	22	
39	9.76 554	18	9.85 567	27	0.14 433	9.90 987	9	21	
40	9.76 572	18	9.85 594	26	0.14 406	9.90 978	9	20	
41	9.76 590	17	9.85 620	27	0.14 380	9.90 969	9	19	
42	9.76 607	18	9.85 647	27	0.14 353	9.90 960	9	18	
43	9.76 625	17	9.85 674	26	0.14 326	9.90 951	9	17	
44	9.76 642	18	9.85 700	27	0.14 300	9.90 942	9	16	
45	9.76 660	17	9.85 727	27	0.14 273	9.90 933	9	15	
46	9.76 677	18	9.85 754	26	0.14 246	9.90 924	9	14	
47	9.76 695	17	9.85 780	27	0.14 220	9.90 915	9	13	
48	9.76 712	18	9.85 807	27	0.14 193	9.90 906	10	12	
49	9.76 730	17	9.85 834	26	0.14 166	9.90 896	9	11	
50	9.76 747	18	9.85 860	27	0.14 140	9.90 887	9	10	
51	9.76 765	17	9.85 887	26	0.14 113	9.90 878	9	9	
52	9.76 782	18	9.85 913	27	0.14 087	9.90 869	9	8	
53	9.76 800	17	9.85 940	27	0.14 060	9.90 860	9	7	
54	9.76 817	18	9.85 967	26	0.14 033	9.90 851	9	6	
55	9.76 835	17	9.85 993	27	0.14 007	9.90 842	10	5	
56	9.76 852	18	9.86 020	26	0.13 980	9.90 832	9	4	
57	9.76 870	17	9.86 046	27	0.13 954	9.90 823	9	3	
58	9.76 887	17	9.86 073	27	0.13 927	9.90 814	9	2	
59	9.76 904	18	9.86 100	26	0.13 900	9.90 805	9	1	
60	9.76 922		9.86 126		0.13 874	9.90 796		0	

	27	26	15
2	5.4	5.2	3.6
3	8.1	7.8	5.4
4	10.8	10.4	7.2
5	13.5	13.0	9.0
6	16.2	15.6	10.8
7	18.9	18.2	12.6
8	21.6	20.8	14.4
9	24.3	23.4	16.2

	17	10
2	3.4	2.0
3	5.1	3.0
4	6.8	4.0
5	8.5	5.0
6	10.2	6.0
7	11.9	7.0
8	13.6	8.0
9	15.3	9.0

	9	8
2	1.8	1.6
3	2.7	2.4
4	3.6	3.2
5	4.5	4.0
6	5.4	4.8
7	6.3	5.6
8	7.2	6.4
9	8.1	7.2

	L Cos	d	L Cot	c d	L Tan	L Sin	d	′

′	L Sin	d	L Tan	cd	L Cot	L Cos	d		Prop. Parts
0	9.76 922	17	9.86 126	27	0.13 874	9.90 796	9	60	
1	9.76 939	18	9.86 153	26	0.13 847	9.90 787	10	59	
2	9.76 957	17	9.86 179	27	0.13 821	9.90 777	9	58	
3	9.76 974	17	9.86 206	26	0.13 794	9.90 768	9	57	
4	9.76 991	18	9.86 232	27	0.13 768	9.90 759	9	56	
5	9.77 009	17	9.86 259	26	0.13 741	9.90 750	9	55	
6	9.77 026	17	9.86 285	27	0.13 715	9.90 741	10	54	
7	9.77 043	18	9.86 312	26	0.13 688	9.90 731	9	53	
8	9.77 061	17	9.86 338	27	0.13 662	9.90 722	9	52	
9	9.77 078	17	9.86 365	27	0.13 635	9.90 713	9	51	
10	9.77 095	17	9.86 392	26	0.13 608	9.90 704	10	50	
11	9.77 112	18	9.86 418	27	0.13 582	9.90 694	9	49	
12	9.77 130	17	9.86 445	26	0.13 555	9.90 685	9	48	
13	9.77 147	17	9.86 471	27	0.13 529	9.90 676	9	47	
14	9.77 164	17	9.86 498	26	0.13 502	9.90 667	10	46	
15	9.77 181	18	9.86 524	27	0.13 476	9.90 657	9	45	
16	9.77 199	17	9.86 551	26	0.13 449	9.90 648	9	44	
17	9.77 216	17	9.86 577	26	0.13 423	9.90 639	9	43	
18	9.77 233	17	9.86 603	27	0.13 397	9.90 630	10	42	
19	9.77 250	18	9.86 630	26	0.13 370	9.90 620	9	41	
20	9.77 268	17	9.86 656	27	0.13 344	9.90 611	9	40	
21	9.77 285	17	9.86 683	26	0.13 317	9.90 602	10	39	
22	9.77 302	17	9.86 709	27	0.13 291	9.90 592	9	38	
23	9.77 319	17	9.86 736	26	0.13 264	9.90 583	9	37	
24	9.77 336	17	9.86 762	27	0.13 238	9.90 574	9	36	
25	9.77 353	17	9.86 789	26	0.13 211	9.90 565	10	35	
26	9.77 370	17	9.86 815	27	0.13 185	9.90 555	9	34	
27	9.77 387	18	9.86 842	26	0.13 158	9.90 546	9	33	
28	9.77 405	17	9.86 868	26	0.13 132	9.90 537	10	32	
29	9.77 422	17	9.86 894	27	0.13 106	9.90 527	9	31	
30	9.77 439	17	9.86 921	26	0.13 079	9.90 518	9	30	
31	9.77 456	17	9.86 947	27	0.13 053	9.90 509	10	29	
32	9.77 473	17	9.86 974	26	0.13 026	9.90 499	9	28	
33	9.77 490	17	9.87 000	27	0.13 000	9.90 490	10	27	
34	9.77 507	17	9.87 027	26	0.12 973	9.90 480	9	26	
35	9.77 524	17	9.87 053	26	0.12 947	9.90 471	9	25	
36	9.77 541	17	9.87 079	27	0.12 921	9.90 462	10	24	
37	9.77 558	17	9.87 106	26	0.12 894	9.90 452	9	23	
38	9.77 575	17	9.87 132	26	0.12 868	9.90 443	9	22	
39	9.77 592	17	9.87 158	27	0.12 842	9.90 434	10	21	
40	9.77 609	17	9.87 185	26	0.12 815	9.90 424	9	20	
41	9.77 626	17	9.87 211	27	0.12 789	9.90 415	10	19	
42	9.77 643	17	9.87 238	26	0.12 762	9.90 405	9	18	
43	9.77 660	17	9.87 264	26	0.12 736	9.90 396	10	17	
44	9.77 677	17	9.87 290	27	0.12 710	9.90 386	9	16	
45	9.77 694	17	9.87 317	26	0.12 683	9.90 377	9	15	
46	9.77 711	17	9.87 343	26	0.12 657	9.90 368	10	14	
47	9.77 728	16	9.87 369	27	0.12 631	9.90 358	9	13	
48	9.77 744	17	9.87 396	26	0.12 604	9.90 349	10	12	
49	9.77 761	17	9.87 422	26	0.12 578	9.90 339	9	11	
50	9.77 778	17	9.87 448	27	0.12 552	9.90 330	10	10	
51	9.77 795	17	9.87 475	26	0.12 525	9.90 320	9	9	
52	9.77 812	17	9.87 501	26	0.12 499	9.90 311	10	8	
53	9.77 829	17	9.87 527	27	0.12 473	9.90 301	9	7	
54	9.77 846	16	9.87 554	26	0.12 446	9.90 292	10	6	
55	9.77 862	17	9.87 580	26	0.12 420	9.90 282	9	5	
56	9.77 879	17	9.87 606	27	0.12 394	9.90 273	10	4	
57	9.77 896	17	9.87 633	26	0.12 367	9.90 263	9	3	
58	9.77 913	17	9.87 659	26	0.12 341	9.90 254	10	2	
59	9.77 930	16	9.87 685	26	0.12 315	9.90 244	9	1	
60	9.77 946		9.87 711		0.12 289	9.90 235		0	
	L Cos	d	L Cot	cd	L Tan	L Sin	d	′	

	27	25	18
2	5.4	5.2	3.6
3	8.1	7.8	5.4
4	10.8	10.4	7.2
5	13.5	13.0	9.0
6	16.2	15.6	10.8
7	18.9	18.2	12.6
8	21.6	20.8	14.4
9	24.3	23.4	16.2

	17	16
2	3.4	3.2
3	5.1	4.8
4	6.8	6.4
5	8.5	8.0
6	10.2	9.6
7	11.9	11.2
8	13.6	12.8
9	15.3	14.4

	10	9
2	2.0	1.8
3	3.0	2.7
4	4.0	3.6
5	5.0	4.5
6	6.0	5.4
7	7.0	6.3
8	8.0	7.2
9	9.0	8.1

′	L Sin	d	L Tan	c d	L Cot	L Cos	d		Prop. Parts
0	9.77 946	17	9.87 711	27	0.12 289	9.90 235	10	**60**	
1	9.77 963	17	9.87 738	26	0.12 262	9.90 225	9	59	
2	9.77 980	17	9.87 764	26	0.12 236	9.90 216	10	58	
3	9.77 997	16	9.87 790	27	0.12 210	9.90 206	9	57	
4	9.78 013	17	9.87 817	26	0.12 183	9.90 197	10	56	
5	9.78 030	17	9.87 843	26	0.12 157	9.90 187	9	**55**	
6	9.78 047	16	9.87 869	26	0.12 131	9.90 178	10	54	
7	9.78 063	17	9.87 895	27	0.12 105	9.90 168	9	53	
8	9.78 080	17	9.87 922	26	0.12 078	9.90 159	10	52	
9	9.78 097	16	9.87 948	26	0.12 052	9.90 149	10	51	
10	9.78 113	17	9.87 974	26	0.12 026	9.90 139	9	**50**	
11	9.78 130	17	9.88 000	27	0.12 000	9.90 130	10	49	
12	9.78 147	16	9.88 027	26	0.11 973	9.90 120	9	48	
13	9.78 163	17	9.88 053	26	0.11 947	9.90 111	10	47	
14	9.78 180	17	9.88 079	26	0.11 921	9.90 101	10	46	
15	9.78 197	16	9.88 105	26	0.11 895	9.90 091	9	**45**	
16	9.78 213	17	9.88 131	27	0.11 869	9.90 082	10	44	
17	9.78 230	16	9.88 158	26	0.11 842	9.90 072	9	43	
18	9.78 246	17	9.88 184	26	0.11 816	9.90 063	10	42	
19	9.78 263	17	9.88 210	26	0.11 790	9.90 053	10	41	
20	9.78 280	16	9.88 236	26	0.11 764	9.90 043	9	**40**	
21	9.78 296	17	9.88 262	27	0.11 738	9.90 034	10	39	
22	9.78 313	16	9.88 289	26	0.11 711	9.90 024	10	38	
23	9.78 329	17	9.88 315	26	0.11 685	9.90 014	9	37	
24	9.78 346	16	9.88 341	26	0.11 659	9.90 005	10	36	
25	9.78 362	17	9.88 367	26	0.11 633	9.89 995	10	**35**	
26	9.78 379	16	9.88 393	27	0.11 607	9.89 985	9	34	
27	9.78 395	17	9.88 420	26	0.11 580	9.89 976	10	33	
28	9.78 412	16	9.88 446	26	0.11 554	9.89 966	10	32	
29	9.78 428	17	9.88 472	26	0.11 528	9.89 956	9	31	
30	9.78 445	16	9.88 498	26	0.11 502	9.89 947	10	**30**	
31	9.78 461	17	9.88 524	26	0.11 476	9.89 937	10	29	
32	9.78 478	16	9.88 550	27	0.11 450	9.89 927	9	28	
33	9.78 494	16	9.88 577	26	0.11 423	9.89 918	10	27	
34	9.78 510	17	9.88 603	26	0.11 397	9.89 908	10	26	
35	9.78 527	16	9.88 629	26	0.11 371	9.89 898	10	**25**	
36	9.78 543	17	9.88 655	26	0.11 345	9.89 888	9	24	
37	9.78 560	16	9.88 681	26	0.11 319	9.89 879	10	23	
38	9.78 576	16	9.88 707	26	0.11 293	9.89 869	10	22	
39	9.78 592	17	9.88 733	26	0.11 267	9.89 859	10	21	
40	9.78 609	16	9.88 759	27	0.11 241	9.89 849	9	**20**	
41	9.78 625	17	9.88 786	26	0.11 214	9.89 840	10	19	
42	9.78 642	16	9.88 812	26	0.11 188	9.89 830	10	18	
43	9.78 658	16	9.88 838	26	0.11 162	9.89 820	10	17	
44	9.78 674	17	9.88 864	26	0.11 136	9.89 810	9	16	
45	9.78 691	16	9.88 890	26	0.11 110	9.89 801	10	**15**	
46	9.78 707	16	9.88 916	26	0.11 084	9.89 791	10	14	
47	9.78 723	16	9.88 942	26	0.11 058	9.89 781	10	13	
48	9.78 739	17	9.88 968	26	0.11 032	9.89 771	10	12	
49	9.78 756	16	9.88 994	26	0.11 006	9.89 761	9	11	
50	9.78 772	16	9.89 020	26	0.10 980	9.89 752	10	**10**	
51	9.78 788	17	9.89 046	27	0.10 954	9.89 742	10	9	
52	9.78 805	16	9.89 073	26	0.10 927	9.89 732	10	8	
53	9.78 821	16	9.89 099	26	0.10 901	9.89 722	10	7	
54	9.78 837	16	9.89 125	26	0.10 875	9.89 712	10	6	
55	9.78 853	16	9.89 151	26	0.10 849	9.89 702	9	**5**	
56	9.78 869	17	9.89 177	26	0.10 823	9.89 693	10	4	
57	9.78 886	16	9.89 203	26	0.10 797	9.89 683	10	3	
58	9.78 902	16	9.89 229	26	0.10 771	9.89 673	10	2	
59	9.78 918	16	9.89 255	26	0.10 745	9.89 663	10	1	
60	9.78 934		9.89 281		0.10 719	9.89 653		**0**	
	L Cos	d	L Cot	c d	L Tan	L Sin	d	′	

	27	26	17
2	5.4	5.2	3.4
3	8.1	7.8	5.1
4	10.8	10.4	6.8
5	13.5	13.0	8.5
6	16.2	15.6	10.2
7	18.9	18.2	11.9
8	21.6	20.8	13.6
9	24.3	23.4	15.3

	16	10	9
2	3.2	2.0	1.8
3	4.8	3.0	2.7
4	6.4	4.0	3.6
5	8.0	5.0	4.5
6	9.6	6.0	5.4
7	11.2	7.0	6.3
8	12.8	8.0	7.2
9	14.4	9.0	8.1

′	L Sin	d	L Tan	c d	L Cot	L Cos	d		Prop. Parts
0	9.78 934	16	9.89 281	26	0.10 719	9.89 653	10	**60**	
1	9.78 950	17	9.89 307	26	0.10 693	9.89 643	10	59	
2	9.78 967	16	9.89 333	26	0.10 667	9.89 633	9	58	
3	9.78 983	16	9.89 359	26	0.10 641	9.89 624	10	57	
4	9.78 999	16	9.89 385	26	0.10 615	9.89 614	10	56	
5	9.79 015	16	9.89 411	26	0.10 589	9.89 604	10	**55**	
6	9.79 031	16	9.89 437	26	0.10 563	9.89 594	10	54	
7	9.79 047	16	9.89 463	26	0.10 537	9.89 584	10	53	
8	9.79 063	16	9.89 489	26	0.10 511	9.89 574	10	52	
9	9.79 079	16	9.89 515	26	0.10 485	9.89 564	10	51	
10	9.79 095	16	9.89 541	26	0.10 459	9.89 554	10	**50**	
11	9.79 111	17	9.89 567	26	0.10 433	9.89 544	10	49	
12	9.79 128	16	9.89 593	26	0.10 407	9.89 534	10	48	
13	9.79 144	16	9.89 619	26	0.10 381	9.89 524	10	47	
14	9.79 160	16	9.89 645	26	0.10 355	9.89 514	10	46	
15	9.79 176	16	9.89 671	26	0.10 329	9.89 504	9	**45**	
16	9.79 192	16	9.89 697	26	0.10 303	9.89 495	10	44	
17	9.79 208	16	9.89 723	26	0.10 277	9.89 485	10	43	
18	9.79 224	16	9.89 749	26	0.10 251	9.89 475	10	42	
19	9.79 240	16	9.89 775	26	0.10 225	9.89 465	10	41	
20	9.79 256	16	9.89 801	26	0.10 199	9.89 455	10	**40**	
21	9.79 272	16	9.89 827	26	0.10 173	9.89 445	10	39	
22	9.79 288	16	9.89 853	26	0.10 147	9.89 435	10	38	
23	9.79 304	15	9.89 879	26	0.10 121	9.89 425	10	37	
24	9.79 319	16	9.89 905	26	0.10 095	9.89 415	10	36	
25	9.79 335	16	9.89 931	26	0.10 069	9.89 405	10	**35**	
26	9.79 351	16	9.89 957	26	0.10 043	9.89 395	10	34	
27	9.79 367	16	9.89 983	26	0.10 017	9.89 385	10	33	
28	9.79 383	16	9.90 009	26	0.09 991	9.89 375	11	32	
29	9.79 399	16	9.90 035	26	0.09 965	9.89 364	10	31	
30	9.79 415	16	9.90 061	25	0.09 939	9.89 354	10	**30**	
31	9.79 431	16	9.90 086	26	0.09 914	9.89 344	10	29	
32	9.79 447	16	9.90 112	26	0.09 888	9.89 334	10	28	
33	9.79 463	15	9.90 138	26	0.09 862	9.89 324	10	27	
34	9.79 478	16	9.90 164	26	0.09 836	9.89 314	10	26	
35	9.79 494	16	9.90 190	26	0.09 810	9.89 304	10	**25**	
36	9.79 510	16	9.90 216	26	0.09 784	9.89 294	10	24	
37	9.79 526	16	9.90 242	26	0.09 758	9.89 284	10	23	
38	9.79 542	16	9.90 268	26	0.09 732	9.89 274	10	22	
39	9.79 558	15	9.90 294	26	0.09 706	9.89 264	10	21	
40	9.79 573	16	9.90 320	26	0.09 680	9.89 254	10	**20**	
41	9.79 589	16	9.90 346	25	0.09 654	9.89 244	11	19	
42	9.79 605	16	9.90 371	26	0.09 629	9.89 233	10	18	
43	9.79 621	15	9.90 397	26	0.09 603	9.89 223	10	17	
44	9.79 636	16	9.90 423	26	0.09 577	9.89 213	10	16	
45	9.79 652	16	9.90 449	26	0.09 551	9.89 203	10	**15**	
46	9.79 668	16	9.90 475	26	0.09 525	9.89 193	10	14	
47	9.79 684	15	9.90 501	26	0.09 499	9.89 183	10	13	
48	9.79 699	16	9.90 527	26	0.09 473	9.89 173	11	12	
49	9.79 715	16	9.90 553	25	0.09 447	9.89 162	10	11	
50	9.79 731	15	9.90 578	26	0.09 422	9.89 152	10	**10**	
51	9.79 746	16	9.90 604	26	0.09 396	9.89 142	10	9	
52	9.79 762	16	9.90 630	26	0.09 370	9.89 132	10	8	
53	9.79 778	15	9.90 656	26	0.09 344	9.89 122	10	7	
54	9.79 793	16	9.90 682	26	0.09 318	9.89 112	11	6	
55	9.79 809	16	9.90 708	26	0.09 292	9.89 101	10	**5**	
56	9.79 825	15	9.90 734	25	0.09 266	9.89 091	10	4	
57	9.79 840	16	9.90 759	26	0.09 241	9.89 081	10	3	
58	9.79 856	16	9.90 785	26	0.09 215	9.89 071	11	2	
59	9.79 872	15	9.90 811	26	0.09 189	9.89 060	10	1	
60	9.79 887		9.90 837		0.09 163	9.89 050		**0**	
	L Cos	d	L Cot	c d	L Tan	L Sin	d	′	

	26	25	17
2	5.2	5.0	3.4
3	7.8	7.5	5.1
4	10.4	10.0	6.8
5	13.0	12.5	8.5
6	15.6	15.0	10.2
7	18.2	17.5	11.9
8	20.8	20.0	13.6
9	23.4	22.5	15.3

	16	15	11
2	3.2	3.0	2.2
3	4.8	4.5	3.3
4	6.4	6.0	4.4
5	8.0	7.5	5.5
6	9.6	9.0	6.6
7	11.2	10.5	7.7
8	12.8	12.0	8 8
9	14.4	13.5	9.9

	10	9
2	2.0	1.8
3	3.0	2.7
4	4.0	3.6
5	5.0	4.5
6	6.0	5.4
7	7.0	6.3
8	8.0	7.2
9	9.0	8.1

′	L Sin	d	L Tan	c d	L Cot	L Cos	d		Prop. Parts
0	9.79 887	16	9.90 837	26	0.09 163	9.89 050	10	60	
1	9.79 903	15	9.90 863	26	0.09 137	9.89 040	10	59	
2	9.79 918	16	9.90 889	25	0.09 111	9.89 030	10	58	
3	9.79 934	16	9.90 914	26	0.09 086	9.89 020	11	57	
4	9.79 950	15	9.90 940	26	0.09 060	9.89 009	10	56	
5	9.79 965	16	9.90 966	26	0.09 034	9.88 999	10	55	
6	9.79 981	15	9.90 992	26	0.09 008	9.88 989	11	54	
7	9.79 996	16	9.91 018	25	0.08 982	9.88 978	10	53	
8	9.80 012	15	9.91 043	26	0.08 957	9.88 968	10	52	
9	9.80 027	16	9.91 069	26	0.08 931	9.88 958	10	51	
10	9.80 043	15	9.91 095	26	0.08 905	9.88 948	11	50	
11	9.80 058	16	9.91 121	26	0.08 879	9.88 937	10	49	
12	9.80 074	15	9.91 147	25	0.08 853	9.88 927	10	48	
13	9.80 089	16	9.91 172	26	0.08 828	9.88 917	11	47	
14	9.80 105	15	9.91 198	26	0.08 802	9.88 906	10	46	
15	9.80 120	16	9.91 224	26	0.08 776	9.88 896	10	45	
16	9.80 136	15	9.91 250	26	0.08 750	9.88 886	11	44	
17	9.80 151	15	9.91 276	25	0.08 724	9.88 875	10	43	
18	9.80 166	16	9.91 301	26	0.08 699	9.88 865	10	42	
19	9.80 182	15	9.91 327	26	0.08 673	9.88 855	11	41	
20	9.80 197	16	9.91 353	26	0.08 647	9.88 844	10	40	
21	9.80 213	15	9.91 379	25	0.08 621	9.88 834	10	39	
22	9.80 228	16	9.91 404	26	0.08 596	9.88 824	11	38	
23	9.80 244	15	9.91 430	26	0.08 570	9.88 813	10	37	
24	9.80 259	15	9.91 456	26	0.08 544	9.88 803	10	36	
25	9.80 274	16	9.91 482	25	0.08 518	9.88 793	11	35	
26	9.80 290	15	9.91 507	26	0.08 493	9.88 782	10	34	
27	9.80 305	15	9.91 533	26	0.08 467	9.88 772	11	33	
28	9.80 320	16	9.91 559	26	0.08 441	9.88 761	10	32	
29	9.80 336	15	9.91 585	25	0.08 415	9.88 751	10	31	
30	9.80 351	15	9.91 610	26	0.08 390	9.88 741	11	30	
31	9.80 366	16	9.91 636	26	0.08 364	9.88 730	10	29	
32	9.80 382	15	9.91 662	26	0.08 338	9.88 720	11	28	
33	9.80 397	15	9.91 688	25	0.08 312	9.88 709	10	27	
34	9.80 412	16	9.91 713	26	0.08 287	9.88 699	11	26	
35	9.80 428	15	9.91 739	26	0.08 261	9.88 688	10	25	
36	9.80 443	15	9.91 765	26	0.08 235	9.88 678	10	24	
37	9.80 458	15	9.91 791	25	0.08 209	9.88 668	11	23	
38	9.80 473	16	9.91 816	26	0.08 184	9.88 657	10	22	
39	9.80 489	15	9.91 842	26	0.08 158	9.88 647	11	21	
40	9.80 504	15	9.91 868	25	0.08 132	9.88 636	10	20	
41	9.80 519	15	9.91 893	26	0.08 107	9.88 626	11	19	
42	9.80 534	16	9.91 919	26	0.08 081	9.88 615	10	18	
43	9.80 550	15	9.91 945	26	0.08 055	9.88 605	11	17	
44	9.80 565	15	9.91 971	25	0.08 029	9.88 594	10	16	
45	9.80 580	15	9.91 996	26	0.08 004	9.88 584	11	15	
46	9.80 595	15	9.92 022	26	0.07 978	9.88 573	10	14	
47	9.80 610	15	9.92 048	25	0.07 952	9.88 563	11	13	
48	9.80 625	16	9.92 073	26	0.07 927	9.88 552	10	12	
49	9.80 641	15	9.92 099	26	0.07 901	9.88 542	11	11	
50	9.80 656	15	9.92 125	25	0.07 875	9.88 531	10	10	
51	9.80 671	15	9.92 150	26	0.07 850	9.88 521	11	9	
52	9.80 686	15	9.92 176	26	0.07 824	9.88 510	11	8	
53	9.80 701	15	9.92 202	25	0.07 798	9.88 499	10	7	
54	9.80 716	15	9.92 227	26	0.07 773	9.88 489	11	6	
55	9.80 731	15	9.92 253	26	0.07 747	9.88 478	10	5	
56	9.80 746	16	9.92 279	25	0.07 721	9.88 468	11	4	
57	9.80 762	15	9.92 304	26	0.07 696	9.88 457	10	3	
58	9.80 777	15	9.92 330	26	0.07 670	9.88 447	11	2	
59	9.80 792	15	9.92 356	25	0.07 644	9.88 436	11	1	
60	9.80 807		9.92 381		0.07 619	9.88 425		0	

	26	25	15
2	5.2	5.0	3.2
3	7.8	7.5	4.8
4	10.4	10.0	6.4
5	13.0	12.5	8.0
6	15.6	15.0	9.6
7	18.2	17.5	11.2
8	20.8	20.0	12.8
9	23.4	22.5	14.4

	15	11	10
2	3.0	2.2	2.0
3	4.5	3.3	3.0
4	6.0	4.4	4.0
5	7.5	5.5	5.0
6	9.0	6.6	6.0
7	10.5	7.7	7.0
8	12.0	8.8	8.0
9	13.5	9.9	9.0

	L Cos	d	L Cot	c d	L Tan	L Sin	d	′

′	L Sin	d	L Tan	c d	L Cot	L Cos	d		Prop. Parts
0	9.80 807	15	9.92 381	26	0.07 619	9.88 425	10	**60**	
1	9.80 822	15	9.92 407	26	0.07 593	9.88 415	11	59	
2	9.80 837	15	9.92 433	25	0.07 567	9.88 404	10	58	
3	9.80 852	15	9.92 458	26	0.07 542	9.88 394	11	57	
4	9.80 867	15	9.92 484	26	0.07 516	9.88 383	11	56	
5	9.80 882	15	9.92 510	25	0.07 490	9.88 372	10	**55**	
6	9.80 897	15	9.92 535	26	0.07 465	9.88 362	11	54	
7	9.80 912	15	9.92 561	26	0.07 439	9.88 351	11	53	
8	9.80 927	15	9.92 587	25	0.07 413	9.88 340	10	52	
9	9.80 942	15	9.92 612	26	0.07 388	9.88 330	11	51	
10	9.80 957	15	9.92 638	25	0.07 362	9.88 319	11	**50**	
11	9.80 972	15	9.92 663	26	0.07 337	9.88 308	10	49	
12	9.80 987	15	9.92 689	26	0.07 311	9.88 298	11	48	
13	9.81 002	15	9.92 715	25	0.07 285	9.88 287	11	47	
14	9.81 017	15	9.92 740	26	0.07 260	9.88 276	10	46	
15	9.81 032	15	9.92 766	26	0.07 234	9.88 266	11	**45**	
16	9.81 047	14	9.92 792	25	0.07 208	9.88 255	11	44	
17	9.81 061	15	9.92 817	26	0.07 183	9.88 244	10	43	
18	9.81 076	15	9.92 843	25	0.07 157	9.88 234	11	42	
19	9.81 091	15	9.92 868	26	0.07 132	9.88 223	11	41	
20	9.81 106	15	9.92 894	26	0.07 106	9.88 212	11	**40**	
21	9.81 121	15	9.92 920	25	0.07 080	9.88 201	10	39	
22	9.81 136	15	9.92 945	26	0.07 055	9.88 191	11	38	
23	9.81 151	15	9.92 971	25	0.07 029	9.88 180	11	37	
24	9.81 166	14	9.92 996	26	0.07 004	9.88 169	11	36	
25	9.81 180	15	9.93 022	26	0.06 978	9.88 158	10	**35**	
26	9.81 195	15	9.93 048	25	0.06 952	9.88 148	11	34	
27	9.81 210	15	9.93 073	26	0.06 927	9.88 137	11	33	
28	9.81 225	15	9.93 099	25	0.06 901	9.88 126	11	32	
29	9.81 240	14	9.93 124	26	0.06 876	9.88 115	10	31	
30	9.81 254	15	9.93 150	25	0.06 850	9.88 105	11	**30**	
31	9.81 269	15	9.93 175	26	0.06 825	9.88 094	11	29	
32	9.81 284	15	9.93 201	26	0.06 799	9.88 083	11	28	
33	9.81 299	15	9.93 227	25	0.06 773	9.88 072	11	27	
34	9.81 314	14	9.93 252	26	0.06 748	9.88 061	10	26	
35	9.81 328	15	9.93 278	25	0.06 722	9.88 051	11	**25**	
36	9.81 343	15	9.93 303	26	0.06 697	9.88 040	11	24	
37	9.81 358	14	9.93 329	25	0.06 671	9.88 029	11	23	
38	9.81 372	15	9.93 354	26	0.06 646	9.88 018	11	22	
39	9.81 387	15	9.93 380	26	0.06 620	9.88 007	11	21	
40	9.81 402	15	9.93 406	25	0.06 594	9.87 996	11	**20**	
41	9.81 417	14	9.93 431	26	0.06 569	9.87 985	10	19	
42	9.81 431	15	9.93 457	25	0.06 543	9.87 975	11	18	
43	9.81 446	15	9.93 482	26	0.06 518	9.87 964	11	17	
44	9.81 461	14	9.93 508	25	0.06 492	9.87 953	11	16	
45	9.81 475	15	9.93 533	26	0.06 467	9.87 942	11	**15**	
46	9.81 490	15	9.93 559	25	0.06 441	9.87 931	11	14	
47	9.81 505	15	9.93 584	26	0.06 416	9.87 920	11	13	
48	9.81 519	14	9.93 610	26	0.06 390	9.87 909	11	12	
49	9.81 534	15	9.93 636	25	0.06 364	9.87 898	11	11	
50	9.81 549	15	9.93 661	26	0.06 339	9.87 887	11	**10**	
51	9.81 563	14	9.93 687	25	0.06 313	9.87 877	10	9	
52	9.81 578	15	9.93 712	26	0.06 288	9.87 866	11	8	
53	9.81 592	14	9.93 738	25	0.06 262	9.87 855	11	7	
54	9.81 607	15	9.93 763	26	0.06 237	9.87 844	11	6	
55	9.81 622	14	9.93 789	25	0.06 211	9.87 833	11	**5**	
56	9.81 636	15	9.93 814	26	0.06 186	9.87 822	11	4	
57	9.81 651	14	9.93 840	25	0.06 160	9.87 811	11	3	
58	9.81 665	15	9.93 865	26	0.06 135	9.87 800	11	2	
59	9.81 680	14	9.93 891	25	0.06 109	9.87 789	11	1	
60	9.81 694		9.93 916		0.06 084	9.87 778		**0**	

	26	25	15
2	5.2	5.0	3.0
3	7.8	7.5	4.5
4	10.4	10.0	6.0
5	13.0	12.5	7.5
6	15.6	15.0	9.0
7	18.2	17.5	10.5
8	20.8	20.0	12.0
9	23.4	22.5	13.5

	14	11	10
2	2.8	2.2	2.0
3	4.2	3.3	3.0
4	5.6	4.4	4.0
5	7.0	5.5	5.0
6	8.4	6.6	6.0
7	9.8	7.7	7.0
8	11.2	8.8	8.0
9	12.6	9.9	9.0

	L Cos	d	L Cot	c d	L Tan	L Sin	d	′

′	L Sin	d	L Tan	c d	L Cot	L Cos	d		Prop. Parts
0	9.81 694		9.93 916		0.06 084	9.87 778		**60**	
1	9.81 709	15	9.93 942	26	0.06 058	9.87 767	11	59	
2	9.81 723	14	9.93 967	25	0.06 033	9.87 756	11	58	
3	9.81 738	15	9.93 993	26	0.06 007	9.87 745	11	57	
4	9.81 752	14	9.94 018	25	0.05 982	9.87 734	11	56	
5	9.81 767	15	9.94 044	26	0.05 956	9.87 723	11	**55**	
6	9.81 781	14	9.94 069	25	0.05 931	9.87 712	11	54	
7	9.81 796	15	9.94 095	26	0.05 905	9.87 701	11	53	
8	9.81 810	14	9.94 120	25	0.05 880	9.87 690	11	52	
9	9.81 825	15	9.94 146	26	0.05 854	9.87 679	11	51	
10	9.81 839	14	9.94 171	25	0.05 829	9.87 668	11	**50**	
11	9.81 854	15	9.94 197	26	0.05 803	9.87 657	11	49	
12	9.81 868	14	9.94 222	25	0.05 778	9.87 646	11	48	
13	9.81 882	14	9.94 248	26	0.05 752	9.87 635	11	47	
14	9.81 897	15	9.94 273	25	0.05 727	9.87 624	11	46	
15	9.81 911	14	9.94 299	26	0.05 701	9.87 613	12	**45**	
16	9.81 926	15	9.94 324	25	0.05 676	9.87 601	11	44	
17	9.81 940	14	9.94 350	26	0.05 650	9.87 590	11	43	
18	9.81 955	15	9.94 375	25	0.05 625	9.87 579	11	42	
19	9.81 969	14	9.94 401	26	0.05 599	9.87 568	11	41	
20	9.81 983	14	9.94 426	25	0.05 574	9.87 557	11	**40**	
21	9.81 998	15	9.94 452	26	0.05 548	9.87 546	11	39	
22	9.82 012	14	9.94 477	25	0.05 523	9.87 535	11	38	
23	9.82 026	14	9.94 503	26	0.05 497	9.87 524	11	37	
24	9.82 041	15	9.94 528	25	0.05 472	9.87 513	11	36	
25	9.82 055	14	9.94 554	26	0.05 446	9.87 501	12	**35**	
26	9.82 069	14	9.94 579	25	0.05 421	9.87 490	11	34	
27	9.82 084	15	9.94 604	25	0.05 396	9.87 479	11	33	
28	9.82 098	14	9.94 630	26	0.05 370	9.87 468	11	32	
29	9.82 112	14	9.94 655	25	0.05 345	9.87 457	11	31	
30	9.82 126	14	9.94 681	26	0.05 319	9.87 446	11	**30**	
31	9.82 141	15	9.94 706	25	0.05 294	9.87 434	12	29	
32	9.82 155	14	9.94 732	26	0.05 268	9.87 423	11	28	
33	9.82 169	14	9.94 757	25	0.05 243	9.87 412	11	27	
34	9.82 184	15	9.94 783	26	0.05 217	9.87 401	11	26	
35	9.82 198	14	9.94 808	25	0.05 192	9.87 390	11	**25**	
36	9.82 212	14	9.94 834	26	0.05 166	9.87 378	12	24	
37	9.82 226	14	9.94 859	25	0.05 141	9.87 367	11	23	
38	9.82 240	14	9.94 884	25	0.05 116	9.87 356	11	22	
39	9.82 255	15	9.94 910	26	0.05 090	9.87 345	11	21	
40	9.82 269	14	9.94 935	25	0.05 065	9.87 334	11	**20**	
41	9.82 283	14	9.94 961	26	0.05 039	9.87 322	12	19	
42	9.82 297	14	9.94 986	25	0.05 014	9.87 311	11	18	
43	9.82 311	14	9.95 012	26	0.04 988	9.87 300	11	17	
44	9.82 326	15	9.95 037	25	0.04 963	9.87 288	12	16	
45	9.82 340	14	9.95 062	25	0.04 938	9.87 277	11	**15**	
46	9.82 354	14	9.95 088	26	0.04 912	9.87 266	11	14	
47	9.82 368	14	9.95 113	25	0.04 887	9.87 255	12	13	
48	9.82 382	14	9.95 139	26	0.04 861	9.87 243	11	12	
49	9.82 396	14	9.95 164	25	0.04 836	9.87 232	11	11	
50	9.82 410	14	9.95 190	26	0.04 810	9.87 221	11	**10**	
51	9.82 424	14	9.95 215	25	0.04 785	9.87 209	12	9	
52	9.82 439	15	9.95 240	25	0.04 760	9.87 198	11	8	
53	9.82 453	14	9.95 266	26	0.04 734	9.87 187	11	7	
54	9.82 467	14	9.95 291	25	0.04 709	9.87 175	12	6	
55	9.82 481	14	9.95 317	26	0.04 683	9.87 164	11	**5**	
56	9.82 495	14	9.95 342	25	0.04 658	9.87 153	11	4	
57	9.82 509	14	9.95 368	26	0.04 632	9.87 141	12	3	
58	9.82 523	14	9.95 393	25	0.04 607	9.87 130	11	2	
59	9.82 537	14	9.95 418	25	0.04 582	9.87 119	11	1	
60	9.82 551		9.95 444	26	0.04 556	9.87 107	12	**0**	
	L Cos	d	L Cot	c d	L Tan	L Sin	d	′	

	26	**25**	**15**
2	5.2	5.0	3.0
3	7.8	7.5	4.5
4	10.4	10.0	6.0
5	13.0	12.5	7.5
6	15.6	15.0	9.0
7	18.2	17.5	10.5
8	20.8	20.0	12.0
9	23.4	22.5	13.5

	14	**12**	**11**
2	2.8	2.4	2.2
3	4.2	3.6	3.3
4	5.6	4.8	4.4
5	7.0	6.0	5.5
6	8.4	7.2	6.6
7	9.8	8.4	7.7
8	11.2	9.6	8.8
9	12.6	10.8	9.9

′	L Sin	d	L Tan	c d	L Cot	L Cos	d		Prop. Parts
0	9.82 551		9.95 444		0.04 556	9.87 107		**60**	
1	9.82 565	14	9.95 469	25	0.04 531	9.87 096	11	59	
2	9.82 579	14	9.95 495	26	0.04 505	9.87 085	11	58	
3	9.82 593	14	9.95 520	25	0.04 480	9.87 073	12	57	
4	9.82 607	14	9.95 545	25	0.04 455	9.87 062	11	56	
5	9.82 621	14	9.95 571	26	0.04 429	9.87 050	12	**55**	
6	9.82 635	14	9.95 596	25	0.04 404	9.87 039	11	54	
7	9.82 649	14	9.95 622	26	0.04 378	9.87 028	11	53	
8	9.82 663	14	9.95 647	25	0.04 353	9.87 016	12	52	
9	9.82 677	14	9.95 672	25	0.04 328	9.87 005	11	51	
10	9.82 691	14	9.95 698	26	0.04 302	9.86 993	12	**50**	
11	9.82 705	14	9.95 723	25	0.04 277	9.86 982	11	49	
12	9.82 719	14	9.95 748	25	0.04 252	9.86 970	12	48	
13	9.82 733	14	9.95 774	26	0.04 226	9.86 959	11	47	
14	9.82 747	14	9.95 799	25	0.04 201	9.86 947	12	46	
15	9.82 761	14	9.95 825	26	0.04 175	9.86 936	11	**45**	
16	9.82 775	13	9.95 850	25	0.04 150	9.86 924	12	44	
17	9.82 788	14	9.95 875	25	0.04 125	9.86 913	11	43	
18	9.82 802	14	9.95 901	26	0.04 099	9.86 902	11	42	
19	9.82 816	14	9.95 926	25	0.04 074	9.86 890	12	41	
20	9.82 830	14	9.95 952	26	0.04 048	9.86 879	11	**40**	
21	9.82 844	14	9.95 977	25	0.04 023	9.86 867	12	39	
22	9.82 858	14	9.96 002	25	0.03 998	9.86 855	12	38	
23	9.82 872	14	9.96 028	26	0.03 972	9.86 844	11	37	
24	9.82 885	13	9.96 053	25	0.03 947	9.86 832	12	36	
25	9.82 899	14	9.96 078	25	0.03 922	9.86 821	11	**35**	
26	9.82 913	14	9.96 104	26	0.03 896	9.86 809	12	34	
27	9.82 927	14	9.96 129	25	0.03 871	9.86 798	11	33	
28	9.82 941	14	9.96 155	26	0.03 845	9.86 786	12	32	
29	9.82 955	14	9.96 180	25	0.03 820	9.86 775	11	31	
30	9.82 968	13	9.96 205	25	0.03 795	9.86 763	12	**30**	
31	9.82 982	14	9.96 231	26	0.03 769	9.86 752	11	29	
32	9.82 996	14	9.96 256	25	0.03 744	9.86 740	12	28	
33	9.83 010	14	9.96 281	25	0.03 719	9.86 728	12	27	
34	9.83 023	13	9.96 307	26	0.03 693	9.86 717	11	26	
35	9.83 037	14	9.96 332	25	0.03 668	9.86 705	12	**25**	
36	9.83 051	14	9.96 357	25	0.03 643	9.86 694	11	24	
37	9.83 065	14	9.96 383	26	0.03 617	9.86 682	12	23	
38	9.83 078	13	9.96 408	25	0.03 592	9.86 670	12	22	
39	9.83 092	14	9.96 433	25	0.03 567	9.86 659	11	21	
40	9.83 106	14	9.96 459	26	0.03 541	9.86 647	12	**20**	
41	9.83 120	14	9.96 484	25	0.03 516	9.86 635	12	19	
42	9.83 133	13	9.96 510	26	0.03 490	9.86 624	11	18	
43	9.83 147	14	9.96 535	25	0.03 465	9.86 612	12	17	
44	9.83 161	14	9.96 560	25	0.03 440	9.86 600	12	16	
45	9.83 174	13	9.96 586	26	0.03 414	9.86 589	11	**15**	
46	9.83 188	14	9.96 611	25	0.03 389	9.86 577	12	14	
47	9.83 202	14	9.96 636	25	0.03 364	9.86 565	12	13	
48	9.83 215	13	9.96 662	26	0.03 338	9.86 554	11	12	
40	9.83 229	14	9.96 687	25	0.03 313	9.86 542	12	11	
50	9.83 242	13	9.96 712	25	0.03 288	9.86 530	12	**10**	
51	9.83 256	14	9.96 738	26	0.03 262	9.86 518	12	9	
52	9.83 270	14	9.96 763	25	0.03 237	9.86 507	11	8	
53	9.83 283	13	9.96 788	25	0.03 212	9.86 495	12	7	
54	9.83 297	14	9.96 814	26	0.03 186	9.86 483	12	6	
55	9.83 310	13	9.96 839	25	0.03 161	9.86 472	11	**5**	
56	9.83 324	14	9.96 864	26	0.03 136	9.86 460	12	4	
57	9.83 338	14	9.96 890	25	0.03 110	9.86 448	12	3	
58	9.83 351	13	9.96 915	25	0.03 085	9.86 436	12	2	
59	9.83 365	14	9.96 940	25	0.03 060	9.86 425	11	1	
60	9.83 378	13	9.96 966	26	0.03 034	9.86 413	12	**0**	
	L Cos	d	L Cot	c d	L Tan	L Sin	d	′	

Prop. Parts			
	26	**25**	**14**
2	5.2	5.0	2.8
3	7.8	7.5	4.2
4	10.4	10.0	5.6
5	13.0	12.5	7.0
6	15.6	15.0	8.4
7	18.2	17.5	9.8
8	20.8	20.0	11.2
9	23.4	22.5	12.6
	13	**12**	**11**
2	2.6	2.4	2.2
3	3.9	3.6	3.3
4	5.2	4.8	4.4
5	6.5	6.0	5.5
6	7.8	7.2	6.6
7	9.1	8.4	7.7
8	10.4	9.6	8.8
9	11.7	10.8	9.9

′	L Sin	d	L Tan	c d	L Cot	L Cos	d		Prop. Parts
0	9.83 378		9.96 966		0.03 034	9.86 413		60	
1	9.83 392	14	9.96 991	25	0.03 009	9.86 401	12	59	
2	9.83 405	13	9.97 016	25	0.02 984	9.86 389	12	58	
3	9.83 419	14	9.97 042	26	0.02 958	9.86 377	12	57	
4	9.83 432	13	9.97 067	25	0.02 933	9.86 366	11	56	
5	9.83 446	14	9.97 092	25	0.02 908	9.86 354	12	55	
6	9.83 459	13	9.97 118	26	0.02 882	9.86 342	12	54	
7	9.83 473	14	9.97 143	25	0.02 857	9.86 330	12	53	
8	9.83 486	13	9.97 168	25	0.02 832	9.86 318	12	52	
9	9.83 500	14	9.97 193	25	0.02 807	9.86 306	12	51	
10	9.83 513	13	9.97 219	26	0.02 781	9.86 295	11	50	
11	9.83 527	14	9.97 244	25	0.02 756	9.86 283	12	49	
12	9.83 540	13	9.97 269	25	0.02 731	9.86 271	12	48	
13	9.83 554	14	9.97 295	26	0.02 705	9.86 259	12	47	
14	9.83 567	13	9.97 320	25	0.02 680	9.86 247	12	46	
15	9.83 581	14	9.97 345	25	0.02 655	9.86 235	12	45	
16	9.83 594	13	9.97 371	26	0.02 629	9.86 223	12	44	
17	9.83 608	14	9.97 396	25	0.02 604	9.86 211	12	43	
18	9.83 621	13	9.97 421	25	0.02 579	9.86 200	11	42	
19	9.83 634	13	9.97 447	26	0.02 553	9.86 188	12	41	
20	9.83 648	14	9.97 472	25	0.02 528	9.86 176	12	40	
21	9.83 661	13	9.97 497	25	0.02 503	9.86 164	12	39	
22	9.83 674	13	9.97 523	26	0.02 477	9.86 152	12	38	
23	9.83 688	14	9.97 548	25	0.02 452	9.86 140	12	37	
24	9.83 701	13	9.97 573	25	0.02 427	9.86 128	12	36	
25	9.83 715	14	9.97 598	25	0.02 402	9.86 116	12	35	
26	9.83 728	13	9.97 624	26	0.02 376	9.86 104	12	34	
27	9.83 741	13	9.97 649	25	0.02 351	9.86 092	12	33	
28	9.83 755	14	9.97 674	25	0.02 326	9.86 080	12	32	
29	9.83 768	13	9.97 700	26	0.02 300	9.86 068	12	31	
30	9.83 781	13	9.97 725	25	0.02 275	9.86 056	12	30	
31	9.83 795	14	9.97 750	25	0.02 250	9.86 044	12	29	
32	9.83 808	13	9.97 776	26	0.02 224	9.86 032	12	28	
33	9.83 821	13	9.97 801	25	0.02 199	9.86 020	12	27	
34	9.83 834	13	9.97 826	25	0.02 174	9.86 008	12	26	
35	9.83 848	14	9.97 851	25	0.02 149	9.85 996	12	25	
36	9.83 861	13	9.97 877	26	0.02 123	9.85 984	12	24	
37	9.83 874	13	9.97 902	25	0.02 098	9.85 972	12	23	
38	9.83 887	13	9.97 927	25	0.02 073	9.85 960	12	22	
39	9.83 901	14	9.97 953	26	0.02 047	9.85 948	12	21	
40	9.83 914	13	9.97 978	25	0.02 022	9.85 936	12	20	
41	9.83 927	13	9.98 003	25	0.01 997	9.85 924	12	19	
42	9.83 940	14	9.98 029	26	0.01 971	9.85 912	12	18	
43	9.83 954	13	9.98 054	25	0.01 946	9.85 900	12	17	
44	9.83 967	13	9.98 079	25	0.01 921	9.85 888	12	16	
45	9.83 980	13	9.98 104	25	0.01 896	9.85 876	12	15	
46	9.83 993	13	9.98 130	26	0.01 870	9.85 864	13	14	
47	9.84 006	14	9.98 155	25	0.01 845	9.85 851	12	13	
48	9.84 020	13	9.98 180	25	0.01 820	9.85 839	12	12	
49	9.84 033	13	9.98 206	26	0.01 794	9.85 827	12	11	
50	9.84 046	13	9.98 231	25	0.01 769	9.85 815	12	10	
51	9.84 059	13	9.98 256	25	0.01 744	9.85 803	12	9	
52	9.84 072	13	9.98 281	25	0.01 719	9.85 791	12	8	
53	9.84 085	13	9.98 307	26	0.01 693	9.85 779	13	7	
54	9.84 098	14	9.98 332	25	0.01 668	9.85 766	12	6	
55	9.84 112	13	9.98 357	25	0.01 643	9.85 754	12	5	
56	9.84 125	13	9.98 383	26	0.01 617	9.85 742	12	4	
57	9.84 138	13	9.98 408	25	0.01 592	9.85 730	12	3	
58	9.84 151	13	9.98 433	25	0.01 567	9.85 718	12	2	
59	9.84 164	13	9.98 458	25	0.01 542	9.85 706	13	1	
60	9.84 177		9.98 484	26	0.01 516	9.85 693		0	
	L Cos	d	L Cot	c d	L Tan	L Sin	d	′	

	26	25	14
2	5.2	5.0	2.8
3	7.8	7.5	4.2
4	10.4	10.0	5.6
5	13.0	12.5	7.0
6	15.6	15.0	8.4
7	18.2	17.5	9.8
8	20.8	20.0	11.2
9	23.4	22.5	12.6

	13	12	11
2	2.6	2.4	2.2
3	3.9	3.6	3.3
4	5.2	4.8	4.4
5	6.5	6.0	5.5
6	7.8	7.2	6.6
7	9.1	8.4	7.7
8	10.4	9.6	8.8
9	11.7	10.8	9.9

′	L Sin	d	L Tan	c d	L Cot	L Cos	d		′
0	9.84 177		9.98 484		0.01 516	9.85 693		**60**	
1	9.84 190	13	9.98 509	25	0.01 491	9.85 681	12	59	
2	9.84 203	13	9.98 534	25	0.01 466	9.85 669	12	58	
3	9.84 216	13	9.98 560	26	0.01 440	9.85 657	12	57	
4	9.84 229	13	9.98 585	25	0.01 415	9.85 645	12	56	
5	9.84 242		9.98 610	25	0.01 390	9.85 632	13	**55**	
6	9.84 255	13	9.98 635	25	0.01 365	9.85 620	12	54	
7	9.84 269	14	9.98 661	26	0.01 339	9.85 608	12	53	
8	9.84 282	13	9.98 686	25	0.01 314	9.85 596	12	52	
9	9.84 295	13	9.98 711	25	0.01 289	9.85 583	13	51	
10	9.84 308		9.98 737	26	0.01 263	9.85 571	12	**50**	
11	9.84 321	13	9.98 762	25	0.01 238	9.85 559	12	49	
12	9.84 334	13	9.98 787	25	0.01 213	9.85 547	12	48	
13	9.84 347	13	9.98 812	25	0.01 188	9.85 534	13	47	
14	9.84 360	13	9.98 838	26	0.01 162	9.85 522	12	46	
15	9.84 373		9.98 863	25	0.01 137	9.85 510	12	**45**	
16	9.84 385	12	9.98 888	25	0.01 112	9.85 497	13	44	
17	9.84 398	13	9.98 913	26	0.01 087	9.85 485	12	43	
18	9.84 411	13	9.98 939	25	0.01 061	9.85 473	12	42	
19	9.84 424	13	9.98 964	25	0.01 036	9.85 460	13	41	
20	9.84 437		9.98 989	25	0.01 011	9.85 448	12	**40**	
21	9.84 450	13	9.99 015	26	0.00 985	9.85 436	12	39	
22	9.84 463	13	9.99 040	25	0.00 960	9.85 423	13	38	
23	9.84 476	13	9.99 065	25	0.00 935	9.85 411	12	37	
24	9.84 489	13	9.99 090	25	0.00 910	9.85 399	12	36	
25	9.84 502		9.99 116	26	0.00 884	9.85 386	13	**35**	
26	9.84 515	13	9.99 141	25	0.00 859	9.85 374	12	34	
27	9.84 528	13	9.99 166	25	0.00 834	9.85 361	13	33	
28	9.84 540	12	9.99 191	25	0.00 809	9.85 349	12	32	
29	9.84 553	13	9.99 217	26	0.00 783	9.85 337	12	31	
30	9.84 566	13	9.99 242	25	0.00 758	9.85 324	13	**30**	
31	9.84 579	13	9.99 267	25	0.00 733	9.85 312	12	29	
32	9.84 592	13	9.99 293	26	0.00 707	9.85 299	13	28	
33	9.84 605	13	9.99 318	25	0.00 682	9.85 287	12	27	
34	9.84 618	12	9.99 343	25	0.00 657	9.85 274	13	26	
35	9.84 630	13	9.99 368	25	0.00 632	9.85 262	12	**25**	
36	9.84 643	13	9.99 394	26	0.00 606	9.85 250	12	24	
37	9.84 656	13	9.99 419	25	0.00 581	9.85 237	13	23	
38	9.84 669	13	9.99 444	25	0.00 556	9.85 225	12	22	
39	9.84 682	13	9.99 469	25	0.00 531	9.85 212	13	21	
40	9.84 694	12	9.99 495	26	0.00 505	9.85 200	12	**20**	
41	9.84 707	13	9.99 520	25	0.00 480	9.85 187	13	19	
42	9.84 720	13	9.99 545	25	0.00 455	9.85 175	12	18	
43	9.84 733	13	9.99 570	25	0.00 430	9.85 162	13	17	
44	9.84 745	12	9.99 596	26	0.00 404	9.85 150	12	16	
45	9.84 758	13	9.99 621	25	0.00 379	9.85 137	13	**15**	
46	9.84 771	13	9.99 646	25	0.00 354	9.85 125	12	14	
47	9.84 784	13	9.99 672	26	0.00 328	9.85 112	13	13	
48	9.84 796	12	9.99 697	25	0.00 303	9.85 100	12	12	
49	9.84 809	13	9.99 722	25	0.00 278	9.85 087	13	11	
50	9.84 822	13	9.99 747	25	0.00 253	9.85 074	13	**10**	
51	9.84 835	13	9.99 773	26	0.00 227	9.85 062	12	9	
52	9.84 847	12	9.99 798	25	0.00 202	9.85 049	13	8	
53	9.84 860	13	9.99 823	25	0.00 177	9.85 037	12	7	
54	9.84 873	13	9.99 848	25	0.00 152	9.85 024	13	6	
55	9.84 885	12	9.99 874	26	0.00 126	9.85 012	12	**5**	
56	9.84 898	13	9.99 899	25	0.00 101	9.84 999	13	4	
57	9.84 911	13	9.99 924	25	0.00 076	9.84 986	13	3	
58	9.84 923	12	9.99 949	25	0.00 051	9.84 974	12	2	
59	9.84 936	13	9.99 975	26	0.00 025	9.84 961	13	1	
60	9.84 949	13	0.00 000	25	0.00 000	9.84 949	12	**0**	
	L Cos	d	L Cot	c d	L Tan	L Sin	d	′	

Prop. Parts

	26	25	14
2	5.2	5.0	2.8
3	7.8	7.5	4.2
4	10.4	10.0	5.6
5	13.0	12.5	7.0
6	15.6	15.0	8.4
7	18.2	17.5	9.8
8	20.8	20.0	11.2
9	23.4	22.5	12.6

	13	12
2	2.6	2.4
3	3.9	3.6
4	5.2	4.8
5	6.5	6.0
6	7.8	7.2
7	9.1	8.4
8	10.4	9.6
9	11.7	10.8

Explanation of Table IV

This table contains the logarithms of the sines and tangents of angles from 0° to 0° 3′ and the cosines and cotangents of angles from 89° 57′ to 90° for every second of the arc to five places of decimals. These are given on the first page of the table. On the following pages are given the logarithms of the sines, cosines, and tangents of all angles from 0° to 2° and of the cosines, sines, and cotangents of all angles from 88° to 90° for each 10 seconds of the arc. As in Table III these logarithms are negative and −10 must be supplied with every logarithm. The use of the table will be illustrated by the following examples:

Ex. 1. Log cos 89° 58′ 36″. Since this angle is greater than 89° 57′, the logarithm can be read from the first page of the table without interpolation. Log cos 89° 58′ 36″ = 6.60985 − 10. This is also the logarithm of the cotangent of the same angle.

Ex. 2. Find log cot 88° 35′ 47″. Since this angle is less than 89° 57′, interpolation is necessary. Log cot 88° 35′ 40″ = 8.38981 − 10. The tabular difference for 10″ is 86 units of the fifth decimal place. Hence log cot 88° 35′ 47″ = (8.38981−10) − .7(.00086) = 8.38921 − 10.

Ex. 3. Find A if log sin A = 7.16430 − 10. A lies between 0° 5′ 0″ and 0° 5′ 10″. Log sin A − log sin 0° 5′ 0″ = .00160 and the tabular difference is .00424. Hence A = 0° 5′ + $\frac{160}{424}$ of 10″ = 0° 5′ 4″ to the nearest second.

TABLE IV

COMMON LOGARITHMS

OF THE

TRIGONOMETRIC FUNCTIONS

FROM 0° TO 2° AND FROM 88° TO 90° FOR EACH TEN SECONDS OF ARC

ALSO

FROM 0° TO 0° 3′ AND FROM 89° 57′ TO 90° FOR EACH SECOND OF ARC

L Sin and L Tan — 0° TO 0° 3′ — L Sin and L Tan

″	0′	1′	2′	″	″	0′	1′	2′	″
0	———	6.46 373	6.76 476	**60**	**30**	6.16 270	6.63 982	6.86 167	**30**
1	4.68 557	6.47 090	6.76 836	59	31	6.17 694	6.64 462	6.86 455	29
2	4.98 660	6.47 797	6.77 193	58	32	6.19 072	6.64 936	6.86 742	28
3	5.16 270	6.48 492	6.77 548	57	33	6.20 409	6.65 406	6.87 027	27
4	5.28 763	6.49 175	6.77 900	56	34	6.21 705	6.65 870	6.87 310	26
5	5.38 454	6.49 849	6.78 248	55	35	6.22 964	6.66 330	6.87 591	25
6	5.46 373	6.50 512	6.78 595	54	36	6.24 188	6.66 785	6.87 870	24
7	5.53 067	6.51 165	6.78 938	53	37	6.25 378	6.67 235	6.88 147	23
8	5.58 866	6.51 808	6.79 278	52	38	6.26 536	6.67 680	6.88 423	22
9	5.63 982	6.52 442	6.79 616	51	39	6.27 664	6.68 121	6.88 697	21
10	5.68 557	6.53 067	6.79 952	**50**	**40**	6.28 763	6.68 557	6.88 969	**20**
11	5.72 697	6.53 683	6.80 285	49	41	6.29 836	6.68 990	6.89 240	19
12	5.76 476	6.54 291	6.80 615	48	42	6.30 882	6.69 418	6.89 509	18
13	5.79 952	6.54 890	6.80 943	47	43	6.31 904	6.69 841	6.89 776	17
14	5.83 170	6.55 481	6.81 268	46	44	6.32 903	6.70 261	6.90 042	16
15	5.86 167	6.56 064	6.81 591	45	45	6.33 879	6.70 676	6.90 306	15
16	5.88 969	6.56 639	6.81 911	44	46	6.34 833	6.71 088	6.90 568	14
17	5.91 602	6.57 207	6.82 230	43	47	6.35 767	6.71 496	6.90 829	13
18	5.94 085	6.57 767	6.82 545	42	48	6.36 682	6.71 900	6.91 088	12
19	5.96 433	6.58 320	6.82 859	41	49	6.37 577	6.72 300	6.91 346	11
20	5.98 660	6.58 866	6.83 170	**40**	**50**	6.38 454	6.72 697	6.91 602	**10**
21	6.00 779	6.59 406	6.83 479	39	51	6.39 315	6.73 090	6.91 857	9
22	6.02 800	6.59 939	6.83 786	38	52	6.40 158	6.73 479	6.92 110	8
23	6.04 730	6.60 465	6.84 091	37	53	6.40 985	6.73 865	6.92 362	7
24	6.06 579	6.60 985	6.84 394	36	54	6.41 797	6.74 248	6.92 612	6
25	6.08 351	6.61 499	6.84 694	35	55	6.42 594	6.74 627	6.92 861	5
26	6.10 055	6.62 007	6.84 993	34	56	6.43 376	6.75 003	6.93 109	4
27	6.11 694	6.62 509	6.85 289	33	57	6.44 145	6.75 376	6.93 355	3
28	6.13 273	6.63 006	6.85 584	32	58	6.44 900	6.75 746	6.93 599	2
29	6.14 797	6.63 496	6.85 876	31	59	6.45 643	6.76 112	6.93 843	1
30	6.16 270	6.63 982	6.86 167	**30**	**60**	6.46 373	6.76 476	6.94 085	**0**
″	59′	58′	57′	″	″	59′	58′	57′	″

L Cos and L Cot — 89° 57′ TO 90° — L Cos and L Cot

Logarithms of the Trigonometric Functions
0° to 0° 20′

′ ″	L Sin	L Tan	L Cos	″ ′	′ ″	L Sin	L Tan	L Cos	″ ′
0 0	— —	— —	10.00000	0 **60**	**10 0**	7.46 373	7.46 373	10.00000	0 **50**
10	5.68 557	5.68 557	10.00000	50	10	7.47 090	7.47 091	10.00000	50
20	5.98 660	5.98 660	10.00000	40	20	7.47 797	7.47 797	10.00000	40
30	6.16 270	6.16 270	10.00000	30	30	7.48 491	7.48 492	10.00000	30
40	6.28 763	6.28 763	10.00000	20	40	7.49 175	7.49 176	10.00000	20
50	6.38 454	6.38 454	10.00000	10	50	7.49 849	7.49 849	10.00000	10
1 0	6.46 373	6.46 373	10.00000	0 **59**	**11 0**	7.50 512	7.50 512	10.00000	0 **49**
10	6.53 067	6.53 067	10.00000	50	10	7.51 165	7.51 165	10.00000	50
20	6.58 866	6.58 866	10.00000	40	20	7.51 808	7.51 809	10.00000	40
30	6.63 982	6.63 982	10.00000	30	30	7.52 442	7.52 443	10.00000	30
40	6.68 557	6.68 557	10.00000	20	40	7.53 067	7.53 067	10.00000	20
50	6.72 697	6.72 697	10.00000	10	50	7.53 683	7.53 683	10.00000	10
2 0	6.76 476	6.76 476	10.00000	0 **58**	**12 0**	7.54 291	7.54 291	10.00000	0 **48**
10	6.79 952	6.79 952	10.00000	50	10	7.54 890	7.54 890	10.00000	50
20	6.83 170	6.83 170	10.00000	40	20	7.55 481	7.55 481	10.00000	40
30	6.86 167	6.86 167	10.00000	30	30	7.56 064	7.56 064	10.00000	30
40	6.88 969	6.88 969	10.00000	20	40	7.56 639	7.56 639	10.00000	20
50	6.91 602	6.91 602	10.00000	10	50	7.57 207	7.57 207	10.00000	10
3 0	6.94 085	6.94 085	10.00000	0 **57**	**13 0**	7.57 767	7.57 767	10.00000	0 **47**
10	6.96 433	6.96 433	10.00000	50	10	7.58 320	7.58 320	10.00000	50
20	6.98 660	6.98 661	10.00000	40	20	7.58 866	7.58 867	10.00000	40
30	7.00 779	7.00 779	10.00000	30	30	7.59 406	7.59 406	10.00000	30
40	7.02 800	7.02 800	10.00000	20	40	7.59 939	7.59 939	10.00000	20
50	7.04 730	7.04 730	10.00000	10	50	7.60 465	7.60 466	10.00000	10
4 0	7.06 579	7.06 579	10.00000	0 **56**	**14 0**	7.60 985	7.60 986	10.00000	0 **46**
10	7.08 351	7.08 352	10.00000	50	10	7.61 499	7.61 500	10.00000	50
20	7.10 055	7.10 055	10.00000	40	20	7.62 007	7.62 008	10.00000	40
30	7.11 694	7.11 694	10.00000	30	30	7.62 509	7.62 510	10.00000	30
40	7.13 273	7.13 273	10.00000	20	40	7.63 006	7.63 006	10.00000	20
50	7.14 797	7.14 797	10.00000	10	50	7.63 496	7.63 497	10.00000	10
5 0	7.16 270	7.16 270	10.00000	0 **55**	**15 0**	7.63 982	7.63 982	10.00000	0 **45**
10	7.17 694	7.17 694	10.00000	50	10	7.64 461	7.64 462	10.00000	50
20	7.19 072	7.19 073	10.00000	40	20	7.64 936	7.64 937	10.00000	40
30	7.20 409	7.20 409	10.00000	30	30	7.65 406	7.65 406	10.00000	30
40	7.21 705	7.21 705	10.00000	20	40	7.65 870	7.65 871	10.00000	20
50	7.22 964	7.22 964	10.00000	10	50	7.66 330	7.66 330	10.00000	10
6 0	7.24 188	7.24 188	10.00000	0 **54**	**16 0**	7.66 784	7.66 785	10.00000	0 **44**
10	7.25 378	7.25 378	10.00000	50	10	7.67 235	7.67 235	10.00000	50
20	7.26 536	7.26 536	10.00000	40	20	7.67 680	7.67 680	10.00000	40
30	7.27 664	7.27 664	10.00000	30	30	7.68 121	7.68 121	10.00000	30
40	7.28 763	7.28 764	10.00000	20	40	7.68 557	7.68 558	9.99999	20
50	7.29 836	7.29 836	10.00000	10	50	7.68 989	7.68 990	9.99999	10
7 0	7.30 882	7.30 882	10.00000	0 **53**	**17 0**	7.69 417	7.69 418	9.99999	0 **43**
10	7.31 904	7.31 904	10.00000	50	10	7.69 841	7.69 842	9.99999	50
20	7.32 903	7.32 903	10.00000	40	20	7.70 261	7.70 261	9.99999	40
30	7.33 879	7.33 879	10.00000	30	30	7.70 676	7.70 677	9.99999	30
40	7.34 833	7.34 833	10.00000	20	40	7.71 088	7.71 088	9.99999	20
50	7.35 767	7.35 767	10.00000	10	50	7.71 496	7.71 496	9.99999	10
8 0	7.36 682	7.36 682	10.00000	0 **52**	**18 0**	7.71 900	7.71 900	9.99999	0 **42**
10	7.37 577	7.37 577	10.00000	50	10	7.72 300	7.72 301	9.99999	50
20	7.38 454	7.38 455	10.00000	40	20	7.72 697	7.72 697	9.99999	40
30	7.39 314	7.39 315	10.00000	30	30	7.73 090	7.73 090	9.99999	30
40	7.40 158	7.40 158	10.00000	20	40	7.73 479	7.73 480	9.99999	20
50	7.40 985	7.40 985	10.00000	10	50	7.73 865	7.73 866	9.99999	10
9 0	7.41 797	7.41 797	10.00000	0 **51**	**19 0**	7.74 248	7.74 248	9.99999	0 **41**
10	7.42 594	7.42 594	10.00000	50	10	7.74 627	7.74 628	9.99999	50
20	7.43 376	7.43 376	10.00000	40	20	7.75 003	7.75 004	9.99999	40
30	7.44 145	7.44 145	10.00000	30	30	7.75 376	7.75 377	9.99999	30
40	7.44 900	7.44 900	10.00000	20	40	7.75 745	7.75 746	9.99999	20
50	7.45 643	7.45 643	10.00000	10	50	7.76 112	7.76 113	9.99999	10
10 0	7.46 373	7.46 373	10.00000	0 **50**	**20 0**	7.76 475	7.76 476	9.99999	0 **40**
′ ″	L Cos	L Cot	L Sin	″ ′	′ ″	L Cos	L Cot	L Sin	″ ′

89° 40′ to 90°

Logarithms of the Trigonometric Functions

0° 20′ to 0° 40′

′	″	L Sin	L Tan	L Cos	″	′	′	″	L Sin	L Tan	L Cos	″	′
20	0	7.76 475	7.76 476	9.99 999	0	**40**	**30**	0	7.94 084	7.94 086	9.99 998	0	**30**
	10	7.76 836	7.76 837	9.99 999	50			10	7.94 325	7.94 326	9.99 998	50	
	20	7.77 193	7.77 194	9.99 999	40			20	7.94 564	7.94 566	9.99 998	40	
	30	7.77 548	7.77 549	9.99 999	30			30	7.94 802	7.94 804	9.99 998	30	
	40	7.77 899	7.77 900	9.99 999	20			40	7.95 039	7.95 040	9.99 998	20	
	50	7.78 248	7.78 249	9.99 999	10			50	7.95 274	7.95 276	9.99 998	10	
21	0	7.78 594	7.78 595	9.99 999	0	**39**	**31**	0	7.95 508	7.95 510	9.99 998	0	**29**
	10	7.78 938	7.78 938	9.99 999	50			10	7.95 741	7.95 743	9.99 998	50	
	20	7.79 278	7.79 279	9.99 999	40			20	7.95 973	7.95 974	9.99 998	40	
	30	7.79 616	7.79 617	9.99 999	30			30	7.96 203	7.96 205	9.99 998	30	
	40	7.79 952	7.79 952	9.99 999	20			40	7.96 432	7.96 434	9.99 998	20	
	50	7.80 284	7.80 285	9.99 999	10			50	7.96 660	7.96 662	9.99 998	10	
22	0	7.80 615	7.80 615	9.99 999	0	**38**	**32**	0	7.96 887	7.96 889	9.99 998	0	**28**
	10	7.80 942	7.80 943	9.99 999	50			10	7.97 113	7.97 114	9.99 998	50	
	20	7.81 268	7.81 269	9.99 999	40			20	7.97 337	7.97 339	9.99 998	40	
	30	7.81 591	7.81 591	9.99 999	30			30	7.97 560	7.97 562	9.99 998	30	
	40	7.81 911	7.81 912	9.99 999	20			40	7.97 782	7.97 784	9.99 998	20	
	50	7.82 229	7.82 230	9.99 999	10			50	7.98 003	7.98 005	9.99 998	10	
23	0	7.82 545	7.82 546	9.99 999	0	**37**	**33**	0	7.98 223	7.98 225	9.99 998	0	**27**
	10	7.82 859	7.82 860	9.99 999	50			10	7.98 442	7.98 444	9.99 998	50	
	20	7.83 170	7.83 171	9.99 999	40			20	7.98 660	7.98 662	9.99 998	40	
	30	7.83 479	7.83 480	9.99 999	30			30	7.98 876	7.98 878	9.99 998	30	
	40	7.83 786	7.83 787	9.99 999	20			40	7.99 092	7.99 094	9.99 998	20	
	50	7.84 091	7.84 092	9.99 999	10			50	7.99 306	7.99 308	9.99 998	10	
24	0	7.84 393	7.84 394	9.99 999	0	**36**	**34**	0	7.99 520	7.99 522	9.99 998	0	**26**
	10	7.84 694	7.84 695	9.99 999	50			10	7.99 732	7.99 734	9.99 998	50	
	20	7.84 992	7.84 993	9.99 999	40			20	7.99 943	7.99 946	9.99 998	40	
	30	7.85 289	7.85 290	9.99 999	30			30	8.00 154	8.00 156	9.99 998	30	
	40	7.85 583	7.85 584	9.99 999	20			40	8.00 363	8.00 365	9.99 998	20	
	50	7.85 876	7.85 877	9.99 999	10			50	8.00 571	8.00 574	9.99 998	10	
25	0	7.86 166	7.86 167	9.99 999	0	**35**	**35**	0	8.00 779	8.00 781	9.99 998	0	**25**
	10	7.86 455	7.86 456	9.99 999	50			10	8.00 985	8.00 987	9.99 998	50	
	20	7.86 741	7.86 743	9.99 999	40			20	8.01 190	8.01 193	9.99 998	40	
	30	7.87 026	7.87 027	9.99 999	30			30	8.01 395	8.01 397	9.99 998	30	
	40	7.87 309	7.87 310	9.99 999	20			40	8.01 598	8.01 600	9.99 998	20	
	50	7.87 590	7.87 591	9.99 999	10			50	8.01 801	8.01 802	9.99 998	10	
26	0	7.87 870	8.87 871	9.99 999	0	**34**	**36**	0	8.02 002	8.02 004	9.99 998	0	**24**
	10	7.88 147	7.88 148	9.99 999	50			10	8.02 203	8.02 205	9.99 998	50	
	20	7.88 423	7.88 424	9.99 999	40			20	8.02 402	8.02 405	9.99 998	40	
	30	7.88 697	7.88 698	9.99 999	30			30	8.02 601	8.02 604	9.99 998	30	
	40	7.88 969	7.88 970	9.99 999	20			40	8.02 799	8.02 801	9.99 998	20	
	50	7.89 240	7.89 241	9.99 999	10			50	8.02 996	8.02 998	9.99 998	10	
27	0	7.89 509	7.89 510	9.99 999	0	**33**	**37**	0	8.03 192	8.03 194	9.99 997	0	**23**
	10	7.89 776	7.89 777	9.99 999	50			10	8.03 387	8.03 390	9.99 997	50	
	20	7.90 041	7.90 043	9.99 999	40			20	8.03 581	8.03 584	9.99 997	40	
	30	7.90 305	7.90 307	9.99 999	30			30	8.03 775	8.03 777	9.99 997	30	
	40	7.90 568	7.90 569	9.99 999	20			40	8.03 967	8.03 970	9.99 997	20	
	50	7.90 829	7.90 830	9.99 999	10			50	8.04 159	8.04 162	9.99 997	10	
28	0	7.91 088	7.91 089	9.99 999	0	**32**	**38**	0	8.04 350	8.04 353	9.99 997	0	**22**
	10	7.91 346	7.91 347	9.99 999	50			10	8.04 540	8.04 543	9.99 997	50	
	20	7.91 602	7.91 603	9.99 999	40			20	8.04 729	8.04 732	9.99 997	40	
	30	7.91 857	7.91 858	9.99 999	30			30	8.04 918	8.04 921	9.99 997	30	
	40	7.92 110	7.92 111	9.99 998	20			40	8.05 105	8.05 108	9.99 997	20	
	50	7.92 362	7.92 363	9.99 998	10			50	8.05 292	8.05 295	9.99 997	10	
29	0	7.92 612	7.92 613	9.99 998	0	**31**	**39**	0	8.05 478	8.05 481	9.99 997	0	**21**
	10	7.92 861	7.92 862	9.99 998	50			10	8.05 663	8.05 666	9.99 997	50	
	20	7.93 108	7.93 110	9.99 998	40			20	8.05 848	8.05 851	9.99 997	40	
	30	7.93 354	7.93 356	9.99 998	30			30	8.06 031	8.06 034	9.99 997	30	
	40	7.93 599	7.93 601	9.99 998	20			40	8.06 214	8.06 217	9.99 997	20	
	50	7.93 842	7.93 844	9.99 998	10			50	8.06 396	8.06 399	9.99 997	10	
30	0	7.94 084	7.94 086	9.99 998	0	**30**	**40**	0	8.06 578	8.06 581	9.99 997	0	**20**

′	″	L Cos	L Cot	L Sin	″	′	′	″	L Cos	L Cot	L Sin	″	′

89° 20′ to 89° 40′

Logarithms of the Trigonometric Functions
0° 40′ to 1° 0′

′	″	L Sin	L Tan	L Cos	″	′	′	″	L Sin	L Tan	L Cos	″	′
40	0	8.06 578	8.06 581	9.99 997	0	**20**	**50**	0	8.16 268	8.16 273	9.99 995	0	**10**
	10	8.06 758	8.06 761	9.99 997	50			10	8.16 413	8.16 417	9.99 995	50	
	20	8.06 938	8.06 941	9.99 997	40			20	8.16 557	8.16 561	9.99 995	40	
	30	8.07 117	8.07 120	9.99 997	30			30	8.16 700	8.16 705	9.99 995	30	
	40	8.07 295	8.07 299	9.99 997	20			40	8.16 843	8.16 848	9.99 995	20	
	50	8.07 473	8.07 476	9.99 997	10			50	8.16 986	8.16 991	9.99 995	10	
41	0	8.07 650	8.07 653	9.99 997	0	**19**	**51**	0	8.17 128	8.17 133	9.99 995	0	**9**
	10	8.07 826	8.07 829	9.99 997	50			10	8.17 270	8.17 275	9.99 995	50	
	20	8.08 002	8.08 005	9.99 997	40			20	8.17 411	8.17 416	9.99 995	40	
	30	8.08 176	8.08 180	9.99 997	30			30	8.17 552	8.17 557	9.99 995	30	
	40	8.08 350	8.08 354	9.99 997	20			40	8.17 692	8.17 697	9.99 995	20	
	50	8.08 524	8.08 527	9.99 997	10			50	8.17 832	8.17 837	9.99 995	10	
42	0	8.08 696	8.08 700	9.99 997	0	**18**	**52**	0	8.17 971	8.17 976	9.99 995	0	**8**
	10	8.08 868	8.08 872	9.99 997	50			10	8.18 110	8.18 115	9.99 995	50	
	20	8.09 040	8.09 043	9.99 997	40			20	8.18 249	8.18 254	9.99 995	40	
	30	8.09 210	8.09 214	9.99 997	30			30	8.18 387	8.18 392	9.99 995	30	
	40	8.09 380	8.09 384	9.99 997	20			40	8.18 524	8.18 530	9.99 995	20	
	50	8.09 550	8.09 553	9.99 997	10			50	8.18 662	8.18 667	9.99 995	10	
43	0	8.09 718	8.09 722	9.99 997	0	**17**	**53**	0	8.18 798	8.18 804	9.99 995	0	**7**
	10	8.09 886	8.09 890	9.99 997	50			10	8.18 935	8.18 940	9.99 995	50	
	20	8.10 054	8.10 057	9.99 997	40			20	8.19 071	8.19 076	9.99 995	40	
	30	8.10 220	8.10 224	9.99 997	30			30	8.19 206	8.19 212	9.99 995	30	
	40	8.10 386	8.10 390	9.99 997	20			40	8.19 341	8.19 347	9.99 995	20	
	50	8.10 552	8.10 555	9.99 996	10			50	8.19 476	8.19 481	9.99 995	10	
44	0	8.10 717	8.10 720	9.99 996	0	**16**	**54**	0	8.19 610	8.19 616	9.99 995	0	**6**
	10	8.10 881	8.10 884	9.99 996	50			10	8.19 744	8.19 749	9.99 995	50	
	20	8.11 044	8.11 048	9.99 996	40			20	8.19 877	8.19 883	9.99 995	40	
	30	8.11 207	8.11 211	9.99 996	30			30	8.20 010	8.20 016	9.99 995	30	
	40	8.11 370	8.11 373	9.99 996	20			40	8.20 143	8.20 149	9.99 995	20	
	50	8.11 531	8.11 535	9.99 996	10			50	8.20 275	8.20 281	9.99 994	10	
45	0	8.11 693	8.11 696	9.99 996	0	**15**	**55**	0	8.20 407	8.20 413	9.99 994	0	**5**
	10	8.11 853	8.11 857	9.99 996	50			10	8.20 538	8.20 544	9.99 994	50	
	20	8.12 013	8.12 017	9.99 996	40			20	8.20 669	8.20 675	9.99 994	40	
	30	8.12 172	8.12 176	9.99 996	30			30	8.20 800	8.20 806	9.99 994	30	
	40	8.12 331	8.12 335	9.99 996	20			40	8.20 930	8.20 936	9.99 994	20	
	50	8.12 489	8.12 493	9.99 996	10			50	8.21 060	8.21 066	9.99 994	10	
46	0	8.12 647	8.12 651	9.99 996	0	**14**	**56**	0	8.21 189	8.21 195	9.99 994	0	**4**
	10	8.12 804	8.12 808	9.99 996	50			10	8.21 319	8.21 324	9.99 994	50	
	20	8.12 961	8.12 965	9.99 996	40			20	8.21 447	8.21 453	9.99 994	40	
	30	8.13 117	8.13 121	9.99 996	30			30	8.21 576	8.21 581	9.99 994	30	
	40	8.13 272	8.13 276	9.99 996	20			40	8.21 703	8.21 709	9.99 994	20	
	50	8.13 427	8.13 431	9.99 996	10			50	8.21 831	8.21 837	9.99 994	10	
47	0	8.13 581	8.13 585	9.99 996	0	**13**	**57**	0	8.21 958	8.21 964	9.99 994	0	**3**
	10	8.13 735	8.13 739	9.99 996	50			10	8.22 085	8.22 091	9.99 994	50	
	20	8.13 888	8.13 892	9.99 996	40			20	8.22 211	8.22 217	9.99 994	40	
	30	8.14 041	8.14 045	9.99 996	30			30	8.22 337	8.22 343	9.99 994	30	
	40	8.14 193	8.14 197	9.99 996	20			40	8.22 463	8.22 469	9.99 994	20	
	50	8.14 344	8.14 348	9.99 996	10			50	8.22 588	8.22 595	9.99 994	10	
48	0	8.14 495	8.14 500	9.99 996	0	**12**	**58**	0	8.22 713	8.22 720	9.99 994	0	**2**
	10	8.14 646	8.14 650	9.99 996	50			10	8.22 838	8.22 844	9.99 994	50	
	20	8.14 796	8.14 800	9.99 996	40			20	8.22 962	8.22 968	9.99 994	40	
	30	8.14 945	8.14 950	9.99 996	30			30	8.23 086	8.23 092	9.99 994	30	
	40	8.15 094	8.15 099	9.99 996	20			40	8.23 210	8.23 216	9.99 994	20	
	50	8.15 243	8.15 247	9.99 996	10			50	8.23 333	8.23 339	9.99 994	10	
49	0	8.15 391	8.15 395	9.99 996	0	**11**	**59**	0	8.23 456	8.23 462	9.99 994	0	**1**
	10	8.15 538	8.15 543	9.99 996	50			10	8.23 578	8.23 585	9.99 994	50	
	20	8.15 685	8.15 690	9.99 996	40			20	8.23 700	8.23 707	9.99 994	40	
	30	8.15 832	8.15 836	9.99 996	30			30	8.23 822	8.23 829	9.99 993	30	
	40	8.15 978	8.15 982	9.99 995	20			40	8.23 944	8.23 950	9.99 993	20	
	50	8.16 123	8.16 128	9.99 995	10			50	8.24 065	8.24 071	9.99 993	10	
50	0	8.16 268	8.16 273	9.99 995	0	**10**	**60**	0	8.24 186	8.24 192	9.99 993	0	**0**

′	″	L Cos	L Cot	L Sin	″	′	′	″	L Cos	L Cot	L Sin	″	′

89° to 89° 20′

′ ″	L Sin	L Tan	L Cos	″ ′	′ ″	L Sin	L Tan	L Cos	″ ′
0 0	8.24 186	8.24 192	9.99 993	0 **60**	**10** 0	8.30 879	8.30 888	9.99 991	0 **50**
10	8.24 306	8.24 313	9.99 993	50	10	8.30 983	8.30 992	9.99 991	50
20	8.24 426	8.24 433	9.99 993	40	20	8.31 086	8.31 095	9.99 991	40
30	8.24 546	8.24 553	9.99 993	30	30	8.31 188	8.31 198	9.99 991	30
40	8.24 665	8.24 672	9.99 993	20	40	8.31 291	8.31 300	9.99 991	20
50	8.24 785	8.24 791	9.99 993	10	50	8.31 393	8.31 403	9.99 991	10
1 0	8.24 903	8.24 910	9.99 993	0 **59**	**11** 0	8.31 495	8.31 505	9.99 991	0 **49**
10	8.25 022	8.25 029	9.99 993	50	10	8.31 597	8.31 606	9.99 991	50
20	8.25 140	8.25 147	9.99 993	40	20	8.31 699	8.31 708	9.99 991	40
30	8.25 258	8.25 265	9.99 993	30	30	8.31 800	8.31 809	9.99 991	30
40	8.25 375	8.25 382	9.99 993	20	40	8.31 901	8.31 911	9.99 991	20
50	8.25 493	8.25 500	9.99 993	10	50	8.32 002	8.32 012	9.99 991	10
2 0	8.25 609	8.25 616	9.99 993	0 **58**	**12** 0	8.32 103	8.32 112	9.99 990	0 **48**
10	8.25 726	8.25 733	9.99 993	50	10	8.32 203	8.32 213	9.99 990	50
20	8.25 842	8.25 849	9.99 993	40	20	8.32 303	8.32 313	9.99 990	40
30	8.25 958	8.25 965	9.99 993	30	30	8.32 403	8.32 413	9.99 990	30
40	8.26 074	8.26 081	9.99 993	20	40	8.32 503	8.32 513	9.99 990	20
50	8.26 189	8.26 196	9.99 993	10	50	8.32 602	8.32 612	9.99 990	10
3 0	8.26 304	8.26 312	9.99 993	0 **57**	**13** 0	8.32 702	8.32 711	9.99 990	0 **47**
10	8.26 419	8.26 426	9.99 993	50	10	8.32 801	8.32 811	9.99 990	50
20	8.26 533	8.26 541	9.99 993	40	20	8.32 899	8.32 909	9.99 990	40
30	8.26 648	8.26 655	9.99 993	30	30	8.32 998	8.33 008	9.99 990	30
40	8.26 761	8.26 769	9.99 993	20	40	8.33 096	8.33 106	9.99 990	20
50	8.26 875	8.26 882	9.99 993	10	50	8.33 195	8.33 205	9.99 990	10
4 0	8.26 988	8.26 996	9.99 992	0 **56**	**14** 0	8.33 292	8.33 302	9.99 990	0 **46**
10	8.27 101	8.27 109	9.99 992	50	10	8.33 390	8.33 400	9.99 990	50
20	8.27 214	8.27 221	9.99 992	40	20	8.33 488	8.33 498	9.99 990	40
30	8.27 326	8.27 334	9.99 992	30	30	8.33 585	8.33 595	9.99 990	30
40	8.27 438	8.27 446	9.99 992	20	40	8.33 682	8.33 692	9.99 990	20
50	8.27 550	8.27 558	9.99 992	10	50	8.33 779	8.33 789	9.99 990	10
5 0	8.27 661	8.27 669	9.99 992	0 **55**	**15** 0	8.33 875	8.33 886	9.99 990	0 **45**
10	8.27 773	8.27 780	9.99 992	50	10	8.33 972	8.33 982	9.99 990	50
20	8.27 883	8.27 891	9.99 992	40	20	8.34 068	8.34 078	9.99 990	40
30	8.27 994	8.28 002	9.99 992	30	30	8.34 164	8.34 174	9.99 990	30
40	8.28 104	8.28 112	9.99 992	20	40	8.34 260	8.34 270	9.99 989	20
50	8.28 215	8.28 223	9.99 992	10	50	8.34 355	8.34 366	9.99 989	10
6 0	8.28 324	8.28 332	9.99 992	0 **54**	**16** 0	8.34 450	8.34 461	9.99 989	0 **44**
10	8.28 434	8.28 442	9.99 992	50	10	8.34 546	8.34 556	9.99 989	50
20	8.28 543	8.28 551	9.99 992	40	20	8.34 640	8.34 651	9.99 989	40
30	8.28 652	8.28 660	9.99 992	30	30	8.34 735	8.34 746	9.99 989	30
40	8.28 761	8.28 769	9.99 992	20	40	8.34 830	8.34 840	9.99 989	20
50	8.28 869	8.28 877	9.99 992	10	50	8.34 924	8.34 935	9.99 989	10
7 0	8.28 977	8.28 986	9.99 992	0 **53**	**17** 0	8.35 018	8.35 029	9.99 989	0 **43**
10	8.29 085	8.29 094	9.99 992	50	10	8.35 112	8.35 123	9.99 989	50
20	8.29 193	8.29 201	9.99 992	40	20	8.35 206	8.35 217	9.99 989	40
30	8.29 300	8.29 309	9.99 992	30	30	8.35 299	8.35 310	9.99 989	30
40	8.29 407	8.29 416	9.99 992	20	40	8.35 392	8.35 403	9.99 989	20
50	8.29 514	8.29 523	9.99 992	10	50	8.35 485	8.35 497	9.99 989	10
8 0	8.29 621	8.29 629	9.99 992	0 **52**	**18** 0	8.35 578	8.35 590	9.99 989	0 **42**
10	8.29 727	8.29 736	9.99 991	50	10	8.35 671	8.35 682	9.99 989	50
20	8.29 833	8.29 842	9.99 991	40	20	8.35 764	8.35 775	9.99 989	40
30	8.29 939	8.29 947	9.99 991	30	30	8.35 856	8.35 867	9.99 989	30
40	8.30 044	8.30 053	9.99 991	20	40	8.35 948	8.35 959	9.99 989	20
50	8.30 150	8.30 158	9.99 991	10	50	8.36 040	8.36 051	9.99 989	10
9 0	8.30 255	8.30 263	9.99 991	0 **51**	**19** 0	8.36 131	8.36 143	9.99 989	0 **41**
10	8.30 359	8.30 368	9.99 991	50	10	8.36 223	8.36 235	9.99 988	50
20	8.30 464	8.30 473	9.99 991	40	20	8.36 314	8.36 326	9.99 988	40
30	8.30 568	8.30 577	9.99 991	30	30	8.36 405	8.36 417	9.99 988	30
40	8.30 672	8.30 681	9.99 991	20	40	8.36 496	8.36 508	9.99 988	20
50	8.30 776	8.30 785	9.99 991	10	50	8.36 587	8.36 599	9.99 988	10
10 0	8.30 879	8.30 888	9.99 991	0 **50**	**20** 0	8.36 678	8.36 689	9.99 988	0 **40**
′ ″	L Cos	L Cot	L Sin	″ ′	′ ″	L Cos	L Cot	L Sin	″ ′

1° 20′ TO 1° 40′

′ ″	L Sin	L Tan	L Cos	″ ′	′ ″	L Sin	L Tan	L Cos	″ ′
20 0	8.36 678	8.36 689	9.99 988	0 **40**	**30** 0	8.41 792	8.41 807	9.99 985	0 **30**
10	8.36 768	8.36 780	9.99 988	50	10	8.41 872	8.41 887	9.99 985	50
20	8.36 858	8.36 870	9.99 988	40	20	8.41 952	8.41 967	9.99 985	40
30	8.36 948	8.36 960	9.99 988	30	30	8.42 032	8.42 048	9.99 985	30
40	8.37 038	8.37 050	9.99 988	20	40	8.42 112	8.42 127	9.99 985	20
50	8.37 128	8.37 140	9.99 988	10	50	8.42 192	8.42 207	9.99 985	10
21 0	8.37 217	8.37 229	9.99 988	0 **39**	**31** 0	8.42 272	8.42 287	9.99 985	0 **29**
10	8.37 306	8.37 318	9.99 988	50	10	8.42 351	8.42 366	9.99 985	50
20	8.37 395	8.37 408	9.99 988	40	20	8.42 430	8.42 446	9.99 985	40
30	8.37 484	8.37 497	9.99 988	30	30	8.42 510	8.42 525	9.99 985	30
40	8.37 573	8.37 585	9.99 988	20	40	8.42 589	8.42 604	9.99 985	20
50	8.37 662	8.37 674	9.99 988	10	50	8.42 667	8.42 683	9.99 985	10
22 0	8.37 750	8.37 762	9.99 988	0 **38**	**32** 0	8.42 746	8.42 762	9.99 984	0 **28**
10	8.37 838	8.37 850	9.99 988	50	10	8.42 825	8.42 840	9.99 984	50
20	8.37 926	8.37 938	9.99 988	40	20	8.42 903	8.42 919	9.99 984	40
30	8.38 014	8.38 026	9.99 987	30	30	8.42 982	8.42 997	9.99 984	30
40	8.38 101	8.38 114	9.99 987	20	40	8.43 060	8.43 075	9.99 984	20
50	8.38 189	8.38 202	9.99 987	10	50	8.43 138	8.43 154	9.99 984	10
23 0	8.38 276	8.38 289	9.99 987	0 **37**	**33** 0	8.43 216	8.43 232	9.99 984	0 **27**
10	8.38 363	8.38 376	9.99 987	50	10	8.43 293	8.43 309	9.99 984	50
20	8.38 450	8.38 463	9.99 987	40	20	8.43 371	8.43 387	9.99 984	40
30	8.38 537	8.38 550	9.99 987	30	30	8.43 448	8.43 464	9.99 984	30
40	8.38 624	8.38 636	9.99 987	20	40	8.43 526	8.43 542	9.99 984	20
50	8.38 710	8.38 723	9.99 987	10	50	8.43 603	8.43 619	9.99 984	10
24 0	8.38 796	8.38 809	9.99 987	0 **36**	**34** 0	8.43 680	8.43 696	9.99 984	0 **26**
10	8.38 882	8.38 895	9.99 987	50	10	8.43 757	8.43 773	9.99 984	50
20	8.38 968	8.38 981	9.99 987	40	20	8.43 834	8.43 850	9.99 984	40
30	8.39 054	8.39 067	9.99 987	30	30	8.43 910	8.43 927	9.99 984	30
40	8.39 139	8.39 153	9.99 987	20	40	8.43 987	8.44 003	9.99 984	20
50	8.39 225	8.39 238	9.99 987	10	50	8.44 063	8.44 080	9.99 983	10
25 0	8.39 310	8.39 323	9.99 987	0 **35**	**35** 0	8.44 139	8.44 156	9.99 983	0 **25**
10	8.39 395	8.39 408	9.99 987	50	10	8.44 216	8.44 232	9.99 983	50
20	8.39 480	8.39 493	9.99 987	40	20	8.44 292	8.44 308	9.99 983	40
30	8.39 565	8.39 587	9.99 987	30	30	8.44 367	8.44 384	9.99 983	30
40	8.39 649	8.39 663	9.99 987	20	40	8.44 443	8.44 460	9.99 983	20
50	8.39 734	8.39 747	9.99 986	10	50	8.44 519	8.44 536	9.99 983	10
26 0	8.39 818	8.39 832	9.99 986	0 **34**	**36** 0	8.44 594	8.44 611	9.99 983	0 **24**
10	8.39 902	8.39 916	9.99 986	50	10	8.44 669	8.44 686	9.99 983	50
20	8.39 986	8.40 000	9.99 986	40	20	8.44 745	8.44 762	9.99 983	40
30	8.40 070	8.40 083	9.99 986	30	30	8.44 820	8.44 837	9.99 983	30
40	8.40 153	8.40 167	9.99 986	20	40	8.44 895	8.44 912	9.99 983	20
50	8.40 237	8.40 251	9.99 986	10	50	8.44 969	8.44 987	9.99 983	10
27 0	8.40 320	8.40 334	9.99 986	0 **33**	**37** 0	8.45 044	8.45 061	9.99 983	0 **23**
10	8.40 403	8.40 417	9.99 986	50	10	8.45 119	8.45 136	9.99 983	50
20	8.40 486	8.40 500	9.99 986	40	20	8.45 193	8.45 210	9.99 983	40
30	8.40 569	8.40 583	9.99 986	30	30	8.45 267	8.45 285	9.99 983	30
40	8.40 651	8.40 665	9.99 986	20	40	8.45 341	8.45 359	9.99 982	20
50	8.40 734	8.40 748	9.99 986	10	50	8.45 415	8.45 433	9.99 982	10
28 0	8.40 816	8.40 830	9.99 986	0 **32**	**38** 0	8.45 489	8.45 507	9.99 982	0 **22**
10	8.40 898	8.40 913	9.99 986	50	10	8.45 563	8.45 581	9.99 982	50
20	8.40 980	8.40 995	9.99 986	40	20	8.45 637	8.45 655	9.99 982	40
30	8.41 062	8.41 077	9.99 986	30	30	8.45 710	8.45 728	9.99 982	30
40	8.41 144	8.41 158	9.99 986	20	40	8.45 784	8.45 802	9.99 982	20
50	8.41 225	8.41 240	9.99 986	10	50	8.45 857	8.45 875	9.99 982	10
29 0	8.41 307	8.41 321	9.99 985	0 **31**	**39** 0	8.45 930	8.45 948	9.99 982	0 **21**
10	8.41 388	8.41 403	9.99 985	50	10	8.46 003	8.46 021	9.99 982	50
20	8.41 469	8.41 484	9.99 985	40	20	8.46 076	8.46 094	9.99 982	40
30	8.41 550	8.41 565	9.99 985	30	30	8.46 149	8.46 167	9.99 982	30
40	8.41 631	8.41 646	9.99 985	20	40	8.46 222	8.46 240	9.99 982	20
50	8.41 711	8.41 726	9.99 985	10	50	8.46 294	8.46 312	9.99 982	10
30 0	8.41 792	8.41 807	9.99 985	0 **30**	**40** 0	8.46 366	8.46 385	9.99 982	0 **20**
′ ″	L Cos	L Cot	L Sin	″ ′	′ ″	L Cos	L Cot	L Sin	″ ′

88° 20′ TO 88° 40′

′ ″	L Sin	L Tan	L Cos	″ ′	′ ″	L Sin	L Tan	L Cos	″ ′
40 0	8.46 366	8.46 385	9.99 982	0 **20**	**50** 0	8.50 504	8.50 527	9.99 978	0 **10**
10	8.46 439	8.46 457	9.99 982	50	10	8.50 570	8.50 593	9.99 978	50
20	8.46 511	8.46 529	9.99 982	40	20	8.50 636	8.50 658	9.99 978	40
30	8.46 583	8.46 602	9.99 981	30	30	8.50 701	8.50 724	9.99 978	30
40	8.46 655	8.46 674	9.99 981	20	40	8.50 767	8.50 789	9.99 977	20
50	8.46 727	8.46 745	9.99 981	10	50	8.50 832	8.50 855	9.99 977	10
41 0	8.46 799	8.46 817	9.99 981	0 **19**	**51** 0	8.50 897	8.50 920	9.99 977	0 **9**
10	8.46 870	8.46 889	9.99 981	50	10	8.50 963	8.50 985	9.99 977	50
20	8.46 942	8.46 960	9.99 981	40	20	8.51 028	8.51 050	9.99 977	40
30	8.47 013	8.47 032	9.99 981	30	30	8.51 092	8.51 115	9.99 977	30
40	8.47 084	8.47 103	9.99 981	20	40	8.51 157	8.51 180	9.99 977	20
50	8.47 155	8.47 174	9.99 981	10	50	8.51 222	8.51 245	9.99 977	10
42 0	8.47 226	8.47 245	9.99 981	0 **18**	**52** 0	8.51 287	8.51 310	9.99 977	0 **8**
10	8.47 297	8.47 316	9.99 981	50	10	8.51 351	8.51 374	9.99 977	50
20	8.47 368	8.47 387	9.99 981	40	20	8.51 416	8.51 439	9.99 977	40
30	8.47 439	8.47 458	9.99 981	30	30	8.51 480	8.51 503	9.99 977	30
40	8.47 509	8.47 528	9.99 981	20	40	8.51 544	8.51 568	9.99 977	20
50	8.47 580	8.47 599	9.99 981	10	50	8.51 609	8.51 632	9.99 977	10
43 0	8.47 650	8.47 669	9.99 981	0 **17**	**53** 0	8.51 673	8.51 696	9.99 977	0 **7**
10	8.47 720	8.47 740	9.99 980	50	10	8.51 737	8.51 760	9.99 976	50
20	8.47 790	8.47 810	9.99 980	40	20	8.51 801	8.51 824	9.99 976	40
30	8.47 860	8.47 880	9.99 980	30	30	8.51 864	8.51 888	9.99 976	30
40	8.47 930	8.47 950	9.99 980	20	40	8.51 928	8.51 952	9.99 976	20
50	8.48 000	8.48 020	9.99 980	10	50	8.51 992	8.52 015	9.99 976	10
44 0	8.48 069	8.48 090	9.99 980	0 **16**	**54** 0	8.52 055	8.52 079	9.99 976	0 **6**
10	8.48 139	8.48 159	9.99 980	50	10	8.52 119	8.52 143	9.99 976	50
20	8.48 208	8.48 228	9.99 980	40	20	8.52 182	8.52 206	9.99 976	40
30	8.48 278	8.48 298	9.99 980	30	30	8.52 245	8.52 269	9.99 976	30
40	8.48 347	8.48 367	9.99 980	20	40	8.52 308	8.52 332	9.99 976	20
50	8.48 416	8.48 436	9.99 980	10	50	8.52 371	8.52 396	9.99 976	10
45 0	8.48 485	8.48 505	9.99 980	0 **15**	**55** 0	8.52 434	8.52 459	9.99 976	0 **5**
10	8.48 554	8.48 574	9.99 980	50	10	8.52 497	8.52 522	9.99 976	50
20	8.48 622	8.48 643	9.99 980	40	20	8.52 560	8.52 584	9.99 976	40
30	8.48 691	8.48 711	9.99 980	30	30	8.52 623	8.52 647	9.99 975	30
40	8.48 760	8.48 780	9.99 979	20	40	8.52 685	8.52 710	9.99 975	20
50	8.48 828	8.48 849	9.99 979	10	50	8.52 748	8.52 772	9.99 975	10
46 0	8.48 896	8.48 917	9.99 979	0 **14**	**56** 0	8.52 810	8.52 835	9.99 975	0 **4**
10	8.48 965	8.48 985	9.99 979	50	10	8.52 872	8.52 897	9.99 975	50
20	8.49 033	8.49 053	9.99 979	40	20	8.52 935	8.52 960	9.99 975	40
30	8.49 101	8.49 121	9.99 979	30	30	8.52 997	8.53 022	9.99 975	30
40	8.49 169	8.49 189	9.99 979	20	40	8.53 059	8.53 084	9.99 975	20
50	8.49 236	8.49 257	9.99 979	10	50	8.53 121	8.53 146	9.99 975	10
47 0	8.49 304	8.49 325	9.99 979	0 **13**	**57** 0	8.53 183	8.53 208	9.99 975	0 **3**
10	8.49 372	8.49 393	9.99 979	50	10	8.53 245	8.53 270	9.99 975	50
20	8.49 439	8.49 460	9.99 979	40	20	8.53 306	8.53 332	9.99 975	40
30	8.49 506	8.49 528	9.99 979	30	30	8.53 368	8.53 393	9.99 975	30
40	8.49 574	8.49 595	9.99 979	20	40	8.53 429	8.53 455	9.99 975	20
50	8.49 641	8.49 662	9.99 979	10	50	8.53 491	8.53 516	9.99 974	10
48 0	8.49 708	8.49 729	9.99 979	0 **12**	**58** 0	8.53 552	8.53 578	9.99 974	0 **2**
10	8.49 775	8.49 796	9.99 979	50	10	8.53 614	8.53 639	9.99 974	50
20	8.49 842	8.49 863	9.99 978	40	20	8.53 675	8.53 700	9.99 974	40
30	8.49 908	8.49 930	9.99 978	30	30	8.53 736	8.53 762	9.99 974	30
40	8.49 975	8.49 997	9.99 978	20	40	8.53 797	8.53 823	9.99 974	20
50	8.50 042	8.50 063	9.99 978	10	50	8.53 858	8.53 884	9.99 974	10
49 0	8.50 108	8.50 130	9.99 978	0 **11**	**59** 0	8.53 919	8.53 945	9.99 974	0 **1**
10	8.50 174	8.50 196	9.99 978	50	10	8.53 979	8.54 005	9.99 974	50
20	8.50 241	8.50 263	9.99 978	40	20	8.54 040	8.54 066	9.99 974	40
30	8.50 307	8.50 329	9.99 978	30	30	8.54 101	8.54 127	9.99 974	30
40	8.50 373	8.50 395	9.99 978	20	40	8.54 161	8.54 187	9.99 974	20
50	8.50 439	8.50 461	9.99 978	10	50	8.54 222	8.54 248	9.99 974	10
50 0	8.50 504	8.50 527	9.99 978	0 **10**	**60** 0	8.54 282	8.54 308	9.99 974	0 **0**
′ ″	L Cos	L Cot	L Sin	″ ′	′ ″	L Cos	L Cot	L Sin	″ ′

TABLE V

Squares, Cubes, Square Roots, and Cube Roots of Numbers from 1 to 100

N	N^2	N^3	$\sqrt{N}$	$\sqrt[3]{N}$	N	N^2	N^3	$\sqrt{N}$	$\sqrt[3]{N}$
1	1	1	1.0000	1.0000	51	2601	132651	7.1414	3.7084
2	4	8	1.4142	1.2599	52	2704	140608	7.2111	3.7325
3	9	27	1.7321	1.4422	53	2809	148877	7.2801	3.7563
4	16	64	2.0000	1.5874	54	2916	157464	7.3485	3.7798
5	25	125	2.2361	1.7100	55	3025	166375	7.4162	3.8030
6	36	216	2.4495	1.8171	56	3136	175616	7.4833	3.8259
7	49	343	2.6458	1.9129	57	3249	185193	7.5498	3.8485
8	64	512	2.8284	2.0000	58	3364	195112	7.6158	3.8709
9	81	729	3.0000	2.0801	59	3481	205379	7.6811	3.8930
10	100	1000	3.1623	2.1544	**60**	3600	216000	7.7460	3.9149
11	121	1331	3.3166	2.2240	61	3721	226981	7.8102	3.9365
12	144	1728	3.4641	2.2894	62	3844	238328	7.8740	3.9579
13	169	2197	3.6056	2.3513	63	3969	250047	7.9373	3.9791
14	196	2744	3.7417	2.4101	64	4096	262144	8.0000	4.0000
15	225	3375	3.8730	2.4662	65	4225	274625	8.0623	4.0207
16	256	4096	4.0000	2.5198	66	4356	287496	8.1240	4.0412
17	289	4913	4.1231	2.5713	67	4489	300763	8.1854	4.0615
18	324	5832	4.2426	2.6207	68	4624	314432	8.2462	4.0817
19	361	6859	4.3589	2.6684	69	4761	328509	8.3066	4.1016
20	400	8000	4.4721	2.7144	**70**	4900	343000	8.3666	4.1213
21	441	9261	4.5826	2.7589	71	5041	357911	8.4261	4.1408
22	484	10648	4.6904	2.8020	72	5184	373248	8.4853	4.1602
23	529	12167	4.7958	2.8439	73	5329	389017	8.5440	4.1793
24	576	13824	4.8990	2.8845	74	5476	405224	8.6023	4.1983
25	625	15625	5.0000	2.9240	75	5625	421875	8.6603	4.2172
26	676	17576	5.0990	2.9625	76	5776	438976	8.7178	4.2358
27	729	19683	5.1962	3.0000	77	5929	456533	8.7750	4.2543
28	784	21952	5.2915	3.0366	78	6084	474552	8.8318	4.2727
29	841	24389	5.3852	3.0723	79	6241	493039	8.8882	4.2908
30	900	27000	5.4772	3.1072	**80**	6400	512000	8.9443	4.3089
31	961	29791	5.5678	3.1414	81	6561	531441	9.0000	4.3267
32	1024	32768	5.6569	3.1748	82	6724	551368	9.0554	4.3445
33	1089	35937	5.7446	3.2075	83	6889	571787	9.1104	4.3621
34	1156	39304	5.8310	3.2396	84	7056	592704	9.1652	4.3795
35	1225	42875	5.9161	3.2711	85	7225	614125	9.2195	4.3968
36	1296	46656	6.0000	3.3019	86	7396	636056	9.2736	4.4140
37	1369	50653	6.0828	3.3322	87	7569	658503	9.3274	4.4310
38	1444	54872	6.1644	3.3620	88	7744	681472	9.3808	4.4480
39	1521	59319	6.2450	3.3912	89	7921	704969	9.4340	4.4647
40	1600	64000	6.3246	3.4200	**90**	8100	729000	9.4868	4.4814
41	1681	68921	6.4031	3.4482	91	8281	753571	9.5394	4.4979
42	1764	74088	6.4807	3.4760	92	8464	778688	9.5917	4.5144
43	1849	79507	6.5574	3.5034	93	8649	804357	9.6437	4.5307
44	1936	85184	6.6332	3.5303	94	8836	830584	9.6954	4.5468
45	2025	91125	6.7082	3.5569	95	9025	857375	9.7468	4.5629
46	2116	97336	6.7823	3.5830	96	9216	884736	9.7980	4.5789
47	2209	103823	6.8557	3.6088	97	9409	912673	9.8489	4.5947
48	2304	110592	6.9282	3.6342	98	9604	941192	9.8995	4.6104
49	2401	117649	7.0000	3.6593	99	9801	970299	9.9499	4.6261
50	2500	125000	7.0711	3.6840	**100**	10000	1000000	10.0000	4.6416

TABLE VI

Conversion Table for Angles

(*a*) Conversion of Degrees, Minutes, and Seconds into Radians

°	Radians	′	Radians	″	Radians
1	0.0174533	1	0.0002909	1	0.0000048
2	0.0349066	2	0.0005818	2	0.0000097
3	0.0523599	3	0.0008727	3	0.0000145
4	0.0698132	4	0.0011636	4	0.0000194
5	0.0872665	5	0.0014544	5	0.0000242
6	0.1047198	6	0.0017453	6	0.0000291
7	0.1221730	7	0.0020362	7	0.0000339
8	0.1396263	8	0.0023271	8	0.0000388
9	0.1570796	9	0.0026180	9	0.0000436
10	0.1745329	10	0.0029089	10	0.0000485
20	0.3490659	20	0.0058178	20	0.0000970
30	0.5235988	30	0.0087266	30	0.0001454
40	0.6981317	40	0.0116355	40	0.0001939
50	0.8726646	50	0.0145444	50	0.0002424
60	1.0471976	60	0.0174533	60	0.0002909
70	1.2217305				
80	1.3962634				
90	1.5707963				

(*b*) Conversion of Radians into Degrees

	Radians	Tenths	Hundredths	Thousandths	Ten Thousandths	Hundred Thousandths
1	57° 17′ 44″.8	5° 43′ 46″.5	0° 34′ 22″.6	0° 3′ 26″.3	0° 0′ 20″.6	0° 0′ 2″.06
2	114° 35′ 29″.6	11° 27′ 33″.0	1° 8′ 45″.3	0° 6′ 52″.5	0° 0′ 41″.3	0° 0′ 4″.13
3	171° 53′ 14″.4	17° 11′ 19″.4	1° 43′ 7″.9	0° 10′ 18″.8	0° 1′ 1″.9	0° 0′ 6″.19
4	229° 10′ 59″.2	22° 55′ 5″.9	2° 17′ 30″.6	0° 13′ 45″.1	0° 1′ 22″.5	0° 9′ 8″.25
5	286° 28′ 44″.0	28° 38′ 52″.4	2° 51′ 53″.2	0° 17′ 11″.3	0° 1′ 43″.1	0° 10′ 0″.31

(*c*) To Change from Minutes and Seconds into the Decimal Parts of a Degree

From Minutes		From Seconds	
1′ = 0°.017	8′ = 0°.133	1″ = 0°.00028	8″ = 0°.00222
2′ = 0°.033	9′ = 0°.150	2″ = 0°.00056	9″ = 0°.00250
3′ = 0°.050	10′ = 0°.167	3″ = 0°.00083	10″ = 0°.00278
4′ = 0°.067	20′ = 0°.333	4″ = 0°.00111	20″ = 0°.00556
5′ = 0°.083	30′ = 0°.500	5″ = 0°.00139	30″ = 0°.00833
6′ = 0°.100	40′ = 0°.667	6″ = 0°.00167	40″ = 0°.01111
7′ = 0°.117	50′ = 0°.833	7″ = 0°.00194	50″ = 0°.01389

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(TABLE VI, *Cont.*)

(*d*) To Change from Decimal Parts of a Degree into Minutes and Seconds

0°.0000 = 0′.000 = 0″	0°.20 = 12′.0 = 12′	0°.60 = 36′.0 = 36′
0°.0001 = 0′.006 = 0″.36	0°.21 = 12′.6 = 12′ 36″	0°.61 = 36′.6 = 36′ 36″
0°.0002 = 0′.012 = 0″.72	0°.22 = 13′.2 = 13′ 12″	0°.62 = 37′.2 = 37′ 12″
0°.0003 = 0′.018 = 1″.08	0°.23 = 13′.8 = 13′ 48″	0°.63 = 37′.8 = 37′ 48″
0°.0004 = 0′.024 = 1″.44	0°.24 = 14′.4 = 14′ 24″	0°.64 = 38′.4 = 38′ 24″
0°.0005 = 0′.030 = 1″.80	0°.25 = 15′.0 = 15′ 0″	0°.65 = 39′.0 = 39′ 0″
0°.0006 = 0′.036 = 2″.16	0°.26 = 15′.6 = 15′ 36″	0°.66 = 39′.6 = 39′ 36″
0°.0007 = 0′.042 = 2″.52	0°.27 = 16′.2 = 16′ 12″	0°.67 = 40′.2 = 40′ 12″
0°.0008 = 0′.048 = 2″.88	0°.28 = 16′.8 = 16′ 48″	0°.68 = 40′.8 = 40′ 48″
0°.0009 = 0′.054 = 3″.24	0°.29 = 17′.4 = 17′ 24″	0°.69 = 41′.4 = 41′ 24″
0°.0010 = 0′.060 = 3″.60	0°.30 = 18′.0 = 18′ 0″	0°.70 = 42′.0 = 42′ 0″
0°.001 = 0′.06 = 3″.6	0°.31 = 18′.6 = 18′ 36″	0°.71 = 42′.6 = 42′ 36″
0°.002 = 0′.12 = 7″.2	0°.32 = 19′.2 = 19′ 12″	0°.72 = 43′.2 = 43′ 12″
0°.003 = 0′.18 = 10″.8	0°.33 = 19′.8 = 19′ 48″	0°.73 = 43′.8 = 43′ 48″
0°.004 = 0′.24 = 14″.4	0°.34 = 20′.4 = 20′ 24″	0°.74 = 44′.4 = 44′ 24″
0°.005 = 0′.30 = 18″.0	0°.35 = 21′.0 = 21′ 0″	0°.75 = 45′.0 = 45′ 0″
0°.006 = 0′.36 = 21″.6	0°.36 = 21′.6 = 21′ 36″	0°.76 = 45′.6 = 45′ 36″
0°.007 = 0′.42 = 25″.2	0°.37 = 22′.2 = 22′ 12″	0°.77 = 46′.2 = 46′ 12″
0°.008 = 0′.48 = 28″.8	0°.38 = 22′.8 = 22′ 48″	0°.78 = 46′.8 = 46′ 48″
0°.009 = 0′.54 = 32″.4	0°.39 = 23′.4 = 23′ 24″	0°.79 = 47′.4 = 47′ 24″
0°.010 = 0′.60 = 36″.0	0°.40 = 24′.0 = 24′ 0″	0°.80 = 48′.0 = 48′ 0″
0°.01 = 0′.6 = 0′ 36″	0°.41 = 24′.6 = 24′ 36″	0°.81 = 48′.6 = 48′ 36″
0°.02 = 1′.2 = 1′ 12″	0°.42 = 25′.2 = 25′ 12″	0°.82 = 49′.2 = 49′ 12″
0°.03 = 1′.8 = 1′ 48″	0°.43 = 25′.8 = 25′ 48″	0°.83 = 49′.8 = 49′ 48″
0°.04 = 2′.4 = 2′ 24″	0°.44 = 26′.4 = 26′ 24″	0°.84 = 50′.4 = 50′ 24″
0°.05 = 3′.0 = 3′ 0″	0°.45 = 27′.0 = 27′ 0″	0°.85 = 51′.0 = 51′ 0″
0°.06 = 3′.6 = 3′ 36″	0°.46 = 27′.6 = 27′ 36″	0°.86 = 51′.6 = 51′ 36″
0°.07 = 4′.2 = 4′ 12″	0°.47 = 28′.2 = 28′ 12″	0°.87 = 52′.2 = 52′ 12″
0°.08 = 4′.8 = 4′ 48″	0°.48 = 28′.8 = 28′ 48″	0°.88 = 52′.8 = 52′ 48″
0°.09 = 5′.4 = 5′ 24″	0°.49 = 29′.4 = 29′ 24″	0°.89 = 53′.4 = 53′ 24″
0°.10 = 6′.0 = 6′ 0″	0°.50 = 30′.0 = 30′ 0″	0°.90 = 54′.0 = 54′ 0″
0°.11 = 6′.6 = 6′ 36″	0°.51 = 30′.6 = 30′ 36″	0°.91 = 54′.6 = 54′ 36″
0°.12 = 7′.2 = 7′ 12″	0°.52 = 31′.2 = 31′ 12″	0°.92 = 55′.2 = 55′ 12″
0°.13 = 7′.8 = 7′ 48″	0°.53 = 31′.8 = 31′ 48″	0°.93 = 55′.8 = 55′ 48″
0°.14 = 8′.4 = 8′ 24″	0°.54 = 32′.4 = 32′ 24″	0°.94 = 56′.4 = 56′ 24″
0°.15 = 9′.0 = 9′ 0″	0°.55 = 33′.0 = 33′ 0″	0°.95 = 57′.0 = 57′ 0″
0°.16 = 9′.6 = 9′ 36″	0°.56 = 33′.6 = 33′ 36″	0°.96 = 57′.6 = 57′ 36″
0°.17 = 10′.2 = 10′ 12″	0°.57 = 34′.2 = 34′ 12″	0°.97 = 58′.2 = 58′ 12″
0°.18 = 10′.8 = 10′ 48″	0°.58 = 34′.8 = 34′ 48″	0°.98 = 58′.8 = 58′ 48″
0°.19 = 11′.4 = 11′ 24″	0°.59 = 35′.4 = 35′ 24″	0°.99 = 59′.4 = 59′ 24″
0°.20 = 12′.0 = 12′ 0″	0°.60 = 36′.0 = 36′ 0″	1°.00 = 60′.0 = 60′ 0″

TABLE VII

Value and Logarithms of Important Constants

Number = N	Value of N	$\log_{10}$ N
π = ratio of circumference to diameter	3.1415927	0.49714987
e = Napierian base	2.7182818	0.43429448
$m = \log_{10} e$ (modulus of common logarithms)	.43429448	9.63778431 − 10
$\frac{1}{m} = \log_e 10$	2.3025851	0.36221569
$\frac{180}{\pi}$ = number of degrees in one radian	57.295780	1.75812263
$\frac{\pi}{180}$ = number of radians in 1°	0.01745329	8.24187737 − 10
g = acceleration due to gravity	32.16 ft. per sec. per sec.	1.5073
centimeters in 1 inch	2.54001	0.404836
inches in 1 centimeter	0.393700	9.595165 − 10
feet in 1 meter	3.28083	0.515984
meters in 1 foot	0.304801	9.484016 − 10
miles in 1 kilometer	0.621370	9.793350 − 10
kilometers in 1 mile	1.60935	0.206651
kilograms in 1 pound	0.453593	9.656666 − 10
pounds in 1 kilogram	2.20462	0.343334
equatorial radius of the earth in miles	3963.3	3.598057
polar radius of the earth in miles	3949.8	3.596575
mean distance of the sun from the earth in miles	92,897,400	7.968004
mean distance of the moon from the earth in miles	238,840	5.3781038

Explanation of Tables VIII, IX, and X

Tables VIII, IX, and X give correct to three decimal places the logarithms of numbers from 1 to 500, the values of the six trigonometric functions for acute angles, and the logarithms of the sines, cosines, tangents, and cotangents of acute angles for every degree of arc. These tables should be used when the data employed do not warrant the reading of results to more than three places in linear measure and to more than tenths of a degree in angular measure, though here as always the degree of accuracy varies in different parts of the table. If four-place tables are desired, the five-place tables may be converted into four-place tables by dropping the figure in the fifth decimal place if it is less than 5, and if it is greater than or equal to 5 by adding 1 to the figure in the fourth place.

Table VIII. The mantissa of any number N greater than 499 may be found by adding .301 to the mantissa of half the number if it is an *even* number. Thus $\log 786 = .301 + \log 393 = .301 + .594 = .895$. If N is odd then $N - 1$ and $N + 1$ are even and we may interpolate between their logarithms which are found as above.

Tables IX and X require no explanation.

TABLE VIII

THREE-PLACE LOGARITHMS OF THE TRIGONOMETRIC FUNCTIONS

For Every Degree of Arc from 0° to 90°

Angle	L Sin	d	L Tan	c d	L Cot	L Cos	d	
0	———		———		———	10.000		**90**
1	8.242	301	8.242	301	1.758	10.000		89
2	8.543	176	8.543	176	1.457	10.000		88
3	8.719	125	8.719	126	1.281	9.999		87
4	8.844		8.845		1.155	9.999		86
		96		97				
5	8.940	79	8.942	80	1.058	9.998		**85**
6	9.019	67	9.022	67	0.978	9.998		84
7	9.086	58	9.089	59	0.911	9.997	1	83
8	9.144	50	9.148	52	0.852	9.996	1	82
9	9.194		9.200		0.800	9.995		81
		46		46			2	
10	9.240	41	9.246	43	0.754	9.993	1	**80**
11	9.281	37	9.289	38	0.711	9.992	2	79
12	9.318	34	9.327	36	0.673	9.990	1	78
13	9.352	32	9.363	34	0.637	9.989	2	77
14	9.384		9.397		0.603	9.987		76
		29		31			2	
15	9.413	27	9.427	29	0.572	9.985	2	**75**
16	9.440	26	9.457	27	0.543	9.983	2	74
17	9.466	24	9.485	27	0.515	9.981	3	73
18	9.490	23	9.512	25	0.488	9.978	2	72
19	9.513		9.537		0.463	9.976		71
		21		24			3	
20	9.534	20	9.561	23	0.439	9.973	3	**70**
21	9.554	20	9.584	22	0.416	9.970	3	69
22	9.574	18	9.606	22	0.394	9.967	3	68
23	9.592	17	9.628	21	0.372	9.964	3	67
24	9.609		9.649		0.351	9.961		66
		17		20			4	
25	9.626	16	9.669	19	0.331	9.957	3	**65**
26	9.642	15	9.688	19	0.312	9.954	4	64
27	9.657	15	9.707	19	0.293	9.950	4	63
28	9.672	14	9.726	18	0.274	9.946	4	62
29	9.686		9.744		0.256	9.942		61
		13		17			4	
30	9.699	13	9.761	18	0.239	9.938	5	**60**
31	9.712	12	9.779	17	0.221	9.933	5	59
32	9.724	12	9.796	17	0.204	9.928	4	58
33	9.736	12	9.813	16	0.187	9.924	5	57
34	9.748		9.829		0.171	9.919		56
		11		16			6	
35	9.759	10	9.845	16	0.155	9.913	5	**55**
36	9.769	10	9.861	16	0.139	9.908	6	54
37	9.779	10	9.877	16	0.123	9.902	5	53
38	9.789	10	9.893	15	0.107	9.897	6	52
39	9.799		9.908		0.092	9.891		51
		9		16			7	
40	9.808	9	9.924	15	0.076	9.884	6	**50**
41	9.817	9	9.939	15	0.061	9.878	7	49
42	9.826	8	9.954	16	0.046	9.871	7	48
43	9.834	8	9.970	15	0.030	9.864	7	47
44	9.842		9.985		0.015	9.857		46
		7		15			8	
45	9.849		10.000		0.000	9.849		**45**
	L Cos	d	L Cot	c d	L Tan	L Sin	d	Angle

TABLE IX

VALUES OF THE TRIGONOMETRIC FUNCTIONS

To Three Decimal Places

Angle	Sin	Tan	Sec	Csc	Cot	Cos	
0	.000	.000	1.000	——	——	1.000	**90**
1	.017	.017	1.000	57.299	57.290	1.000	89
2	.035	.035	1.001	28.654	28.636	.999	88
3	.052	.052	1.001	19.107	19.081	.999	87
4	.070	.070	1.002	14.336	14.301	.998	86
5	.087	.087	1.004	11.474	11.430	.996	**85**
6	.105	.105	1.006	9.567	9.514	.995	84
7	.122	.123	1.008	8.206	8.144	.993	83
8	.139	.141	1.010	7.185	7.115	.990	82
9	.156	.158	1.012	6.392	6.314	.988	81
10	.174	.176	1.015	5.759	5.671	.985	**80**
11	.191	.194	1.019	5.241	5.145	.982	79
12	.208	.213	1.022	4.810	4.705	.978	78
13	.225	.231	1.026	4.445	4.331	.974	77
14	.242	.249	1.031	4.134	4.011	.970	76
15	.259	.268	1.035	3.864	3.732	.966	**75**
16	.276	.287	1.040	3.628	3.487	.961	74
17	.292	.306	1.046	3.420	3.271	.956	73
18	.309	.325	1.051	3.236	3.078	.951	72
19	.326	.344	1.058	3.072	2.904	.946	71
20	.342	.364	1.064	2.924	2.747	.940	**70**
21	.358	.384	1.071	2.790	2.605	.934	69
22	.375	.404	1.079	2.669	2.475	.927	68
23	.391	.424	1.086	2.559	2.356	.921	67
24	.407	.445	1.095	2.459	2.246	.914	66
25	.423	.466	1.103	2.366	2.145	.906	**65**
26	.438	.488	1.113	2.281	2.050	.899	64
27	.454	.510	1.122	2.203	1.963	.891	63
28	.469	.532	1.133	2.130	1.881	.883	62
29	.485	.554	1.143	2.063	1.804	.875	61
30	.500	.577	1.155	2.000	1.732	.866	**60**
31	.515	.601	1.167	1.942	1.664	.857	59
32	.530	.625	1.179	1.887	1.600	.848	58
33	.545	.649	1.192	1.836	1.540	.839	57
34	.559	.675	1.206	1.788	1.483	.829	56
35	.574	.700	1.221	1.743	1.428	.819	**55**
36	.588	.727	1.236	1.701	1.376	.809	54
37	.602	.754	1.252	1.662	1.327	.799	53
38	.616	.781	1.269	1.624	1.280	.788	52
39	.629	.810	1.287	1.589	1.235	.777	51
40	.643	.839	1.305	1.556	1.192	.766	**50**
41	.656	.869	1.325	1.524	1.150	.755	49
42	.669	.900	1.346	1.494	1.111	.743	48
43	.682	.933	1.367	1.466	1.072	.731	47
44	.695	.966	1.390	1.440	1.036	.719	46
45	.707	1.000	1.414	1.414	1.000	.707	**45**
	Cos	**Cot**	**Csc**	**Sec**	**Tan**	**Sin**	**Angle**

TABLE X

THREE-PLACE LOGARITHMS OF NUMBERS

From 1 to 500

N	0	1	2	3	4	5	6	7	8	9
1	000	041	079	114	146	176	204	230	255	279
2	301	322	342	362	380	398	415	431	447	462
3	477	491	505	519	531	544	556	568	580	591
4	602	613	623	633	643	653	663	672	681	690
5	699	708	716	724	732	740	748	756	763	771
6	778	785	792	799	806	813	820	826	833	839
7	845	851	857	863	869	875	881	886	892	898
8	903	908	914	919	924	929	934	940	944	949
9	954	959	964	968	973	978	982	987	991	996
10	000	004	009	013	017	021	025	029	033	037
11	041	045	049	053	057	061	065	068	072	076
12	079	083	086	090	093	097	100	104	107	111
13	114	117	121	124	127	130	134	137	140	143
14	146	149	152	155	158	161	164	167	170	173
15	176	179	182	185	188	190	193	196	199	201
16	204	207	210	212	215	218	220	223	225	228
17	230	233	236	238	241	243	246	248	250	253
18	255	258	260	262	265	267	270	272	274	276
19	279	281	283	286	288	290	292	294	297	299
20	301	303	305	307	310	312	314	316	318	320
21	322	324	326	328	330	332	334	336	338	340
22	342	344	346	348	350	352	354	356	358	360
23	362	364	365	367	369	371	373	375	377	378
24	380	382	384	386	387	389	391	393	394	396
25	398	400	401	403	405	407	408	410	412	413
26	415	417	418	420	422	423	425	427	428	430
27	431	433	435	436	438	439	441	442	444	446
28	447	449	450	452	453	455	456	458	459	461
29	462	464	465	467	468	470	471	473	474	476
30	477	479	480	481	483	484	486	487	489	490
31	491	493	494	496	497	498	500	501	502	504
32	505	507	508	509	511	512	513	515	516	517
33	519	520	521	522	524	525	526	528	529	530
34	531	533	534	535	537	538	539	540	542	543
35	544	545	547	548	549	550	551	553	554	555
36	556	558	559	560	561	562	563	565	566	567
37	568	569	571	572	573	574	575	576	577	579
38	580	581	582	583	584	585	587	588	589	590
39	591	592	593	594	595	597	598	599	600	601
40	602	603	604	605	606	607	609	610	611	612
41	613	614	615	616	617	618	619	620	621	622
42	623	624	625	626	627	628	629	630	631	632
43	633	634	635	636	637	638	639	640	641	642
44	643	644	645	646	647	648	649	650	651	652
45	653	654	655	656	657	658	659	660	661	662
46	663	664	665	666	667	667	668	669	670	671
47	672	673	674	675	676	677	678	679	679	680
48	681	682	683	684	685	686	687	688	688	689
49	690	691	692	693	694	695	695	696	697	698

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